CAREER POINT

JEE Main Online Exam 2020

Questions & Answer

9th January 2020 | Shift - II

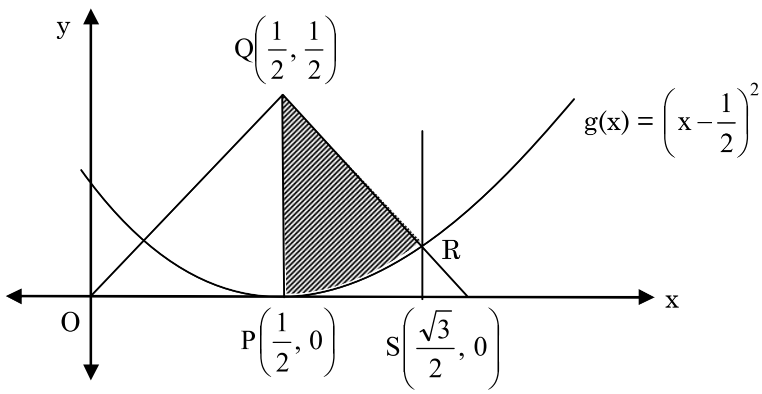
MATHEMATICS

Q.1 Given : $f(x) = \begin{cases} x & ; \quad 0 \le x < \frac{1}{2} \\ \frac{1}{2} & ; \quad x = \frac{1}{2} \\ 1 - x & ; \quad \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is -

(1) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (2) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (3) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (4) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

Ans. **[4]**

Sol. Co-ordinates of $P\left(\frac{1}{2}, 0\right)$, $Q\left(\frac{1}{2}, \frac{1}{2}\right)$, $R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$ and $S\left(\frac{\sqrt{3}}{2}, 0\right)$



Required area = Area of trapezium PQRS $- \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left[\left(x - \frac{1}{2}\right)^3\right]_{1/2}^{\sqrt{3}/2}$$

$$= \frac{1}{2}\left(\frac{\sqrt{3} - 1}{2}\right)\left(\frac{3 - \sqrt{3}}{2}\right) - \frac{1}{3}\left[\left(\frac{\sqrt{3} - 1}{2}\right)^3 - 0\right]$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

Q.2 Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to -

(1) 26 (2) 24 (3) 25 (4) 28

Ans. **[3]**

Sol. Roots of equation $ax^2 - 2bx + 5 = 0$ are α, α

$\therefore \alpha + \alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$ ...(i)

and $\alpha^2 = \frac{5}{a}$ ...(ii)

from eq. (i) & (ii)

$\frac{b^2}{a^2} = \frac{5}{a} \Rightarrow b^2 = 5a$ ...(iii) $(a \neq 0)$

Roots of equation $x^2 - 2bx - 10 = 0$ are α, β

$\therefore$ $\alpha + \beta = 2b$

and $\alpha\beta = -10$

Now, $\alpha = \frac{b}{a}$ is root of equation $x^2 - 2bx - 10 = 0$

$\therefore \frac{b^2}{a^2} - \frac{2b^2}{a} - 10 = 0$

$\Rightarrow \frac{5}{a} - 10 - 10 = 0$ $(\because b^2 = 5a)$

$\Rightarrow a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$

So, $\alpha^2 = \frac{b^2}{a^2} = 20$ and $\beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

Q.3 If A = {x $\in$ R : |x| < 2} and B = {x $\in$ R : |x – 2| $\geq$ 3}; then -

(1) $A \cup B = R - (2, 5)$ (2) $A \cap B = (-2, -1)$ (3) $B - A = R - (-2, 5)$ (4) $A - B = [-1, 2)$

Ans. **[3]**

Sol. A = {x $\in$ R : |x| < 2}

$A \in (-2, 2)$

and B = {x $\in$ R : |x – 2| $\geq$ 3}

$B \in (-\infty, -1] \cup [5, \infty)$

$A \cup B \in (-\infty, 2) \cup [5, \infty)$

$A \cap B \in (-2, -1]$

$B - A \in (-\infty, -2] \cup [5, \infty)$ or $R - (-2, 5)$

$A - B \in (-1, 2)$

Q.4 Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to-

(1) 175 (2) 225 (3) 300 (4) 150

Ans. [4]

