# JEE Main Online Exam 2020

## Questions & Solution

### 9th January 2020 | Shift - I

## MATHEMATICS

**Q.1** In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is

(1) $\frac{11}{16}$ (2) $\frac{15}{16}$ (3) $\frac{9}{16}$ (4) $\frac{13}{16}$

**Ans.** **[1]**

**Sol.** Possibilities that the second A card appears before the third B card are

= AA + ABA + BAA + ABBA + BBAA + BABA

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4$

$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

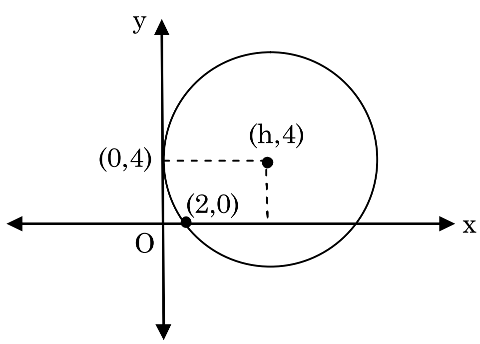
$= \frac{11}{16}$

**Q.2** A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle?

(1) $3x - 4y - 24 = 0$ (2) $4x - 3y + 17 = 0$ (3) $4x + 3y - 8 = 0$ (4) $3x + 4y - 6 = 0$

**Ans.** **[4]**

**Sol.** Let the centre of circle is (h, 4) and r = h




$\therefore$ equation of circle $(x - h)^2 + (y - 4)^2 = h^2$

it passes through (2, 0)

$\therefore (2 - h)^2 + 16 = h^2$

$\Rightarrow h = 5$

$\therefore$ centre (5, 4) and r = 5.

By option 3x + 4y – 6 = 0 is a tangent to the circle ($\because$ p = r)

$$p = \left|\frac{15+16-6}{\sqrt{9+16}}\right| = 5$$

**Q.3** If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in $R^3$, then α + β is equal to

(1) 0 (2) –10 (3) 10 (4) 2

**Ans.** **[3]**

**Sol.** Plane intersect in a line

$$\therefore \Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$\Rightarrow 1(7\alpha + 25) - 4(\alpha + 5) - 2(5 - 7) = 0$

$\Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$

$\Rightarrow 3\alpha + 9 = 0$

$\Rightarrow \alpha = -3$

Also $\Delta_z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$$

$\Rightarrow 1(35 - 5\beta) - 4(5 - \beta) + 1(5 - 7) = 0$

$\Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

**Q.4** A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 $cm^3$/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is

(1) $\frac{1}{18\pi}$ (2) $\frac{1}{54\pi}$ (3) $\frac{1}{36\pi}$ (4) $\frac{5}{6\pi}$

**Ans.** **[1]**

**Sol.** Let the thickness = x cm

Total volume $V = \frac{4}{3}\pi(10 + x)^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\, 3(10+x)^2 \frac{dx}{dt}$$

Given that $\frac{dV}{dt} = 50\ cm^3/min$, $x = 5$ cm

$$\Rightarrow 50 = \frac{4}{3}\pi\, 3(10+5)^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{18\pi}\ cm/min.$$

**Q.5** If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adj\ B|}{|C|}$ is equal to :

(1) 16 (2) 8 (3) 2 (4) 72

**Ans.** **[2]**

**Sol.** $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 6$

$|adj\ B| = |adj\ adj\ A| = |A|^{(n-1)^2} = |A|^4 = 36 \times 36$

$|C| = 27|A| = 27 \times 6$

$$\frac{|adj\ B|}{|C|} = \frac{36 \times 36}{27 \times 6} = 8$$

**Q.6** If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x} & ; \quad x < 0 \\ b & ; \quad x = 0 \\ \frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} & ; \quad x > 0 \end{cases}$ is continuous at x = 0, then a + 2b is equal to :

(1) –2 (2) –1 (3) 1 (4) 0

**Ans.** **[4]**

**Sol.** L.H.L. $= \lim_{x \to 0} \frac{\sin(a+2)x + \sin x}{x}$

$= a + 3$

R.H.L. $= \lim_{x \to 0} \frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} = \lim_{x \to 0} \frac{(1+3x)^{1/3} - 1}{x}$

$$= \lim_{x \to 0} \frac{\frac{1}{3}(1+3x)^{-2/3}.\ 3}{1} = 1$$

f(0) = b

b = 1 and a = –2, So a + 2b = 0

**Q.7** The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$ is equal to

(1) $2\pi^2$ (2) $4\pi$ (3) $\pi^2$ (4) $2\pi$

**Ans.** [3]

**Sol.** $I = \int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$ ....(1)

$I = \int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$ ....(2)

