CAREER POINT

JEE Main Online Exam 2019

Questions & Answer

7th January 2019 | Shift - II

MATHS

Q.1 The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, where k is an integer, is:

(1) 6 (2) 3 (3) 2 (4) 4

Ans. **[4]**

Sol. $6.{}^{35}C_r = (k^2 - 3)\ {}^{36}C_{r+1}$

$6.{}^{35}C_r = (k^2 - 3).\ \frac{36}{r+1}\ {}^{35}C_r.$

$\Rightarrow k^2 - 3 = \frac{r+1}{6}$

$k \in I \Rightarrow$ r can be

(1) $r = 5 \Rightarrow k = \pm 2$

(2) $r = 35 \Rightarrow k = \pm 3$

4 ordered pairs (5, 2), (5, –2), (35, 3), (35, 3)

Q.2 The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|}\ dx = 5$, is:

(1) $\log_e \sqrt{2}$ (2) $\log_e\left(\frac{3}{2}\right)$ (3) $\log_e\left(\frac{4}{3}\right)$ (4) $\log_e 2$

Ans. **[4]**

Sol. $4\alpha \int_{-1}^{2} e^{-\alpha|x|}\ dx = 5$

$4\alpha \int_{-1}^{0} e^{\alpha x} dx + \int_{0}^{2} e^{-\alpha x} dx\ = 5$

$\Rightarrow 4\alpha \left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + 4\alpha\left(\frac{e^{-\alpha x}}{-\alpha}\right)_{0}^{2} = 5$

$\Rightarrow 4\alpha\left(\frac{1-e^{-\alpha}}{\alpha} - \frac{e^{-2\alpha}-1}{-\alpha}\right) = 5$

Let $e^{-\alpha} = t$

$\therefore\ 4t^2 + 4t - 3 = 0$

$\Rightarrow t = \frac{1}{2}$

$e^{-\alpha} = \frac{1}{2}$

$\alpha = \ell n\ 2$

Q.3 Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim\limits_{x\to 0}\left(2+\dfrac{f(x)}{x^3}\right) = 4$, then which one of the following is not ture?

(1) f is an odd function

(2) $f(1) - 4f(-1) = 4$

(3) x = 1 is a point of minima and x = –1 Is a point of maxima of f

(4) x = 1 is a point of maxima and x = –1 Is a point of minimum of f

Ans. **[4]**

Sol. $f'(x) = 0$ at $x = 1, -1$, also 0 is a repeated root.

$\therefore f'(x) = a(x + 1)(x - 1)x^2$

$= a(x^2 - 1)x^2$

$f'(x) = ax^4 - x^2$

$f(x) = \dfrac{ax^5}{5} - \dfrac{ax^3}{3} + C$

$\because f(0) = 0 \Rightarrow c = 0$

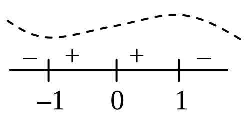
$\lim\limits_{x\to 0}\dfrac{f(x)}{x^3} = 2$

$\Rightarrow 0 - \dfrac{a}{3} = 2$

$\Rightarrow a = -6$

$\therefore f(x) = -\dfrac{6}{5}x^5 + 2x^3$

$\therefore f'(x) = -6(x^2 - 1)(x^2)$



Minima at x = –1

Maxima at x = 1

Q.4 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to:

(1) $\left(-\dfrac{3}{2}, 3\vec{c}\times\vec{b}\right)$ (2) $\left(-\dfrac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (3) $\left(\dfrac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (4) $\left(\dfrac{3}{2}, 3\vec{a}\times\vec{c}\right)$

Ans. **[2]**

Sol. $\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = 0$

$3 + 2\,(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -\dfrac{3}{2}$

$\lambda = -\dfrac{3}{2}$

$\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$= \vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+(-\vec{a}-\vec{b})\times\vec{a}$

$\vec{d} = 3(\vec{a}\times\vec{b})$

Q.5 Let y = y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$.

Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{\sqrt{5}}{2}$ (3) $-\frac{\sqrt{5}}{4}$ (4) $-\frac{\sqrt{5}}{2}$

Ans. **[4]**

Sol. $y.\sqrt{1-x^2} = k - x.\sqrt{1-y^2}$

On differentiating

$y.\frac{-2x}{2\sqrt{1-x^2}} + \sqrt{1-x^2}.y' = +\frac{x.2yy'}{2\sqrt{1-y^2}} - \sqrt{1-y^2}$

Put $x = \frac{1}{2}$, $y = -\frac{1}{4}$ & $x.y = -\frac{1}{8}$

On solving we get

$y' = \frac{-\sqrt{5}}{2}$

Q.6 The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$ is:

(1) 210 (2) 120 (3) 330 (4) 420

Ans. **[3]**

Sol. $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$

$(1 + x)^{10}\frac{\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$

$\Rightarrow (1 + x)^{11} - x^{11}$

Coefficient of x^7 is ${}^{11}C_7 = 330$

Q.7 Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$, then which one of the following statements is not true?