Sol. Let the G.P. is a, ar, ar^2 ....

$$\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + ... + a_{201} = 200$$

$$\Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \quad ...(1)$$

$$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + ... + a_{200} = 100$$

$$\Rightarrow \frac{ar(r^{200}-1)}{r^2-1} = 100 \quad ...(2)$$

dividing (1) by (2)

we get, r = 2

adding both eq. (1) & (2)

$$\Rightarrow a_2 + a_3 + a_4 + a_5 + ... + a_{201} = 300$$

$$\Rightarrow r(a_1 + a_2 + ... + a_{200}) = 300$$

$$\Rightarrow a_1 + a_2 + ... + a_{200} = \frac{300}{r}$$

$$\Rightarrow \sum_{n=1}^{200} a_1 = \frac{300}{2} = 150$$

Q.5 If z be a complex number satisfying |Re(z)| + |Im(z)| = 4, then |z| cannot be -

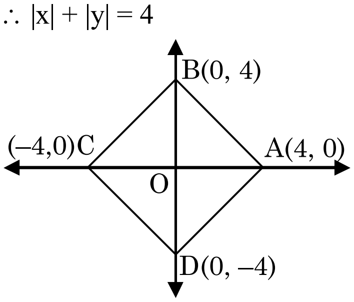
(1) $\sqrt{10}$ (2) $\sqrt{8}$ (3) $\sqrt{7}$ (4) $\sqrt{\frac{17}{2}}$

Ans. [3]

Sol. Let z = x + iy

given that |Re(z)| + |Im(z)| = 4

$\therefore$ |x| + |y| = 4



Maximum value of $|z| = 4$

Minimum value of $|z|$ = perpendicular distance of line AB from (0, 0)

$= 2\sqrt{2}$

So, $|z| \in [2\sqrt{2}, 4]$

So, $|z|$ can not be $\sqrt{7}$

Q.6 If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then -

(1) $y(1 - x) = 1$ (2) $x(1 - y) = 1$ (3) $y(1 + x) = 1$ (4) $x(1 + y) = 1$

Ans. **[1]**

Sol. $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$

$\Rightarrow x = 1 - \tan^2\theta + \tan^4\theta - \tan^6\theta$ ......

$\Rightarrow x = \frac{1}{1+\tan^2\theta} = \cos^2\theta$ ...(1)

and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$y = 1 + \cos^2\theta + \cos^4\theta + \cos^6\theta + ...$

$y = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

$\Rightarrow \sin^2\theta = \frac{1}{y}$ ....(2)

Adding (1) & (2), we get,

$x + \frac{1}{y} = 1$

$\Rightarrow$ $y(1 - x) = 1$

Q.7 Let [t] denote the greatest integer $\le$ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2] \sin(\pi x)$ is discontinuous, when x is equal to -

(1) $\sqrt{A+1}$ (2) $\sqrt{A}$ (3) $\sqrt{A+21}$ (4) $\sqrt{A+5}$

Ans. **[1]**

Sol. $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$

$\Rightarrow \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow \lim_{x\to 0}\left(4 - x\left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow A = 4$

Now, $f(x) = [x^2] \sin(\pi x)$ is continuous at every integer point but discontinuous at non integer points then by options, $\sqrt{A+1}$ is correct answer.

Q.8 If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively -

(1) F, F (2) T, F (3) T, T (4) F, T

Ans. [3]

Sol. $p \to (p \wedge \sim q)$ will be false only when p is true and $(p \wedge \sim q)$ is false.

So, p = T, q = T

Q.9 Let a function $f : [0, 5] \to R$ be continuous, $f(1) = 3$ and F be defined as $F(x) = \int_1^x t^2 g(t)\,dt$, where $g(t) = \int_1^t f(u)\,du$. Then for the function F, the point x = 1 is

(1) not a critical point (2) a point of local maxima

(3) a point of local minima (4) a point of inflection

Ans. [3]

Sol. $F(x) = \int_1^x t^2 g(t)\,dt$

$\Rightarrow F'(x) = x^2 g(x)$

at $F'(1) = (1)\,g(1) = 0$ $\{\because g(1) = 0\}$

Now, $F''(x) = 2xg(x) + x^2 g'(x)$

$F''(1) = 2g(1) + g'(1)$

$F''(1) = 0 + g'(1)$ $\{g'(t) = f(t);\ \ g'(1) = f'(1) = 3\}$

$F''(1) = 3$

So, at x = 1, $F'(1) = 0$ but $F''(1) > 0$

$\therefore$ For the function f(x), x = 1 is a point of local minima.

