INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY 2018-2019

Date of Examination : 25th November 2018

Time : 02 : 00 pm to 04 : 00 pm

[Ques. Paper Code : A422]

(Total Marks : 240)

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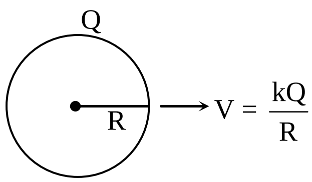
1. There is a uniformly charged non-conducting solid sphere made of material of dielectric constant 1. If the electric potential at infinity is taken to be zero, then the potential at its surface is V. If we take the electric potential at its surface to be zero, then the potential at the centre will be

(a) 3V/2 (b) V/2 (c) V (d) Zero

Ans. **[b]**

Sol.



When V at ∞ is zero

Now V at surface is taken to be zero.

So, V′ at centre

$$V' = \frac{3}{2}\frac{kQ}{R} - \frac{kQ}{R} = \frac{1}{2}\frac{kQ}{R}$$

$$\Rightarrow V' = \frac{V}{2}$$

2. Suppose 5 cos x + 12 cos y = 13. The maximum possible value of 5 sin x + 12 sin y is -

(a) $\sqrt{13}$ (b) $\sqrt{120}$ (c) $\sqrt{240}$ (d) 13

Ans. **[b]**

Sol. $5 \cos x + 12 \cos y = 13$ … (1)

$5 \sin x + 12 \sin y = a$ … (2)

$(1)^2 + (2)^2$

$25 + 144 + 120 (\cos x \cos y + \sin x \sin y) = (13)^2 + a^2$

$a^2 = 120 \cos (x - y)$

$a^2 \le 120$

$a \le \sqrt{120}$

3. If speed of light (c), acceleration due to gravity (g) and pressure (P) are taken to be fundamental units, then dimension of universal gravitational constant (G) is -

(a) CgP^{-3} (b) $C^2g^3P^{-2}$ (c) $C^0g^2P^{-1}$ (d) $C^2g^2P^{-2}$

Ans. **[c]**

Sol. $C = LT^{-1}$

$g = LT^{-2}$

$P = MLT^{-2}L^{-2} = ML^{-1}T^{-2}$

$F = \frac{Gm_1m_2}{R^2}$ $G = \frac{FR^2}{m_1m_2} = M^{-1}L^3T^{-2}$

Let $C^a g^b P^c = G$

$\Rightarrow [LT^{-1}]^a [LT^{-2}]^b [ML^{-1}T^{-2}]^c = M^{-1}L^3T^{-2}$

$L^{a+b-c} = L^3$ $\Rightarrow a + b - c = 3$ … (1)

$T^{-a-2b-2c} = T^{-2}$ $\Rightarrow -a - 2b - 2c = -2$ … (2)

$M^c = M^{-1}$ $c = -1$ …(3)

$a + b = 2$

$-a - 2b = -4$ $-b = -2$

$\Rightarrow b = 2$

$a = 0$

$\Rightarrow G = C^0 g^2 P^{-1}$

4. Let $f(x) = \begin{cases} \frac{\pi}{2}\sin x, & \text{for } 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$. Then

(a) no where continuous $(0, \pi)$

(b) continuous on $(0, \pi)$ except at $x = \frac{\pi}{2}$

(c) continuous on $(0, \pi)$, but nowhere differentiable

(d) differentiable at all points of $(0, \pi)$

Ans. **[d]**

Sol. $f'\left(\frac{\pi}{2} - h\right) \lim_{h\to 0} \frac{\frac{\pi}{2}\sin\left(\frac{\pi}{2} - h\right) - \frac{\pi}{2}}{-h}$

$= \lim_{h\to 0} \frac{\frac{\pi}{2}[\cos h - 1]}{-h}$

$= \lim_{h\to 0} \frac{\pi}{2} \frac{2\sin^2 h/2}{h} = 0$

Clearly $f'\left(\frac{\pi}{2}^+\right) = 0$

$\therefore$ L.H.D. = R.H.D.

5. A wave propagating along X-axis is represented by $y = a\sin(At - Bx + C)$ where y is the displacement of the particle, a the amplitude of the wave and t is the time. If A, B and C are three constants then the dimension of $\left(\frac{abc}{A}\right)$ is the same as that of

(a) Length (b) Mass (c) Time (d) Velocity

Ans. **[c]**

Sol. $y = a\sin(At - Bx + C)$

$Bx = \text{const. } B = L^{-1}$

$C = M^0L^0T^0$

$A = T^{-1},\ a = L^1$

$\frac{abc}{A} = \frac{LL^{-1}}{T^{-1}} = T^1$

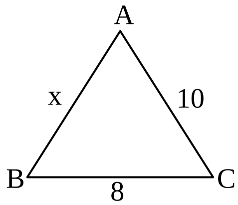
Time

6. The sides of a triangle are 8, 10, x where x is a positive integer. The number of possible values of x for which triangle becomes acute is -

(a) 6 (b) 5 (c) 4 (d) 3

Ans. **[a]**

Sol.



$\therefore\ 8 + x > 10$

$x > 2$ …(1)

Also $8 + 10 > x$

$x < 18$ … (2)

$\cos B > 0$ & $\cos C > 0$

$x^2 + 64 - 100 > 0$ $64 + 100 - x^2 > 0$

$x^2 > 36$ $x^2 < 164$

$x > 6$ $x < \sqrt{164}$

$\therefore\ 6 < x < \sqrt{164}$

$x \in Z^+$

$x = 7, 8, 9, 10, 11, 12$

7. The speed (v in m/s) and time (t in second) for an object moving along a straight line are related as $t^2 - 2\sqrt{2}\,vt + 50 = 0$. The possible values of v is

(a) $v \geq 5$ m/s only (b) $v \geq 10$ m/s only (c) $v \geq 15$ m/s only (d) $v \geq 25$ m/s only

Ans. **[a]**

Sol. $t^2 - 2\sqrt{2}v\,t + 50 = 0$

For t to be real

$D \geq 0$

$(-2\sqrt{2}\,v)^2 - 4 \,.\, 1 \,.\, 50 \geq 0$

$8v^2 - 200 \geq 0$ $8v^2 \geq 200$

$v^2 \geq 25$ $v \geq 5$ m/s

8. There are n teachers in a school and all possible 4 member committees are formed. Among these, exactly $\frac{1}{20}$th part of committees have 2 fixed members. The sum of the digits of n is -

(a) 8 (b) 7 (c) 6 (d) 5

Ans. **[b]**

Sol. $\frac{1}{20}\ ({}^{n}C_4) = {}^{n-2}C_2$

$$\frac{n(n-1)(n-2)(n-3)}{24} = 20 \times \frac{(n-2)(n-3)}{2}$$

$$n(n-1) = \frac{20 \times 24}{2}$$

$n(n - 1) = 240$

$n = 16$

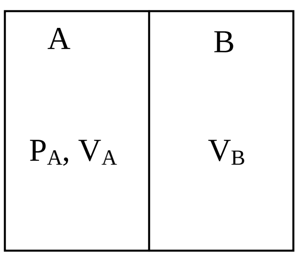
Sum of digit = 6 + 1 = 7

9. A chamber is enclosed in a thermally insulated cover and a partition wall separates it into two parts A and B. Part A is filled up with an ideal gas at pressure p_A and as a volume V_A. The other part (part B) is evacuated and has a volume v_B. Assume this part to be vacuum. The partition wall is now removed. When the equilibrium is set in. The pressure p in the entire chamber is

(a) $p = p_A$ (b) $p = \frac{p_A(v_A + v_B)}{v_B}$ (c) $p = \frac{p_A v_A}{v_A + v_B}$ (d) $p = \frac{p_A v_B}{v_A + v_B}$

Ans. **[c]**

Sol.