Eq. (1) + (2)

$2I = \int_0^{2\pi} \frac{2\pi\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$

$I = 2\pi\int_0^{\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$

$I = 4\pi\int_0^{\pi/2} \frac{\sin^8 x}{\sin^8 x + \cos^8 x}\,dx$ ...(3)

$I = 4\pi\int_0^{\pi/2} \frac{\cos^8 x}{\cos^8 x + \sin^8 x}\,dx$ ...(4)

Eq. (3) + (4)

$2I = 4\pi\int_0^{\pi/2} dx = \pi^2$

**Q.8** The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to : (where C is a constant of integration)

(1) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7} + C$ (2) $-\left(\frac{x-3}{x+4}\right)^{-1/7} + C$ (3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7} + C$ (4) $\left(\frac{x-3}{x+4}\right)^{1/7} + C$

**Ans.** [4]

**Sol.** $\int \frac{1}{(x+4)^{8/7}(x-3)^{6/7}}\,dx$

$= \int \frac{dx}{(x+4)^2\left(\frac{x-3}{x+4}\right)^{6/7}}$

Put $\frac{x-3}{x+4} = t$

$$\left\{\frac{(x+4)-(x-3)}{(x+4)^2}\right\}dx = dt$$

$$= \frac{1}{7}\int \frac{dt}{t^{6/7}}$$

$$= \frac{1}{7}\left(\frac{t^{1/7}}{1/7}\right) + C$$

$$= \left(\frac{x-3}{x+4}\right)^{1/7} + C$$

**Q.9** Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than :

(1) $\frac{b+a}{b-a}$ (2) 1 (3) $\frac{c-a}{b-c}$ (4) $\frac{b-c}{c-a}$

**Ans.** **[3]**

**Sol.** Use LMVT for $x \in [a, c]$

$$\frac{f(c)-f(a)}{c-a} = f'(\alpha),\ \alpha \in (a, c)$$

also use LMVT for $x \in [c, b]$

$$\frac{f(b)-f(c)}{b-c} = f'(\beta);\ \beta \in (c, b)$$

given : $f''(x) < 0$

$\Rightarrow f'(x)$ is decreasing $f'(\alpha) < f'(\beta)$

$$\frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c} \qquad (\because f(x) \text{ is increasing})$$

**Q.10** The number of real roots of the equation $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

(1) 4 (2) 1 (3) 3 (4) 2

**Ans.** **[2]**

**Sol.** Let $e^x = t \in (0, \infty)$

given eq. $\Rightarrow t^4 + t^3 - 4t^2 + t + 1 = 0$

$$t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$$

Let $t + \frac{1}{t} = z$

$(z^2 - 2) + (z) - 4 = 0$

$z^2 + z - 6 = 0$

$z = -3, 2$

$\Rightarrow z = 2 \Rightarrow e^x + e^{-x} = 2$

x = 0 is only solution.

**Q.11** Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is :

(1) $\sqrt{10}$  (2) $2\sqrt{3}$  (3) $\frac{15}{4}$  (4) $\frac{7}{2}$

**Ans.** [4]

**Sol.** $|z - i| = |z + 2i|$ (let $z = x + iy$)

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y = -\frac{1}{2}$

again $|z| = \frac{5}{2}$

$x^2 + y^2 = \frac{25}{4}$

$\Rightarrow x^2 = 6$

$z = \pm\sqrt{6} - \frac{1}{2}i$

$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \frac{7}{2}$

**Q.12** The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is

(1) $\frac{1}{4}$  (2) $\frac{1}{2\sqrt{2}}$  (3) $\frac{1}{2}$  (4) $\frac{1}{\sqrt{2}}$

**Ans.** [2]

**Sol.** $\cos^3\left(\frac{\pi}{8}\right).\sin\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right).\cos\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)$

$= \cos^3\frac{\pi}{8}.\sin\frac{\pi}{8} + \sin^3\frac{\pi}{8}.\cos\frac{\pi}{8}$

$= \cos\frac{\pi}{8}.\sin\frac{\pi}{8}\left(\cos^2\frac{\pi}{8}.\sin^2\frac{\pi}{8}\right)$

$$= \frac{1}{2}\left(2\cos\frac{\pi}{8}.\sin\frac{\pi}{8}\right)$$

$$= \frac{1}{2}\sin\frac{\pi}{4}$$

$$= \frac{1}{2\sqrt{2}}$$

**Q.13** If $e_1$ and $e_2$ are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4} = 1$ respectively and $(e_1, e_2)$ is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to