Q.10 A random variable X has the following probability distribution :

X :	1	2	3	4	5
P(X) :	K^2	2K	K	2K	$5K^2$

Then P(X > 2) is equal to -

(1) $\frac{1}{36}$ (2) $\frac{1}{6}$ (3) $\frac{7}{12}$ (4) $\frac{23}{36}$

Ans. [4]

Sol. $\sum_{i=1}^{5} P(X) = 1$

$\Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0$

$\Rightarrow K = \frac{1}{6},\ K = -1$ (rejected)

So, $K = \frac{1}{6}$

$P(X > 2) = K + 2K + 5K^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36}$

$= \frac{23}{36}$

Q.11 If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e |f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to -

(1) $(1, 1 - \tan\theta)$ (2) $(-1, 1 - \tan\theta)$ (3) $(-1, 1 + \tan\theta)$ (4) $(1, 1 + \tan\theta)$

Ans. **[3]**

Sol. $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$= \int \frac{d\theta}{\cos^2\theta\left(\frac{2\tan\theta}{1-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}\right)}$

$= \int \frac{\sec^2\theta\,(1-\tan^2\theta)}{(1+\tan\theta)^2}\,d\theta$

Put $\tan\theta = t$

$\Rightarrow \sec^2\theta\,d\theta = dt$

$= \int \frac{(1-t^2)}{(1+t)^2}\,dt = \int\left(\frac{1-t}{1+t}\right)dt$

$= \int\left(-1 + \frac{2}{1+t}\right)dt$

$= -t + 2\log_e |1 + t| + C$

$= -\tan\theta + 2\log_e |1 + \tan\theta| + C$

$\therefore\ \lambda = -1$ and $f(\theta) = 1 + \tan\theta$

Q.12 The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$, then its eccentricity is -

(1) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (2) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (3) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (4) $\sqrt{\frac{5}{6}}$

Ans. **[1]**

Sol. Let the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$

Given that $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $y = mx \pm \sqrt{a^2m^2 + b^2}$ ....(1)

Given tangent is $x + 6y = 8$

$\Rightarrow \quad y = -\frac{1}{6}x + \frac{8}{6}$ ...(2)

from eq. (1) & (2)

$m = -\frac{1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \quad a^2\left(\frac{1}{36}\right) + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \quad a^2 = 16$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4/3}{16}} = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}$

Q.13 In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to -

(1) 16 : 1 (2) 1 : 8 (3) 8 : 1 (4) 1 : 16

Ans. **[1]**

Sol. General term $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

$T_{r+1} = {}^{16}C_r\left(\frac{1}{\cos\theta}\right)^{16-r}\left(\frac{1}{\sin\theta}\right)^r x^{16-2r}$

term is independent of x

$\therefore\ 16 - 2r = 0$

$\Rightarrow r = 8$

$T_9 = {}^{16}C_8\left(\frac{1}{\cos\theta}\right)^8\left(\frac{1}{\sin\theta}\right)^8$

$T_9 = {}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$

$\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right] \Rightarrow 2\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

For least value $\Rightarrow$ sin2θ should be maximum

$\therefore\ 2\theta = \frac{\pi}{2}$

So, $\ell_1 = {}^{16}C_8(2^8)$ ...(1)

Again, $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right] \Rightarrow 2\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$

Minimum value, $\ell_2 = {}^{16}C_8 \dfrac{2^8}{(1/\sqrt{2})^8} = {}^{16}C_8 2^{12}$ ...(2)

$$\frac{\ell_2}{\ell_1} = \frac{2^4}{1} = \frac{16}{1}$$

Q.14 If x = 2 sinθ – sin2θ and y = 2cosθ – cos2θ, θ ∈ [0, 2π], then $\dfrac{d^2y}{dx^2}$ at θ = π is -