Applying conservation of moles

$n_A + n_B = n$

Initially $n_A = \frac{P_A V_A}{RT}$, $n_B = 0$

Finally $n = \frac{P(V_A + V_B)}{RT}$

$$\frac{P_A V_B}{RT} = \frac{P(V_A + V_B)}{RT}$$

$$\Rightarrow P = \frac{P_A V_B}{V_A + V_B}$$

10. Let $(1 + x - 3x^2)^{2018} = a_0 + a_1x + a_2x^2 + ... + a_{4036}x^{4036}$. The last digit of $a_0 + a_2 + a_4 + ... a_{4036}$ is

(a) 0 (b) 5 (c) 7 (d) 9

Ans. **[b]**

Sol. Put x = 1

$1 = a_0 + a_1 + a_2 \ldots\ldots + a_{4036}$ … (1)

Put x = –1

$3^{2018} = a_0 - a_1 + a_2 - a_3 \ldots\ldots a_{4036}$ … (2)

Add (1) & (2)

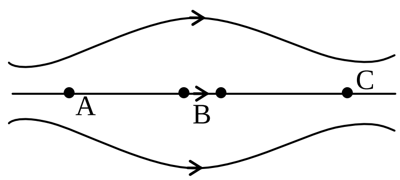
$3^{2018} + 1 = 2(a_0 + a_2 + a_4 \ldots\ldots)$

$\frac{3^{2018}+1}{2} = a_0 + a_2 + a_4 \ldots\ldots$

$\therefore$ Last digit of $3^{2018} = 3^2 . 3^{2016} = 9 \times 1 = 9$

$\therefore$ Last digit of $\frac{3^{2018}+1}{2} = 5$

11. The figure shows some of the field lines of an electric field. The figure suggests that



(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$ (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

Ans. **[c]**

Sol. $E_A = E_C > E_B$ (From theory)

12. The value of the integral $\int_0^2 x\cos(\pi\{x\})\,dx$, where {x} denotes the fractional part of x, is

(a) 0 (b) $\frac{4}{\pi^2}$ (c) $\frac{-4}{\pi^2}$ (d) $\frac{-2}{\pi^2}$

Ans. **[c]**

Sol. $\int_0^2 x\cos\pi(x-[x])\,dx$

$= \int_0^1 x\cos\pi x\;dx + \int_1^2 x\cos(\pi x - \pi)\,dx$

$= \frac{-2}{\pi^2} - \frac{2}{\pi^2}$

$= \frac{-4}{\pi^2}$

13. The moment of the force F = 4**i** + 5**j** – 6**k** acting at the point (2, 0, –3) and about the axis passing through a point (2, –2, –2) is given by

(a) –7**i** – 4**j** – 8**k** (b) –7**i** – 8**j** – 4**k** (c) –4**i** – **j** – 8**k** (d) –8**i** – 4**j** – 7**k**

Ans. **[a]**

Sol. A(2, –2, –2) B(2, 0, –3)

$\vec{r} = \overrightarrow{AB} = 2\hat{j} - \hat{k}$

$\vec{\tau} = (2\hat{j} - \hat{k}) \times (4\hat{i} + 5\hat{j} - 6\hat{k})$

$\vec{\tau} = -7\hat{i} - 4\hat{j} - 8\hat{k}$

14. If α β γ are the roots of $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = 0$, then $\dfrac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2}$ equals

(a) $\frac{1}{7}$ (b) 7 (c) $\frac{1}{6}$ (d) 6

Ans. **[b]**

Sol. $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = 0$

$x(x^2 - 4) - (x - 2) + 2(2 - x) = 0$

$x^3 - 4x - x + 2 + 4 - 2x = 0$

$x^3 - 7x + 6 = 0$

$(x - 1)(x^2 + x - 6) = 0$

$(x - 1)(x + 3)(x - 2) = 0$

$x = 1, 2, -3$

$\therefore \dfrac{\alpha^4 + \beta^4 + \gamma^4}{\alpha^2 + \beta^2 + \gamma^2} = \dfrac{1 + 16 + 81}{1 + 4 + 9} = \dfrac{98}{14} = 7$

15. If all nuclear reactions in the sun now were to suddenly stop for ever, then

(a) Distances between planets and sun would decrease

(b) Angular momentum of planets would increase

(c) Inner planets will be engulfed by the sun

(d) Speed of rotation of the sun about its own axis would increase

Ans. **[d]**

Sol. By theory

16. Three well known stars (a) Procyon (b) Antares and (c) Vega are respectively in the constellation

(a) Orion, Sagittarius and Scorpios (b) Orion, Taurus and Ursa major

(c) Canis minor, Scorpius and Lyra (d) Scorpios, Canes minor and Leo

Ans. **[c]**

Sol.

17. One gram of Radium, with atomic weight 226, emits 4×10^{10} particles per second. The half-life of Radium is

(a) 4.6×10^{10} s (b) 4.6×10^{9} s (c) 4.6×10^{12} s (d) 4.6×10^{14} s

Ans. [a]

Sol. $\frac{dN}{dt} = 4 \times 10^{10}$, $\frac{dN}{dt} = -\lambda N$

$N = \frac{1}{226} N_A = \frac{6.023 \times 10^{23}}{226}$

$\Rightarrow 4 \times 10^{10} = \lambda \times \frac{6.023 \times 10^{23}}{226} \Rightarrow \lambda = \frac{4 \times 10^{10} \times 226}{6.023 \times 10^{23}}$

$\because t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693 \times 6.023 \times 10^{23}}{4 \times 10^{10} \times 226}$

$= 0.0046 \times 10^{13}$

$= 4.6 \times 10^{10}$

18. Let $\langle a_n \rangle_{n \geq 0}$ be a geometric progression with common ratio r, | r | < 1. Let $s_1 = \sum_{k=0}^{\infty} a_k$, $s_2 = \sum_{k=0}^{\infty} a_{2k}$ and $s_3 = \sum_{k=0}^{\infty} a_{3k}$. Suppose $\frac{s_1}{s_2} = \frac{5}{4}$. Then $\frac{s_2}{s_3}$ equals

(a) $\frac{5}{4}$ (b) $\frac{25}{24}$ (c) $\frac{21}{20}$ (d) $\frac{9}{10}$

Ans. [c]

Sol. $s_1 = a_0 + a_1 + a_2$ ....

$s_1 = \frac{a_0}{1-r}$

$s_2 = a_0 + a_2 + a_4$ ....

$s_2 = \frac{a_0}{1-r^2}$

$s_3 = a_0 + a_3 + a_6$ ....

$s_3 = \frac{a_0}{1-r^3}$

$\therefore \frac{s_1}{s_2} = \frac{5}{4}$ given

$\Rightarrow \frac{1-r^2}{1-r} = \frac{5}{4}$

$r = \frac{1}{4}$

Now $\frac{s_2}{s_3} = \frac{1-r^3}{1-r^2} = \frac{1+r+r^2}{1+r} = \frac{1+\frac{1}{4}+\frac{1}{16}}{1+\frac{1}{4}} = \frac{21}{5 \times 4} = \frac{21}{20}$

19. An electric dipole of moment p is lying on a plane in a uniform electric field E_0 with the dipole axis along the field. The dipole on the plane is rotated by an angle 60º keeping its centre of mass fixed. The potential energy of the dipole in its new position will be

(a) $-pE_0$ (b) $-(pE_0)/2$ (c) $-(pE_0)/3$ (d) $-(p/E_0)/4$

Ans. **[b]**

Sol. $\because$ P.E. = $-\vec{P}.\vec{E}$

So, $U = -PE_0 \cos 60^\circ = \frac{-PE_0}{2}$

20. Let $I_1 = \int_0^1 \frac{dx}{1+\sqrt[3]{x}}$ and $I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}}$. Then $4I_1 + 3I_2$ equals

(a) 3 (b) 4 (c) 6 (d) 7

Ans. **[b]**

Sol. $I_1 = \int_0^1 \frac{dx}{1+x^{1/3}}$

$x = t^3$

$dx = 3t^2\, dt$

$= \int_0^1 \frac{3t^2 dt}{1+t}$

$= 3\left[\int_0^1 \frac{(t^2-1)\,dt}{1+t} + \int_0^1 \frac{dt}{1+t}\right]$

$= 3\left[\int_0^1 (t-1)dt + (\ell n(1+t))_0^1\right]$

$= 3\left[\left(\frac{1}{2}-1\right) + \ell n 2\right]$

$= \frac{-3}{2} + 3\ell n 2$

$4I_1 = -6 + 12\, \ell n\, 2$

$I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}}$

$x = t^4$

$dx = 4t^3\, dt$

$I_2 = 4\int \frac{t^3 dt}{1+t}$

$= 4\left[\int_0^1 \frac{(t^3+1)}{t+1}dt - \int_0^1 \frac{dt}{t+1}\right]$

$= 4\left[\int_0^1 (t^2 - t + 1)dt - \int_0^1 \frac{dt}{t+1}\right]$

$= 4\left[\left(\frac{1}{3} - \frac{1}{2} + 1\right) - \ell n 2\right]$

$I_2 = 4\left[\frac{5}{6} - \ell n 2\right]$

$3I_2 = 10 - 12\, \ell n\, 2$

$\therefore 4I_1 + 3I_2 = 4$

21. Imagine a planet of same mass as that of the earth but having a radius twice of that of the earth. A simple pendulum located at some point on its equator failed to show any oscillation when given a small displacement from its equilibrium position. The time taken by this planet to spin once about its own axis is

(a) nearly 2 hours (b) nearly 4 hours (c) nearly 6 hours (d) nearly 8 hours

Ans. **[b]**

Sol. For $T = \infty$

$g' = 0$

$\because\ g' = g - \omega^2 R\cos^2\theta$

for $g' = 0$ at equation

$w = \sqrt{\frac{g}{R}}$

$T = 2\pi\sqrt{\frac{R}{g}}$

Here, $R = 2Re$

$g = ge/4$

$\Rightarrow T = 237$ min

$T \approx 4$ hrs

22. Let ABCD be a rectangle. Let E be a point on the diagonal AC at a distance 16 from the side AB and let DE = 15. Then the area of the rectangle ABCD to the nearest integer is