(1) 16 (2) 14 (3) 15 (4) 17

**Ans.** **[1]**

**Sol.** $e_1 = \sqrt{1-\frac{4}{18}} = \frac{\sqrt{7}}{3}$

$e_2 = \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$

$(e_1, e_2)$ lies on $15x^2 + 3y^2 = k$

$\therefore\ 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right) = k$

$\Rightarrow\ k = 16$

**Q.14** If $f'(x) = \tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to

(1) $\frac{\pi-1}{4}$ (2) $\frac{1}{4}$ (3) $\frac{\pi+2}{4}$ (4) $\frac{\pi+1}{4}$

**Ans.** **[4]**

**Sol.** $f'(x) = \tan^{-1}(\sec x + \tan x);\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$f'(x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left\{\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right\}$$

$$f'(x) = \tan^{-1}\left\{\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right\}$$

$$f'(x) = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\} = \frac{\pi}{4}+\frac{x}{2}$$

integrating on both side

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$

given that f(0) = 0

$\Rightarrow c = 0$

So, $f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}$

**Q.15** Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \ldots. x_{10} - 3$, then the ordered pair (μ, λ) is equal to

(1) (3, 6) (2) (6, 3) (3) (3, 3) (4) (6, 6)

**Ans.** **[3]**

**Sol.** Mean of $(x_i - 5) = \frac{\Sigma(x_i - 5)}{10} = 1$

Now, mean of $(x_i - 3)$ = mean of $\{(x_i - 5) + 2\} = 1 + 2$

$\lambda = 3$

$\mu$ = variance of $(x_i - 5) = \frac{\Sigma(x_i - 5)^2}{10} - (\text{mean})^2$

$= \frac{40}{10} - (1)^2 = 3$

**Q.16** Negation of the statement : '$\sqrt{5}$ is an integer or 5 is irrational ' is

(1) $\sqrt{5}$ is not an integer or 5 is not irrational (2) $\sqrt{5}$ is irrational or 5 is an integer

(3) $\sqrt{5}$ is an integer and 5 is irrational (4) $\sqrt{5}$ is not an integer and 5 is not irrational

**Ans.** **[4]**

**Sol.** $\sqrt{5}$ is not an integer and 5 is not irrational

**Q.17** If the number of five digit numbers with distinct digits and 2 at the 10[th] place is 336 k, then k is equal to

(1) 7 (2) 4 (3) 6 (4) 8

**Ans.** **[4]**

**Sol.**

| | | | | |
|---|---|---|---|---|
| | | | 2 | |

$8 \times 8 \times 7 \quad \times 6 = 2688 = 336k$

$\Rightarrow k = 8$

**Q.18** If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)\,dx$ is equal to :

(1) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$  (2) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

(3) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$  (4) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

**Ans.** **[4]**

**Sol.** $\int_0^1 (a+bx+cx^2)\,dx = \left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)_0^1 = a+\frac{b}{2}+\frac{c}{3}$

$f(1) = a + b + c$

$f(0) = a$

$f\left(\frac{1}{2}\right) = a+\frac{b}{2}+\frac{c}{4}$

Now, $\frac{1}{6}\left[f(0)+f(1)+4f\left(\frac{1}{2}\right)\right] = \frac{1}{6}(6a + 3b + 2c) = a+\frac{b}{2}+\frac{c}{3}$

**Q.19** Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :

(1) (–9, –6)  (2) (9, 7)  (3) (–9, –7)  (4) (7, 6)

**Ans.** **[1]**

**Sol.** Centroid D $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) \equiv (2, 2)$

Point of intersection of lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$ is P $\left(-\frac{1}{5}, \frac{2}{5}\right)$

So, equation of line DP is $8x - 11y + 6 = 0$

Point (–9, –6) satisfy this equation.