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $-\frac{3}{4}$ (4) $-\frac{3}{8}$

Ans. **[Bonus]**

Sol. x = 2 sinθ – sin2θ

$$\Rightarrow \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

and y = 2 cosθ – cos 2θ

$$\Rightarrow \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\text{Now, } \frac{dy}{dx} = \frac{2(\sin 2\theta - \sin\theta)}{(\cos\theta - \cos 2\theta)} = \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}.\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot\frac{3\theta}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\,\text{cosec}^2\frac{3\theta}{2}\left(\frac{3}{2}\right)\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{3}{2}\left(\text{cosec}^2\frac{3\theta}{2}\right)\left(\frac{1}{2\cos\theta - 2\cos 2\theta}\right)$$

at θ = π

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2}\right)(1)\left(\frac{1}{-2-2}\right) = \frac{3}{8}$$

Q.15 If $\dfrac{dy}{dx} = \dfrac{xy}{x^2+y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is -

(1) $\frac{1}{2}\sqrt{3}\,e$ (2) $\sqrt{2}\,e$ (3) $\frac{e}{\sqrt{2}}$ (4) $\sqrt{3}\,e$

Ans. **[4]**

Sol. $\dfrac{dy}{dx} = \dfrac{xy}{x^2+y^2}$

Put y = vx

then $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x(vx)}{x^2 + v^2x^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$\Rightarrow \int \frac{1+v^2}{v^3}\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2v^2} + \log v = -\log x + C$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \log\left(\frac{y}{x}\right) = -\log x + C \qquad ...(1)$$

put x = 1, y = 1

$$\Rightarrow C = -\frac{1}{2}$$

from eq. (1)

$$-\frac{1}{2}\frac{x^2}{y^2} + \log\left(\frac{y}{x}\right) = -\log x - \frac{1}{2}$$

Put $y = e \Rightarrow -\frac{x^2}{2e^2} + \log\left(\frac{e}{x}\right) + \log x = -\frac{1}{2}$

$$\Rightarrow x^2 = 3e^2$$

$$\Rightarrow x = \pm\sqrt{3}\,e$$

$$\Rightarrow x = \sqrt{3}\,e$$

Q.16 Let a – 2b + c = 1. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then -

(1) $f(-50) = -1$ (2) $f(50) = 1$ (3) $f(50) = -501$ (4) $f(-50) = 501$

Ans. **[2]**

Sol. $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$R_1 \to R_1 - 2R_2 + R_3$

$$f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

f(x) = 1

$\therefore$ f(50) = 1

Q.17 If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is -

(1) $\frac{945}{2^{10}}$ (2) $\frac{965}{2^{11}}$ (3) $\frac{945}{2^{11}}$ (4) $\frac{965}{2^{10}}$

Ans. [1]

Sol. Total ways $\Rightarrow n = 4^{10}$

Number of ways placing exactly 2 and 3 balls in two of these boxes

$= {}^4C_2 \times \frac{5!}{2!\ 3!} \times 2! \times {}^{10}C_5 \times 2^5$

Required probability $= \frac{945}{2^{10}}$

Q.18 The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) no solution

(2) infinitely many solutions, (x, y, z) satisfying y = 2z

(3) only the trivial solution

(4) infinitely many solutions, (x, y, z) satisfying x = 2z

Ans. [4]

Sol. $7x + 6y - 2z = 0$ ....(1)

$3x + 4y + 2z = 0$ ....(2)

$x - 2y - 6z = 0$ ....(3)

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$\Delta = 7(-24 + 4) - 6(-18 - 2) - 2(-6 - 4) = 0$

$\Delta = 0$

$\therefore$ infinite non-trival solution exist

to eliminate y we operate eq. (1) – (2) + (3)

$5x = 10z$

$x = 2z$

Q.19 If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is -

(1) $2x + y - 24 = 0$ (2) $x + 2y + 8 = 0$ (3) $x - 2y + 8 = 0$ (4) $2x - y - 24 = 0$

Ans. [3]

Sol. $y^2 = 8x$ (a = 2)