(a) 468 (b) 469 (c) 470 (d) 471

Ans. **[This question has been deleted by IAPT]**

23. An alloy of two metals is formed by taking their equal masses and it was found to float on mercury (density 13.6 g cm^{-3}) with 52.7% above the mercury surface. When an alloy is formed by taking equal volumes of these two metals it was found to float on mercury with 51.5% of its volume below the surface of mercury. The densities of the two metals in g cm^{-3} are closest to

(a) 6 and 8 (b) 5 and 9 (c) 4.5 and 9.5 (d) 4 and 10

Ans. **[b]**

Sol. In case of flotation

$$\frac{V_s}{V} = \frac{\rho_o}{\rho_\ell}$$

$V_s \rightarrow$ Volume submerged

$V \rightarrow$ Total volume

$\rho_o \rightarrow$ Density of object

$\rho_\ell \rightarrow$ Density of fluid

Let density of alloys be ρ_1 & ρ_2

When equal masses are mixed then

$$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = (1 - 0.527) \times 13.6 \qquad \text{….(1)}$$

When equal volume are mixed then

$$\rho_{min} = \frac{\rho_1 + \rho_2}{2} = 0.515 \times 13.6 \qquad \text{…..(2)}$$

On solving (1) & (2)

$\rho_1 = 5,\ \rho_2 = 9$

24. If n is the number of functions f : {a, b, c, d} → {a, b, c, d} such that no more than two elements in the domain of f have the same image, then

(a) $n \le 100$ (b) $100 < n \le 150$ (c) $150 < n \le 200$ (d) $n > 200$

Ans. **[d]**

Sol. f : {a, b, c, d} → {a, b, c, d}

no same image	To element have same image and two element have distinct	Two same and other two same
$= 4! = 24$	$\left(\left(\frac{4!}{2!\,1!\,1!}\right)\times\frac{1}{2!}\right)\times(4\times3\times2)$	$\left(\frac{4!}{2!\,2!}\times\frac{1}{2!}\right)\times 4\times 3$
	$=\frac{24}{2\times2}\times4\times3\times2$	$=\frac{24}{8}\times4\times3$
	$= 24 \times 6 = 144$	$= 36$

Total = 24 + 144 + 36 = 204

25. On the rechargeable batteries of 1.5 V often used for digital cameras one can find 2300 mAh or 2800 mAh or something similar is written. This connected to the

(a) power that the battery can provide (b) current that can be drawn from the battery

(c) total charge that the battery can supply (d) time for which the battery can be used

Ans. **[c]**

Sol. mAh i.e. $I \times t = Q$

So, Option (c)

26. The planet in which sun appears to rise in the west is

(a) Venus (b) Uranus (c) Saturn (d) Mercury

Ans. **[a]**

Sol. Option (a)

27. Apart from the earth, *Aurora* phenomena are observed on which of the following planet(s)

(a) Venus (b) Mars (c) Mercury (d) Jupiter

Ans. **[d]**

Sol. Option (d)

28. The sum of the last three digits in the expansion of 5^{2018} is

(a) 8 (b) 9 (c) 13 (d) 14

Ans. **[c]**

Sol. $(25)^x$ when $x \ge 2$

last three digits are 625

so that sum of last three digits

is 6 + 2 + 5 = 13

29. If the wavelength of the incident light changes from 400 nm to 300 nm the stopping potential for photoelectrons emitted from the surface of a material becomes

(a) 0.56 V lower (b) 1.04 V higher (c) 0.34 V lower (d) 0.56 V higher

Ans. **[b]**

Sol. $eV_S = \frac{hc}{\lambda} - \phi$

$eV_{S_1} = \frac{hc}{400} - \phi$ …..(1)

$eV_{S_2} = \frac{hc}{300} - \phi$ ….(2)

On subtracting

$e(V_{S_1} - V_{S_2}) = \frac{hc}{400} - \frac{hc}{300}$

$V_{S_1} - V_{S_2} = \frac{hc}{e}\left(\frac{300-400}{120000}\right)$

$\Rightarrow V_{S_2} - V_{S_1} = \frac{hc}{1200e} = \frac{12400\ eV}{1200\,e}$

$V_{S_2} - V_{S_1} = 1.04$ V higher

30. Find the integer closest to the integral $\int_0^6 x^{\{\sqrt{x}\}}dx$, where {x} denotes the largest integer not exceeding x.

(a) 58 (b) 59 (c) 60 (d) 61

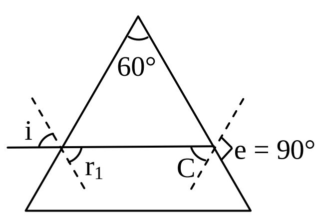
Ans. **[Bonus]**

31. A ray of light enters a glass prism of refractive index 1.55. The cross section of the prism is an equilateral triangle. The emergent ray comes out of the other refracting surface at the grazing angle. The angle of incidence on the first surface is about

(a) 30.7º (b) 28.2º (c) 37.6º (d) 41.2º

Ans. **[a]**

Sol.



$\mu = 1.55$, A = 60°

$\angle e = 90° \Rightarrow r_2 = C$

$r_1 + r_2 = A = 60°$

$r_1 = A - r_2 = (60 - C)$

By Snell's Law 1 sin i = 1.55 sin (60–C)

sin i = 1.55 (sin 60° cos C – cos 60° sinC)

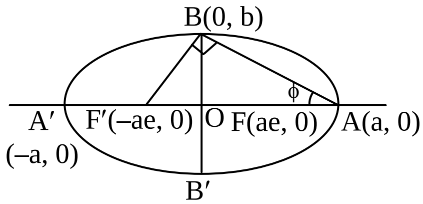
on solving $i \simeq 31°$

32. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ be an ellipse with major axis AA′ and minor axis BB′. Let F and F′ be the foci of the ellipse, with F between A and F′. Suppose ABF′ forms a right-angled triangle. Let e denote the eccentricity of the ellipse. If ϕ denotes ∠FAB, then $\tan^2(\phi)$ is equal to

(a) $\sqrt{e}$ (b) e (c) e^2 (d) 1 + e

Ans. **[b]**

Sol.



$\tan\phi = \frac{b}{a}$

$\therefore \tan^2\phi = \frac{b^2}{a^2}$ … (1)

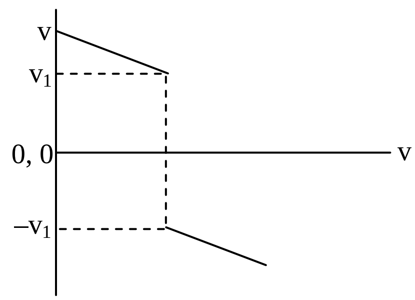
Also AB ⊥ BF′

$\frac{b}{-a} \times \frac{b}{ae} = -1$

$b^2 = a^2 e$ … (2)

$\frac{b^2}{a^2} = e$

33. The following graph shows a velocity versus time graph for a ball. Which explanation best fits the motion of the ball as shown by the graph ?



(a) The ball falls from a height, is caught, and is thrown down with a greater velocity

(b) The ball rises to a height, hits the ceiling, and falls down

(c) The ball falls from a height, hits the floor and bounces up.

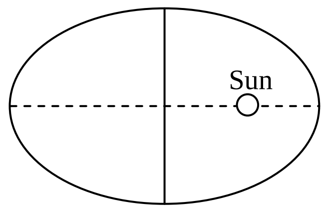
(d) The ball rises to a height, is caught, and then is thrown down with the same velocity.

Ans. **[b]**

Sol. Option (d)

For answering the questions 34 and 37 read the next few lines. Our knowledge of planetary systems is based on the wealth of observations by Copernicus, Tycho Brahe, Johannes Kepler spread over more than a century. Newton may have got some clues about his famous law of universal gravitation, at least the inverse square nature of distance law, from the painstaking work of his predecessors.

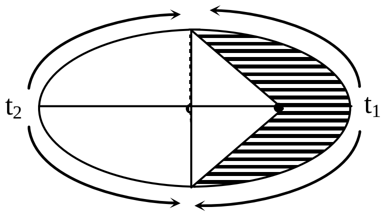
34. According to Kepler's first law, planets go round the Sun in elliptic orbits. If orbit of the earth of eccentricity e around Sun is divided into two halves by the minor axis, the difference in times spent in the two halves of the orbit is



(a) $2e/\pi$ year (b) e/π year (c) $e/(1 - e)$ year (d) $2e^2/(1 - e^2)$ year

Ans. **[a]**

Sol.