**Q.20** The product $2^{\frac{1}{4}}\cdot 4^{\frac{1}{16}}\cdot 8^{\frac{1}{48}}\cdot 16^{\frac{1}{128}}$.... to ∞ is equal to :

(1) $2^{1/2}$  (2) 2  (3) 1  (4) $2^{1/4}$

**Ans.** **[1]**

**Sol.** $2^{\frac{1}{4}}\cdot 4^{\frac{1}{16}}\cdot 8^{\frac{1}{48}}\cdot 16^{\frac{1}{128}}$....

$= 2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+......\infty}$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+......\infty}$

$= 2^{\left(\frac{1/4}{1-(1/2)}\right)} = 2^{1/2}$

**Q.21** The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is _____

**Ans.** **[8]**

**Sol.** Let A (1, – 1, 3), B(2, – 4, 11), C (–1, 2, 3) & D (3, –2, 10)

$\overrightarrow{AB} = \hat{i} - 3\hat{j} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ $= \dfrac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|} = \dfrac{4+12+56}{\sqrt{16+16+49}} = \dfrac{72}{9} = 8$

**Q.22** If for $x \ge 0$, $y = y(x)$ is the solution of the differential equation, $(x + 1)dy = ((x + 1)^2 + y - 3)\,dx$, $y(2) = 0$, then $y(3)$ is equal to _____

**Ans.** **[3]**

**Sol.** $(1 + x)\dfrac{dy}{dx} - y = (1 + x)^2 - 3$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{1+x} = (1 + x) - \dfrac{3}{1+x}$

$\text{I.F.} = -e^{-\int \frac{1}{1+x}dx} = \dfrac{1}{1+x}$

$\therefore$ solution is $y\left(\dfrac{1}{1+x}\right) = \int\left((1+x) - \dfrac{3}{1+x}\right)\dfrac{1}{1+x}dx$

$\dfrac{y}{1+x} = x + \dfrac{3}{1+x} + c$

Put $x = 2$, $y = 0 \Rightarrow c = -3$

$\Rightarrow \dfrac{y}{1+x} = x + \dfrac{3}{1+x} - 3$

Put $x = 3 \Rightarrow \dfrac{y}{4} = 3 + \dfrac{3}{4} - 3 \Rightarrow y = 3$

**Q.23** The coefficient of $x^4$ in the expansion of $(1 + x + x^2)^{10}$ is _____

**Ans.** **[615]**

**Sol.** General term $= \dfrac{10!}{\alpha!\,\beta!\,\gamma!}x^{\beta+2\gamma}$

For coeff. of $x^4 \Rightarrow \beta + 2\gamma = 4 \qquad (\alpha + \beta + \gamma = n)$

$\gamma = 0, \beta = 4, \alpha = 6 \Rightarrow \dfrac{10!}{6!\,4!\,0!} = 210$

$\gamma = 1, \beta = 2, \alpha = 7 \Rightarrow \dfrac{10!}{7!\,2!\,1!} = 360$

$\gamma = 2, \beta = 0, \alpha = 8 \Rightarrow \dfrac{10\,!}{8!\,0!\,2!} = 45$

Total = 615

**Q.24** The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is _____

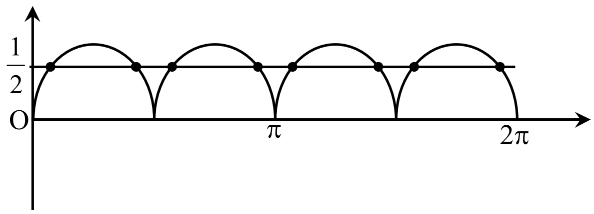
**Ans.** **[8]**

**Sol.** $\log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow \log_{1/2}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x||\cos x| = \left(\frac{1}{2}\right)^2$

$\Rightarrow |\sin 2x| = \frac{1}{2}$




8 solutions

**Q.25** If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$ , $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in R)$ are coplanar and $3(\vec{p}\cdot\vec{q})^2 - \lambda\left|\vec{r}\times\vec{q}\right|^2 = 0$, then the value of $\lambda$ is _____

**Ans.** **[1]**

**Sol.** $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$

$\Rightarrow (a + 1) + a + a = 0$

$\Rightarrow a = -\frac{1}{3}$

$\vec{p} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k}$

$\vec{q} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

$\vec{r} = -\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

$\vec{p}\cdot\vec{q} = \frac{1}{9}(-2-2+1) = -\frac{1}{3}$

$\vec{r}\times\vec{q} = \frac{1}{9}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$

$= \frac{1}{9}(3\hat{i} + 3\hat{j} + 3\hat{k}) = \frac{\hat{i}+\hat{j}+\hat{k}}{3}$

$|\vec{r}\times\vec{q}|^2 = \frac{1}{3}$

$\Rightarrow 3(\vec{p}\cdot\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$

$\Rightarrow 3\left(\frac{1}{9}\right) - \lambda\left(\frac{1}{3}\right) = 0$

$\Rightarrow \lambda = 1$