Let one end of focal chord is $(at^2, 2at) = \left(\frac{1}{2}, -2\right)$

$\Rightarrow 2at = -2$

$\Rightarrow t = -1/2$

other end of focal chord will be $\left(\frac{a}{t^2}, -\frac{2a}{t}\right) \equiv (8, 8)$

Now, tangent at B(8, 8)

$\Rightarrow y(8) = 8\left(\frac{x+8}{2}\right)$

$\Rightarrow x - 2y + 8 = 0$

Q.20 Let f and g be differentiable functions on R such that fog is the identity function. If for some a, b $\in$ R, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to -

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) 5 (4) 1

Ans. **[1]**

Sol. fog is an identity function

$\therefore$ fog(x) = x

$\Rightarrow f'\{g(x)\}\, g'(x) = 1$

put x = a

$\Rightarrow f'\{g(a)\}\, g'(a) = 1$

$\Rightarrow f'(b)\,(5) = 1$

$\Rightarrow f'(b) = \frac{1}{5}$

Q.21 If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ ($\lambda \in$ R) is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ____________.

Ans. **[3]**

Sol. Required distance = perpendicular distance of plane $23x - 10y - 2z + 48 = 0$ either from (–1, 3, –1) or (–3, –2, 1)

$$\Rightarrow \left|\frac{-23-30+2+48}{\sqrt{(23)^2+(10)^2+(2)^2}}\right| = \frac{k}{\sqrt{633}}$$

$$\Rightarrow \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}}$$

$\therefore$ k = 3

Q.22 If $C_r = {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \ldots + (101) \cdot C_{25} = 2^{25} \cdot k$, then k is equal to ___________.

Ans. **[51]**

Sol. $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \ldots + (101) \cdot C_{25}$

$$= \sum_{r=0}^{25} (4r+1)\, {}^{25}C_r = 4\sum_{r=0}^{25} r.\, {}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$$

$= 4\sum_{r=0}^{25} r.\frac{25}{r}\,{}^{24}C_{r-1} + 2^{25}$

$= 100 \,.\, 2^{24} + 2^{25}$

$= 2^{25}(50 + 1)$

$\therefore$ k = 51

Q.23 The number of terms common to the two A.P.'s 3, 7, 11, ….. 407 and 2, 9, 16, …. 709 is ___________.

Ans. **[14]**

Sol. First A.P. is 3, 7, 11, 15, 19, 23, ... 407

Second A.P. is 2, 9, 16, 23, ... 709

First common term = 23

Common difference d = L.C.M. (4, 7) = 28

Last term $\le$ 407

$\Rightarrow 23 + (n-1)(28) \le 407$

$\Rightarrow n \le 14.7$

So, n = 14

Q.24 If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ______________.

Ans. **[36]**

Sol. $C_1 = (3, 0)$, $C_2(0, 4)$

$r_1 = \sqrt{9+0-8} = 1$; $r_2 = \sqrt{16-16+k} = \sqrt{k}$

Two circles touch each other

$\therefore C_1C_2 = |r_1 \pm r_2|$

$5 = |1 \pm \sqrt{k}\,|$

$\Rightarrow 1 + \sqrt{k} = 5$ or $\sqrt{k} - 1 = 5$

$\Rightarrow$ k = 16 or k = 36

Maximum value of k = 36

Q.25 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b}\cdot\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to _______.

Ans. **[30]**

Sol. $|\vec{a}\times(\vec{b}\times\vec{c})| = |\vec{a}||\vec{b}\times\vec{c}|\sin\frac{\pi}{2}$

$= |\vec{a}||\vec{b}||\vec{c}|\sin\frac{\pi}{3}$

$$= (\sqrt{3})\,(5)\,|\vec{c}|.\frac{\sqrt{3}}{2}$$

$$= \frac{15}{2}|\vec{c}| \qquad ...(1)$$

$$\Rightarrow \cos\frac{\pi}{3} = \frac{\vec{b}\,.\,\vec{c}}{|\vec{b}||\vec{c}|}$$

$$\Rightarrow \frac{1}{2} = \frac{10}{5|\vec{c}|} \Rightarrow |\vec{c}| = 4$$

from eq. (1)

$$|\vec{a}\times(\vec{b}\times\vec{c})| = \frac{15}{2}\times 4 = 30$$