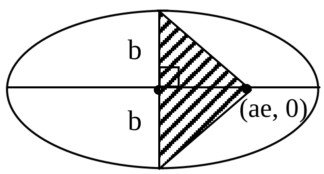


$$\frac{dA}{dt} = \text{constant}$$

$t_1 + t_2 = 1$ year

$A_1 \rightarrow$ shaded area

$$\frac{t_1}{t_2} = \frac{A_1}{A_2}$$



Area of shaded triangle = aeb

Area of ellipse = πab

Area of half ellipse = $\frac{\pi ab}{2}$

$\Rightarrow$ Area $A_1 = \frac{\pi ab}{2} - aeb$

$A_2 = \pi ab - A_1$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\pi ab - 2aeb}{2(\pi ab - A_1)} = \frac{\pi ab - 2aeb}{2\left(\pi ab - \frac{\pi ab}{2} + aeb\right)}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\pi ab - 2aeb}{2\left(\frac{\pi ab}{2} + aeb\right)} = \frac{\pi ab - 2aeb}{\pi ab + 2aeb}$$

$$\frac{A_1}{A_2} = \frac{\pi - 2e}{\pi + 2e}$$

$\Rightarrow \frac{t_1}{t_2} = \frac{\pi - 2e}{\pi + 2e}$

So, $t_1 = \frac{\pi - 2e}{\pi + 2e} \cdot t_2$

$t_1 + t_2 = 1$

$\frac{\pi - 2e}{\pi + 2e} t_2 + t_2 = 1$

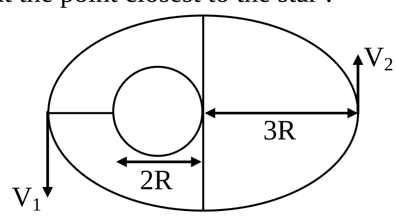
$t_2\left(1 + \frac{\pi - 2e}{\pi + 2e}\right) = 1$

$t_2\left(\frac{2\pi}{\pi + 2e}\right) = 1 \quad \Rightarrow \quad t_2 = \frac{\pi + 2e}{2\pi}$

Now $\quad t_2 - t_1 = \; t_2 - \frac{\pi - 2e}{\pi + 2e} t_2 \Rightarrow \; t_2\left(1 - \frac{\pi - 2e}{\pi + 2e}\right) = t_2\left(\frac{4e}{\pi + 2e}\right)$

So, $t_2 - t_1 = \frac{\pi + 2e}{2\pi} \cdot \frac{4e}{\pi + 2e} = \frac{2e}{\pi}$

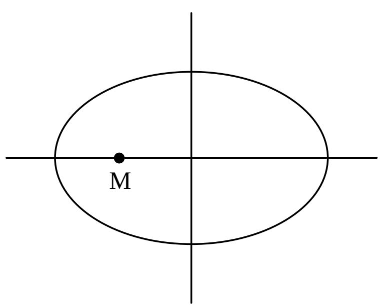
35. A planet goes around a star of mass M and radius R in an orbit of semi major axis 3R, with the distances as shown. What is the velocity V_1 at the point closest to the star :



(a) $(GM/2R)^{1/2}$ (b) $(2GM/3R)^{1/2}$ (c) $(4GM/3R)^{1/2}$ (d) $(GM/6R)^{1/2}$

Ans. **[b]**

Sol.



$mv_2 4R = mv_1 2R$

$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{-GMm}{6R}$

$\frac{1}{2}mv_1^2 = \frac{3GMm}{6R} - \frac{GMm}{6R}$

$= \frac{GMm}{3R}$

$v_1 = \sqrt{\frac{2GM}{3R}}$

36. What are the eccentricity and length of semi minor axis in the orbit in Q.34 ?

(a) 0.30, 2.50R (b) 0.33,2.00R (c) 0.33, 2.83R (d) 0.25, 2.75R

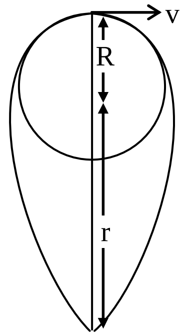
Ans. **[This question has been deleted by IAPT]**

37. If the earth of mass M is assumed to be a sphere of 6400 Km, with what velocity must a projectile be fired from the earth's surface in order that its subsequent path may be an ellipse with major axis 80,000 Km ? [Take the product GM = 4.0×10^{14} m^3s^2]

(a) 10.70 Km/s (b) 11.20 Km/s (c) 9.50 Km/s (d) 11.70 Km/s

Ans. **[a]**

Sol.



$$-\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{r}+\frac{1}{2}mv'^2$$

$$v^2 - v'^2 = 2GM\left[\frac{1}{R}-\frac{1}{r}\right]$$

$$vR = v'r$$

$$v' = \frac{vR}{r}$$

$$v^2 - \frac{v^2R^2}{r^2} = 2GM\left[\frac{1}{R}-\frac{1}{r}\right] \qquad (r + R) = 80000$$

$$v^2\left[\frac{r^2-R^2}{r^2}\right] = 2GM\left[\frac{r-R}{rR}\right] \qquad r = 73600$$

$$v^2\left[\frac{r+R}{r^2}\right] = 2GM\left[\frac{r-R}{rR}\right]$$

$$v^2 = \sqrt{\frac{2GM.r}{R(r+R)}} = \sqrt{\frac{2GM.73600}{6400\times80000\times10^3}}$$

$$= \sqrt{\frac{2\times4\times10^{14}\times73600}{6400\times8\times10^7}}$$

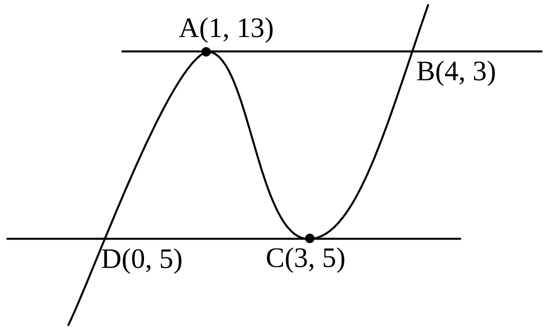
$$= \sqrt{11.5\times10^7} = \sqrt{115\times10^6} = 10.7 \text{ km/s}$$

38. Consider the cubic curve $y = 2x^3 - 12x^2 + 18x + 5$. Let A and C be its extremum points. The tangents at A and C to the curve intersect it again at two other points B and D respectively. The area of the quadrilateral ABCD is -

(a) 12 (b) 24 (c) 36 (d) 48

Ans. **[b]**

Sol.



$y = 2x^3 - 12x^2 + 18x + 5$ …(1)

$\frac{dy}{dx} = 6x^2 - 24x + 18 = 0$

$x^2 - 4x + 3 = 0$

x = 1, 3

$\therefore$ At x = 1, y = 13

x = 3, y = 5

Tangent at A is y = 13, put in (1)

we get x = 1, 4

B(4, 13)

Tangent at C is y = 5

Put in (1) we get

x = 0, 3

D(0, 5)

$\therefore$ Area quad ABCD = $\frac{1}{2}(3 + 3) \times 8 = 24$

39. A crater on the surface of the moon has a diameter of 80 km. If the distance to earth and moon is 3.78×10^5 km then the visual angle in degree is

(a) 0.012 (b) 0.021 (c) 0.019 (d) 0.026

Ans. **[a]**

Sol. $\because \theta = \frac{\text{Arc}}{\text{Radius}}$ (in radians)

$\theta = \frac{80}{3.78 \times 10^5} \times \frac{180}{3.14}$ (in degrees)

$\theta = 0.012^\circ$

40. A K-type star in the main sequence has a luminosity of 0.40 times the luminosity of sun. This star is observed to have a flux of 6.23×10^{-4} Wm^{-2}. The distance (in parsec) to this star is about (ignore atmospheric effects, luminosity of sun is 3.8×10^{26} Wm^{-2} and 1 parsec is 3.08×10^{16} Km)

(a) 45 pc (b) 4.5 pc (c) 450 pc (d) 0.45 pc

Ans. **[Bonus]**

Sol. $L_{star} = 0.4\, L_{sun}$

$4\pi R^2 \times \phi = 0.4 \times 3.8 \times 10^{26}$ m

$\Rightarrow R = 0.14 \times 10^{15}$ m

Now 1 parsec = 3.08×10^{16} m

$\Rightarrow 0.14 \times 10^{15} = \frac{1}{3.08\times10^{16}} \times 0.14 \times 10^{15}$

R = 0.0045 pc

41. Sun is at a mean distance of about 27,000 light years from the centre of the Milky way galaxy and completes one revolution about the galactic centre in about 225 million years. The linear speed of Sun is

(a) 160 km s^{-1} (b) 230 km s^{-1} (c) 30 km s^{-1} (d) 80 km s^{-1}

Ans. **[b]**

Sol. $T = \frac{2\pi r}{v}$

$225 \times 10^6 \times 1 \text{ year} = \frac{2\pi\times27000\times3\times10^8\times1\text{year}}{v}$

$v = \frac{2\pi\times27000\times3\times10^8}{225\times10^6}$

$= 2260 \times 10^2$ m/s

v = 230 km s^{-1}

42. Light from the nearest star 'proxima centauri' takes 4.24 light years to reach earth. The stellar parallax of this star is about

(a) 1.30 s (b) 0.77 s (c) 13.8 s (d) 0.24 s

Ans. **[b]**

Sol. stellar Parallax = $\frac{1\text{AU}}{\text{distance}}$

$= \frac{1.49\times10^{11}}{4.24\times9.461\times10^{15}}$ (in radians)

= 0.77 (in sec)

43. A block of conductor with its area equal to 'A' and thickness 'b' is placed between the plates of a parallel plate capacitor without touching either of the plates. If the area of the plates of the capacitor be 'A' each and 'd' be the separation between the plates then the capacitance of the system after the introduction of the block is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 A}{d\left(1+\frac{b}{d}\right)}$ (c) $\frac{\varepsilon_0 A}{d\left(1-\frac{b}{d}\right)}$ (d) $\frac{\varepsilon_0 A}{d\left[1-\left(\frac{b}{d}\right)^2\right]}$

Ans. **[c]**

Sol. $\because C = \frac{A\epsilon_0}{d-b+\frac{b}{\infty}}$

$C = \frac{A\epsilon_0}{d\left(1-\frac{b}{d}\right)}$

44. The number of real solution of the equation $|x - |x - |x - 4||| = x^2 - 4x$ is

(a) 0 (b) 1 (c) 2 (d) more than 2

Ans. **[c]**

Sol.

$|x - |x - |x - 4||| = x^2 - 4x$

Case $x \geq 4$:

$|x - |x - (x - 4)|| = x^2 - 4x$

$|x - 4| = x^2 - 4x$

$x - 4 = x^2 - 4x$

$x^2 - x - 4x + 4 = 0$

$x (x - 1) - 4 (x - 1) = 0$

$x - 1 = 0 \quad x - 4 = 0$

$x = 1$ (wrong) $\quad x = 4$ Ans.

Case $x < 4$:

$|x - |x - (4 - x)|| = x^2 - 4x$

$|x - |2x - 4|| = x^2 - 4x$

Case $2 \leq x < 4$:

$|x - 2x + 4| = x^2 - 4x$

$|4 - x| = x^2 - 4x$

$4 - x = x^2 - 4x$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

$(x - 4) (x + 1) = 0$

$x = 4$ wrong

$x = -1$ wrong

Case $x < 2$:

$|x - (4 - 2x)| = x^2 - 4x$

$|3x - 4| = x^2 - 4x$

Case $4/3 \leq x < 2$:

$x^2 - 7x + 4 = 0$

$x = \dfrac{7 + \sqrt{33}}{2}$ wrong

$x = \dfrac{7 - \sqrt{33}}{2}$

Case $x < 4/3$:

$x^2 - x - 4 = 0$

$x = \dfrac{1 + \sqrt{17}}{2}$

$x = \dfrac{1 - \sqrt{17}}{2}$ Ans.

45. A body of mass 1.0 kg is pulled along a rough horizontal surface by a horizontal force of 5N for 10 s starting from rest. If the kinetic friction is $\mu_k = 0.40$, the amount of heat generated is equal to (assuming $g = 10\ ms^{-2}$)

(a) 190 J (b) 200 J (c) 210 J (d) 205 J

Ans. **[b]**

Sol. Net force on body

$F_{net} = 5 - 0.4 \times 1 \times 10$

$ma = 1$ N

$a = 1 m/s^2$

$S = \dfrac{1}{2} \times 1 \times (10)^2 = 50$ m

$\therefore$ Heat generated = work done against friction

$= 4 \times 50 = 200$ J

46. Six dice are rolled simultaneously. The probability of getting at least four identical numbers is

(a) $\dfrac{2250}{6^6}$ (b) $\dfrac{2436}{6^6}$ (c) $\dfrac{2535}{6^6}$ (d) $\dfrac{2738}{6^6}$

Ans. **[b]**

Sol. P(getting at least four identical number)

$$= {}^6C_4 . \left\{ {}^6C_1 . \left(\frac{1}{6}\right)^4 \right\} \left\{\frac{5}{6}\right\} \left\{\frac{5}{6}\right\} + {}^6C_5 \left({}^6C_1 \left(\frac{1}{6}\right)^5 \right) \times \frac{5}{6} + {}^6C^6 \left({}^6C_1 \left(\frac{1}{6}\right)^6 \right)$$

$$= \frac{2436}{6^6}$$

47. The ceiling of a long hall is 45 m high. The maximum horizontal distance that a ball thrown with a speed of 50 ms^{-1} can go without hitting the ceiling is nearly equal to (g = 10 ms^{-2})

(a) 250 m (b) 240 m (c) 230 m (d) 300 m

Ans. **[b]**

Sol. H = 45 m u = 50 m/s

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$45 \times 2 \times 10 = (50)^2 \sin^2\theta$

$50 \sin\theta = 30$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(50)^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{10}$$

$$= \frac{2500 \times 2 \times 3 \times 4}{250}$$

$\Rightarrow R = 240$ m

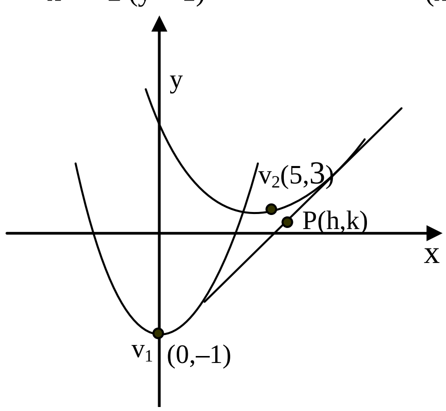
48. The tangents drawn from a certain point P to the parabola $2y = x^2 - 2$ are also tangents to the parabola $4y = x^2 - 10x + 37$. The sum of the coordinates of P is

(a) 10 (b) 6 (c) 0 (d) –10

Ans. **[d]**

Sol. $\mathbf{v_1}$: $2y + 2 = x^2$ $\mathbf{v_2}$: $4y = (x-5)^2 + 12$

$x^2 = 2(y+1)$ $(x-5)^2 = 4(y-3)$



Tangent on v_1 is

$$y + 1 = mx - \frac{m^2}{2}$$

$$y = mx - \left(1 + \frac{m^2}{2}\right)$$

$$k = mh - \left(1 + \frac{m^2}{2}\right)$$

$\Rightarrow 2k = 2mh - 2 - m^2$

$m^2 - 2mh + 2k + 2 = 0$ .... (1)

Tangent on v_2 : $y - 3 = m(x - 5) - m^2$

$k = mh + 3 - 5m - m^2$

$m^2 + (5 - h)m + k - 3 = 0$ ..... (2)

$\therefore$ Both roots are same

$\frac{1}{1} = \frac{-2h}{5-h} = \frac{2k+2}{k-3}$

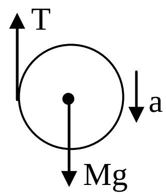
$h = -5, k = -5$

49. A yo-yo of mass 'M' and radius of the inner hub 'r' is completely wound with a string. It is allowed to start unwinding with zero downward initial velocity. The moment of inertia of the yo-yo about an axis passing through its centre of mass and normal to the discs is I. The acceleration with which the yo-yo falls when $I = Mr^2$ can be given by

(a) a = g (b) a = g/2 (c) a = 2g/3 (d) a = g/4

Ans. **[b]**

Sol.



$Mg - T = Ma$ ......(1)

$T.r = I\,\alpha$

$T.\,r = Mr^2\left(\frac{a}{r}\right)$

$T = Ma$ ...... (2)

from eq. (1) & (2)

$Mg - Ma = Ma$

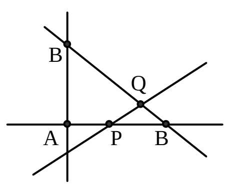
$a = \frac{g}{2}$

50. What is the least possible length of a line segment that cuts a triangle with sides 3, 4, 5 in to two geometrical figures having equal area ?

(a) $\sqrt{12}$ (b) $\sqrt{6}$ (c) $\sqrt{5}$ (d) 2

Ans. **[d]**

Sol.



$\frac{1}{2} \times 4 \times 3 \times \frac{1}{2} = \frac{1}{2} \times BP \times BQ \times \sin B$

$\Rightarrow BP\ BQ = \frac{6}{\sin B}$

$\cos B = \frac{(BP)^2 + (BQ)^2 - (PQ)^2}{2BP \cdot BQ}$

$\Rightarrow (PQ)^2 = (BP)^2 + (BQ)^2 - 12\cot B$

$$(PQ)^2_{min} = \frac{12}{\sin B} - \frac{12\cos B}{\sin B} = \frac{12(1-\cos B)}{\sin B}$$

$(PQ)^2_{min}$ = when cos B = $\frac{4}{5}$, sin B = $\frac{3}{5}$

$$(PQ)^2_{min} = \frac{12(1-\frac{4}{5})}{\frac{3}{5}} = \frac{12\times\frac{1}{5}}{\frac{3}{5}}$$

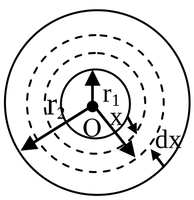
$(PQ)_{min} = 2$

51. A plane spiral of N turns, having the radii of internal and external loops as r_1 and r_2 carries a current I. The magnetic induction at the centre of the spiral will be

(a) $\frac{\mu_0 NI}{(r_2-r_1)}\ln\frac{r_2}{r_1}$ (b) $\frac{\mu_0 NI}{2(r_2-r_1)}\ln\frac{r_2}{r_1}$ (c) $\frac{\mu_0 NI}{(r_2-r_1)}\ln\frac{r_1}{r_2}$ (d) $\frac{\mu_0 NI}{2(r_2-r_1)}\ln\frac{r_1}{r_2}$

Ans. **[b]**

Sol.



Let dN be the no. of turns in elemental portion

$$dN = \frac{N}{(r_2-r_1)}.dx$$

$$dB = \frac{\mu_0 i}{2x}.\,dN$$

$$= \frac{\mu_0 i}{2x}\frac{N}{(r_2-r_1)}\,dx$$

solving, we get

$$B = \frac{\mu_0 NI}{2(r_2-r_1)}\ \ln\frac{r_2}{r_1}$$

52. The number of nonzero real solutions of the equations $x^{x+y} = y^3$, $y^{x+y} = x^{12}$ is

(a) 0 (b) 1 (c) 2 (d) more than 2

Ans. **[d]**

Sol.

$x^{x+y} = y^3$ | $y^{x+y} = x^{12}$

$(x + y) \log x = 3 \log y$ | $(x + y) \log y = 12 \log x$

$\frac{\log x}{\log y} = \frac{3}{x+y}$ ....(i) | $\frac{\log x}{\log y} = \frac{x+y}{12}$ ...(ii)

from (i) and (ii)

$$\frac{3}{x+y} = \frac{x+y}{12}$$

$x + y = \pm 6$ ....(iii)

$$\because \frac{\log x}{\log y} = \pm\frac{1}{2}$$

$y = x^{\pm 2}$

put $y = x^2$ in (iii)

$x^2 + x \pm 6 = 0$

$x^2 + x - 6 = 0$ has two real solutions

$\boxed{x = 2, -3}$

but $x^2 + x + 6 = 0$ will have imaginary solutions

Also put $y = x^{-2}$ in (iii)

$x + \frac{1}{x^2} = \pm 6$

$x^3 - 6x^2 + 1 = 0$ or $x^3 + 6x^2 + 1 = 0$

has at least one real solution   has at least one real solution

53. Two identical circular coils are carrying currents i_1 and i_2 are suspended from a torsion free cotton thread in placed in a region of uniform magnetic field B. Each time the coils are given a small angular displacement from their respective equilibrium positions. The time period of the small torsional oscillations were found to be T_1 and T_2. The ratio $\frac{T_1}{T_2}$ would be

(a) $\frac{i_1}{i_2}$ (b) $\frac{i_2}{i_1}$ (c) $\sqrt{\frac{i_1}{i_2}}$ (d) $\sqrt{\frac{i_2}{i_1}}$

Ans. **[d]**

Sol. For a torsional pendulum -

$\tau \propto \frac{1}{\sqrt{i}}$

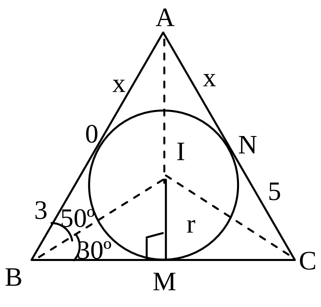
$\Rightarrow \frac{\tau_1}{\tau_2} = \sqrt{\frac{i_2}{i_1}}$

54. A triangle has a side of length 8 units, one of the angles of the triangle on this side is 60º. If the inradius of the triangle is $\sqrt{3}$ units, the perimeter of the triangle is

(a) $15\sqrt{3}$ (b) 24   (c) $12\sqrt{3}$ (d) 20

Ans. **[d]**

Sol.



r = $\sqrt{3}$

$\tan 30º = \frac{\sqrt{3}}{BM}$

BM = 3 = BO

CM = 5 = CN

Now

$$\cos 60^\circ = \frac{64+(3+x)^2-(5+x)^2}{2\times 8\times(3+x)}$$

24 + 8x = 64 + (8 + 2x) (–2)

24 + 8x = 64 – 16 – 4x

12x = 24

x = 2

$\therefore$ Perimeter = 8 + 7 + 5 = 20

55. Two cells with emfs E_1 and E_2 have internal resistances r_1 and r_2 respectively. The two cells are connected in series with an external resistance and the current through the external resistance is found to be 1.5 A. When the polarities of the cells are reversed this current is found to be 0.5A. The ratio of the emfs of the cells is

(a) 2.5 (b) 1.5 (c) 2 (d) 4

Ans. **[c]**

Sol. For I case -

$$I = \frac{(E_1+E_2)}{(r_1+r_2)+R} = 1.5$$

$E_1 + E_2 = 1.5\,(r_1 + r_2 + R)$ ....(1)

For II case -

$E_1 - E_2 = 0.5\,(r_1 + r_2 + R)$ .... (2)

$$\frac{eq(1)}{eq(2)} \Rightarrow \frac{E_1+E_2}{E_1-E_2} = \frac{1.5}{0.5} = 3$$

$E_1 + E_2 = 3E_1 - 3E_2$

$4E_2 = 2E_1$

$$\frac{E_1}{E_2} = \frac{2}{1}$$

56. A points P(8, 4) divides a chord, lying completely in the first quadrant, of a parabola $y^2 = 4x$ in the ratio 1 : 4. The mid-point of the chord has coordinates

(a) (17.5, 8) (b) (18.5, 7) (c) (19.5, 6) (d) (20.5, 5)

Ans. **[b]**

Sol. $A(t_1^2, 2t)$ $B(t_2^2, 2t_2)$

$\frac{4t_1^2+t_2^2}{5} = 8$ $\frac{8t_1+2t_2}{5} = 4$

$4t_1^2 + t_2^2 = 40$ ...(i) $4t_1 + t_2 = 10$

$t_2 = (10 - 4t_1)$ ....(ii)

put in (i)

$4t_1^2 + (10 - 4t_1)^2 = 40$

solve it

$t_1 = 1$ $t_2 = 6$

$t_1 = 3$ $t_2 = -2 \rightarrow$ wrong

i.e. $t_1 = 1$ $t_2 = 6$

A(1, 2) B(36, 12)

midpoint (37/2, 14/2)

= (18.5, 7) Ans.

57. The de Broglie wavelength associated with neutrons with thermal equilibrium with matter at temperatures 300 K and at 400 K are in the ratio close to

(a) 1 : 1 (b) 1.15 : 1 (c) 1 : 2.3 (d) 1 : 2.8

Ans. **[b]**

Sol. $\because\ \lambda \propto \frac{1}{\sqrt{T}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{400}{300}} = 1.15$$

58. The sum of all real values of α for which the equation $x^3 - 7x + \alpha = 0$ has two real roots differing by 1 is

(a) 0 (b) 6 (c) 12 (d) –12

Ans. **[a]**

Sol. $x^3 - 7x + \alpha = 0$

Let roots are x_1, $x_1 + 1$, x_2

$\therefore\ 2x_1 + x_2 + 1 = 0$ … (1)

$x_1(x_1 + 1) + x_1x_2 + x_2(x_1 + 1) = -7$

$x_1^2 + 2x_1x_2 + x_2 + x_1 = -7$

$x_1^2 + 2x_1(-2x_1 - 1) - (1 + x_1) = -7$ (using 1)

$x_1^2 + x_1 - 2 = 0$

$x_1 = -2, 1$

$\therefore\ \alpha = 6, -6$

59. Which of the following physical quantities has the unit volt-second

(a) Energy (b) Electric flux (c) Magnetic flux (d) Inductance

Ans. **[c]**

Sol. Fact based.

60. A die is rolled 5 times. The probability that there are at least two equal numbers among the outcomes obtained is

(a) $\frac{319}{324}$ (b) $\frac{49}{54}$ (c) $\frac{13}{18}$ (d) $\frac{4}{9}$

Ans. **[b]**

Sol. P(at least two equal number) = 1 – P(None show same no.)

$$= 1 - \frac{6 \times 5 \times 4 \times 3 \times 2}{6 \times 6 \times 6 \times 6 \times 6}$$

$$= 1 - \frac{5}{54} = \frac{49}{54}$$

61. The wave length of H_α line from hydrogen discharge tube in a laboratory is 656 nm. The corresponding radiation received from two galaxies A and B have wavelengths of 648 nm and 688 nm respectively. Then

(a) A is approaching the earth with a speed of 2.4×10^4 kms^{-1}

(b) B approaching the earth with a speed of 1×10^4 kms^{-1}

(c) A is receding from the earth with a speed of 3.6×10^4 kms^{-1}

(d) B is receding the earth with a speed of 1.5×10^4 ksm^{-1}

Ans. **[d]**

Sol. $\because \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

for A $\frac{8 \times 3 \times 10^8}{656} = v$

$v = 3.6 \times 10^3$ km/s

for B $v = 1.5 \times 10^4$ km/s

62. The correct sequence of the objects in the ascending order of distance from the sun, is

(a) Kupier belt, Uranus, asteroid belt and oort cloud

(b) Uranus, asteroid belt, oort cloud and kupier belt

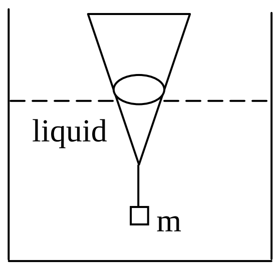
(c) Oort cloud, asteroid belt, Uranus and kupier belt

(d) asteroid belt, Uranus, Kupier belt, and Oort cloud

Ans. **[d]**

Sol. Fact based

63. A cone of height h is floating in a liquid upside down with a mass m attached to it as shown in the figure. Water reaches a height of h/2 at equilibrium. The cone is now given a small downward push and is found to oscillate about its mean position. If friction is ignored the frequency of this oscillation is



(a) $\frac{1}{2\pi}\sqrt{\frac{g}{h}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2g}{h}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6g}{h}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{9g}{h}}$

Ans. **[c]**

Sol. For equilibrium

$\sigma \times \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times \frac{h}{2} . g = mg$

$\sigma\pi\left(\frac{r}{2}\right)^2 \times g = \frac{6m}{h}$

New, change in Bouyant force = ma

$\frac{6m}{h}.gx = ma$

$a = \frac{6g}{h}x$

$f = \frac{1}{2\pi}\sqrt{\frac{6g}{h}}$

64. The number of solutions of $1 - \sin^4 x - 2\cos^4 x = 0$ in the interval $[0, 2\pi]$ is

(a) 6 (b) 4 (c) 2 (d) 0

Ans. **[a]**

Sol. $(1 - \sin^4 x) - 2\cos^4 x = 0$

$(\cos^2 x)(1 + \sin^2 x) - 2\cos^4 x = 0$

$\cos^2 x = 0$	$1 + \sin^2 x - 2\cos^2 x = 0$
$x = \pi/2$	$1 + \sin^2 x - 2(1 - \sin^2 x) = 0$
$x = 3\pi/2$	$1 + \sin^2 x - 2 + 2\sin^2 x = 0$

$\sin^2 x = \frac{1}{3}$

$\sin x = \pm \frac{1}{\sqrt{3}}$

four solutions

Total six solutions

65. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere ?

(a) Angular momentum (b) Rotational kinetic energy

(c) Moment of inertia (d) Angular velocity

Ans. **[a]**

Sol. $\because \tau_{ext} = 0$

L = constant

66. If n is the least positive integer such that $\binom{n-1}{5} + \binom{n-1}{7} < \binom{n}{7}$, the sum of digits of n is

(a) 6 (b) 5 (c) 4 (d) 3

Ans. **[c]**

Sol. ${}^{n-1}C_5 + {}^{n-1}C_7 < {}^{n}C_7$

$$\frac{(n-1)!}{5!(n-6)!} + \frac{(n-1)!}{7!(n-8)!} < \frac{n!}{7!(n-7)!} \Rightarrow \frac{1}{(n-6)(n-7)} + \frac{1}{7\times 6} < \frac{n}{7\times 6(n-7)}$$

$$\frac{42 + (n-6)(n-7)}{7\times 6\times (n-6)(n-7)} < \frac{n}{7\times 6(n-7)}$$

$42 + n^2 - 6n - 7n + 42 < n(n-6) \Rightarrow 42 + n^2 - 6n - 7n + 42 < n^2 - 6n$

$7n > 84$

$n > 12$

i.e. $n = 13$

sum of digits = 1 + 3 = 4

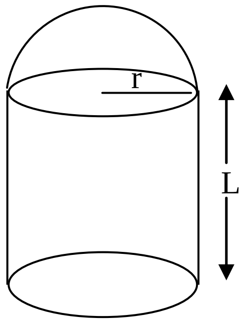
67. The flat surface of a solid hemisphere of radius r is cemented to one flat surface of a cylinder (of identical material) of radius r and length L. If the total mass is M, moment of inertia of the combination about the axis of the cylinder will be

(a) $Mr^2 \dfrac{\frac{L}{2} + \frac{4r}{15}}{L + \frac{2r}{3}}$ (b) $Mr^2 \dfrac{\frac{L}{3} + \frac{4r}{5}}{L + \frac{2r}{3}}$ (c) $Mr^2 \dfrac{\frac{L}{3} + \frac{3r}{5}}{\frac{L}{2} + \frac{r}{3}}$ (d) $Mr^2 \dfrac{\frac{L}{6} + \frac{2r}{5}}{L + \frac{4r}{3}}$

Ans. **[a]**

Sol. $\rho \to$ density of material

$$\rho = \frac{M}{\frac{2}{3}\pi r^3 + \pi r^2 L}$$



$I = I_{Hemisphere} + I_{cylinder}$

$$= \frac{2}{5}\left(\rho \times \frac{2}{3}\pi r^3\right) r^2 + \frac{\rho \times \pi r^2 L \times r^2}{2}$$

After putting value of ρ & solving, we get

$$I = Mr^2 \left[\frac{\frac{L}{2} + \frac{4}{15}r}{L + 2\frac{r}{3}}\right]$$

68. The limit $\lim_{x\to\infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$

(a) does not exist (b) is $\frac{1}{2}$ (c) is 2 (d) is *l*n 2

Ans. **[b]**

Sol. $\lim_{x\to\infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$

$$= \lim_{x\to\infty} \frac{x + \sqrt{x + \sqrt{x}} - x}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{x^{1/2}\sqrt{1 + \frac{1}{\sqrt{x}}}}{x^{1/2}\left(\left\{1 + \frac{1}{\sqrt{x}}\sqrt{1 + \frac{1}{\sqrt{x}}}\right\} + 1\right)} = \frac{1}{1+1} = \frac{1}{2}$$

69. An electron is moving with uniform velocity along a line in the plane of the paper. It is now subjected to a uniform magnetic field B perpendicular to the plane of the paper and going into it. The electron will move in a circular path in the plane of the paper in

(a) clockwise direction with time period proportional to B.

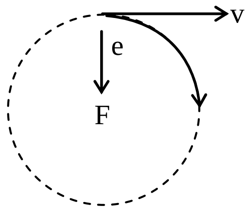
(b) anticlockwise direction with time period inversely proportional to B.

(c) clockwise direction with time period inversely proportional to B.

(d) anticlockwise direction with time period proportional to B.

Ans. **[c]**

Sol.



$q = -e$

$\vec{v} = v\hat{i}$

$\vec{B} = -B\hat{k}$

$\vec{F} = q(\vec{v} \times \vec{B})$

$= -e[v\hat{i} \times (-B\hat{k}) \Rightarrow evB[-\hat{j}]$

Motion : clockwise

$T = \frac{2\pi M}{qB} \Rightarrow T \propto \frac{1}{B}$

70. Let $s_n = 1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + ... + n\left(1+\frac{1}{n}\right)^n$. Then $\sum_{n=1}^{\infty}\frac{1}{2^{2\sqrt{S_n}}}$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) 1

Ans. **[This question has been deleted by IAPT]**

71. A loudspeaker emits sound at a maximum audible level of 130 dB when measured directly from a distance of 1 metre. If the safe limit of audible sound to our ears is 90 dB, a listener must stand directly at a minimum distance of

(a) 1.44 m (b) e^2 m (c) 100 m (d) 2.09 m

Ans. **[c]**

Sol. $L_1 = 90$ dB $r_1 = ?$

$L_2 = 130$ dB $r_2 = 1$m

$L_2 - L_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$

$\frac{130-90}{10} = \log_{10}\left(\frac{I_2}{I_1}\right) \Rightarrow 10^4 = \frac{r_1^2}{r_2^2} \quad \left(\because I \propto \frac{1}{r^2}\right)$

$\Rightarrow r_1 = 100\, r_2$

$r_1 = 100$ m

72. The diameter of radio telescope, working at a wavelength of $\lambda = 1$ cm, with the same resolution as optical telescope of diameter D = 10 cm is

(a) 2 m (b) 2 km (c) 20 km (d) 200 km

Ans. **[b]**

Sol. Resolution $= 1.22\frac{\lambda}{d}$

$1.22 \times \frac{0.01}{d} = 1.22 \times \frac{5\times10^{-7}}{0.1}$

$d = 2000$ m $= 2$ km

73. In a binary system, the apparent magnitude of the primary star is 1.0 and that of the secondary star is 2.0. The maximum combined magnitude of this system is

(a) 3 (b) 1.5 (c) 1 (d) 0.64

Ans. **[d]**

Sol. $m_f = -2.5 \log_{10}[10^{-m_1 \times 0.4} + 10^{-m_2 \times 0.4}]$

Here, $m_1 = 1$; $m_2 = 2$

$m_f \approx 0.64$

74. Suppose the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (b < a) at the point $\left(ae, -\frac{b^2}{a}\right)$ makes an angle of 30º with x-axis. Then $\frac{b^2}{a^2}$ equals

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans. **[c]**

Sol. Equation of tangent

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

Slope of tangent

$$= -\frac{b\cos\theta}{a\sin\theta} = \tan 30^\circ \qquad \frac{\cos\theta}{\sin\theta} = -\frac{a}{b} \times \frac{1}{\sqrt{3}}$$

$$a\cos\theta = ae \qquad b\sin\theta = -\frac{b^2}{a}$$

$$\frac{a}{b}\frac{\cos\theta}{\sin\theta} = \frac{a^2 e}{-b^2}$$

$$\frac{a^2}{-b^2\sqrt{3}} = -\frac{a^2 e}{-b^2}$$

$$e = \frac{1}{\sqrt{3}}$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{3} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{2}{3}$$

75. A piece of strong magnet is suspended from a helical spring made of a non magnetic material and oscillates in a vertical plane with a time period of T on the surface of the earth. If this is taken to the mooon then it will oscillate

(a) with a time period $T_1 > T$ as the value of 'g' is smaller on the moon

(b) with a time period $T_1 < T$ as the value of 'g' is smaller on the moon

(c) with a time period $T_1 < T$ as there is no magnetic field on the moon

(d) with the same time period as the spring and the suspended body are the same on the moon

Ans. **[d]**

Sol. $\because T = 2\pi\sqrt{\frac{m}{k}}$

'T' is independent of 'g'

76. The number of triples (a, b, c) of natural numbers satisfying the equation $\frac{5}{12} = \frac{1}{a} + \frac{1}{ab} + \frac{1}{abc}$ is

(a) 7 (b) 8 (c) 9 (d) 12

Ans. **[a]**

Sol. $\frac{5}{12} = \frac{1}{a} + \frac{1}{ab} + \frac{1}{abc}$

$5a = 12 + \frac{(c+1)12}{bc}$

c = 1 $\Rightarrow 5a = 12 + \frac{24}{b}$; b = 3, 8

c = 2 $\Rightarrow 5a = 12 + \frac{18}{b}$; b = 1, 6

c = 3 $\Rightarrow 5a = 12 + \frac{16}{b}$; b = 2

c = 4 $\Rightarrow 5a = 12 + \frac{15}{b}$; b = 5

c = 6 $\Rightarrow 5a = 12 + \frac{14}{b}$; b not possible

c = 12 $\Rightarrow 5a = 12 + \frac{13}{b}$; b = 1

= 7 ways

77. A 1.5 times magnified real image of an object is obtained when it is placed 16 cm away from a thin convex lens. Now a thin concave lens is placed in contact with the convex lens keeping the object undisturbed and an image of same magnification is formed by the combination. The focal length of the concave lens is

(a) 8 cm (b) 10 cm (c) 12 cm (d) 16 cm

Ans. **[c]**

Sol. For convex lens

$u = -16$

$m = -1.5$

$m = \frac{v}{u} \Rightarrow v = 24$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1}$

$\frac{1}{24} + \frac{1}{16} = \frac{1}{f_1}$ ...... (1)

with concave lens

$m = +1.5$

$m = \frac{v}{u}$

$v = -24$

$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$

$\Rightarrow -\frac{1}{24} + \frac{1}{16} = \frac{1}{16} + \frac{1}{24} + \frac{1}{f_2}$

$\Rightarrow \frac{1}{f_2} = -\frac{1}{12}$

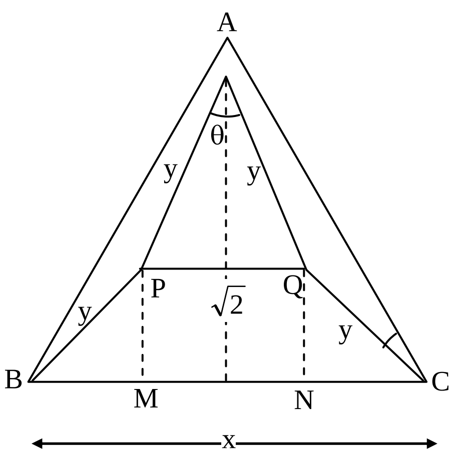
$\Rightarrow f_2 = -12$ cm

78. Let ABC be an equilateral triangle with side x. Two points P and Q are inside ABC such that PQ is parallel to BC and AP = AQ = PB = QC = $\sqrt{3}$ + 1 and PQ = $\sqrt{2}$. Then x equals

(a) $4\sqrt{2} + 2\sqrt{6}$ (b) $2\sqrt{2} + \sqrt{6}$ (c) $2\sqrt{3} + \sqrt{6}$ (d) $2\sqrt{6} + \sqrt{3}$

Ans. **[b]**

Sol.



$y = \sqrt{3} + 1$

$PQ = \sqrt{2}$

$\cos\theta = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}+1)^2 - 2}{2(\sqrt{3}+1)^2}$

$= \frac{2(4+2\sqrt{3}) - 2}{2(4+2\sqrt{3})}$

$\cos\theta = \frac{6+4\sqrt{3}}{4(2+\sqrt{3})}$

$\cos\theta = \frac{3+2\sqrt{3}}{2(2+\sqrt{3})} = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$

$\angle BAP = 15^\circ$

$\angle PBM = 45^\circ$

$\cos 45^\circ = \frac{BM}{y} \Rightarrow \frac{1}{\sqrt{2}} = \frac{BM}{\sqrt{3}+1}$

$BM = \frac{\sqrt{3}+1}{\sqrt{2}} = CN$

$x = \sqrt{2} + \frac{2(\sqrt{3}+1)}{\sqrt{2}}$

$= \sqrt{6} + 2\sqrt{2}$

79. Critical velocity, drift velocity, escape velocity, and rms velocity are the different types of velocities that we come across in the same order while discussing

(a) viscosity, electron motion in solids, gravitation, surface tension respectively

(b) motion of gas molecules, viscosity, gravitation, electron motion in solids respectively

(c) sound propagation, gravitation, motion of gas molecules, colour of light respectively

(d) viscosity, electron motion in solids, gravitation, motion of gas molecules respectively

Ans. **[d]**

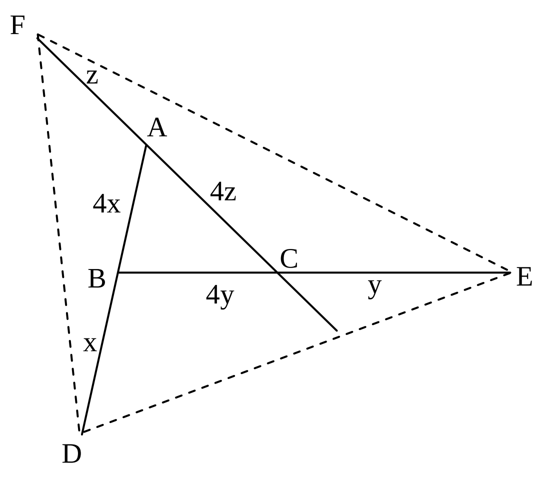
Sol. Fact based

80. In a triangle ABC, AB is extended to D such that AB : BD = 4 : 1 ; BC is extended to E such that BC : CE = 4 : 1 ; and CA is extended to F such that CA : AF = 4 : 1. The ratio of the area of triangle DEF to that of ABC is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{15}{8}$ (d) $\frac{31}{16}$

Ans. **[d]**

Sol.



Let arΔABC = Δ

ar $\Delta DEF = \frac{1}{2} 5zy \sin C + \frac{1}{2} 5xy \sin A + \frac{1}{2} 5xy \sin B + \Delta$

ar $\Delta DEF = \frac{5}{2}[zy \sin C + xz \sin A + xy \sin B] + \Delta$ ...(i)

In ΔABC

$\Delta = \frac{1}{2} 4z.4y \sin C$

$$zy \sin C = \frac{\Delta}{8} \qquad ...(ii)$$

Also $\Delta = \frac{1}{2} 4x.4z \sin A$

$$xz \sin A = \frac{\Delta}{8} \qquad ...(iii)$$

and $xy \sin B = \frac{\Delta}{8}$ ...(iv)

put in (i)

$$\text{ar } \Delta DEF = \frac{5}{2}\left[\frac{3\Delta}{8}\right] + \Delta$$

$$\frac{\text{ar } \Delta DEF}{\text{ar } \Delta ABC} = \frac{31}{16}$$