(1) $p_5 = p_2 . p_3$ (2) $p_3 = p_5 - p_4$

(3) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$ (4) $p_5 = 11$

Ans. **[1]**

Sol. $p_5 = \alpha^5 + \beta^5$

$\because \alpha^2 = \alpha + 1$

$= (\alpha + 1)^2.\alpha + (\beta + 1)^2.\beta$

$= (\alpha^2 + 2\alpha + 1)\alpha + (\beta^2 + 2\beta + 1)\beta$

$= (3\alpha + 2)\alpha + (3\beta + 2)\beta$

$= 3\alpha^2 + 2\alpha + 3\beta^2 + 2\beta$

$= 5\alpha + 5\beta + 6$ $[\because \alpha + \beta = 1]$

$= 5(1) + 6 = 11$

$p_2 = \alpha^2 + \beta^2 = \alpha + \beta + 2 = 3$

$p_3 = \alpha^3 + \beta^3 = (\alpha + 1).\alpha + (\beta + 1).\beta$

$= \alpha^2 + \beta^2 + \alpha + \beta$

$= \alpha + \beta + 3$

$= 1 + 3 = 4$

Hence $p_5 \ne p_2.p_3$

Q.8 If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 +...... is (102)m, then m is equal to:

(1) 20 (2) 25 (3) 10 (4) 5

Ans. **[1]**

Sol. 3 + 4 + 8 + 9 + 13 + 14 +.......upto 40 terms

$\Rightarrow$ 7 + 17 + 27 +.......20 terms

$S = \frac{20}{2}[2 \times 7 + 19 \times 10]$

$= 102 \times 20 = 102$ m

$\therefore$ m = 20

Q.9 The area (in sq. units) of the region $\{(x, y) \in R^2 \,|4x^2 \le y \le 8x + 12\}$ is:

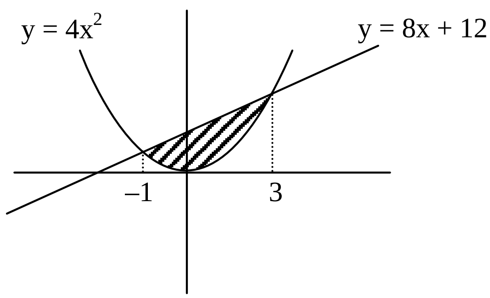
(1) $\frac{128}{3}$ (2) $\frac{127}{3}$ (3) $\frac{124}{3}$ (4) $\frac{125}{3}$

Ans. **[1]**

Sol. For point of intersection

$4x^2 = 8x + 12$

$x = -1, 3$



$\text{Area} = \int_{-1}^{3}[(8x+12)-4x^2]\,dx$

$= \left[\frac{8x^2}{2}+12x-\frac{4x^3}{3}\right]_{-1}^{3}$

$= (36 + 36 - 36) - (4 - 12 + \frac{4}{3})$

$= \frac{128}{3}$

Q.10 Let a_1, a_2, a_3, ... be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:

(1) $\frac{511}{3}$ (2) –171 (3) –513 (4) 171

Ans. **[2]**

Sol. $a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4$ ....(i)

$a_3 + a_4 = 16 \Rightarrow a_1r^2 + a_1r^3 = 16$ ....(ii)

equation (i) ÷ (ii)

$\Rightarrow \frac{1}{r^2} = \frac{1}{4}$

$\Rightarrow r = \pm 2$

$r = 2 \Rightarrow a_1 = \frac{4}{3}$

$r = -2 \Rightarrow a_1 = -4$

$\sum_{i=1}^{9} a_i = \frac{a(r^9 - 1)}{(r-1)} = (-4)\frac{((-2)^9 - 1)}{(-2-1)}$

$= \frac{4}{3}(-513) = 4\lambda$

$\Rightarrow \lambda = -171$

Q.11 If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to:

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{9}$ (3) $\frac{\pi}{3} + \frac{1}{6}$ (4) $\frac{2\pi}{3}$

Ans. **[1]**

Sol. $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$

$2\frac{\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$

$2\sin^2\theta - 5\sin\theta + 2 = 0$

$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$ only

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\downarrow \quad \downarrow$

$\theta_1 \quad \theta_2$

$\int_{\pi/6}^{5\pi/6} \cos^2 3\theta \, d\theta = \int_{\pi/6}^{5\pi/6} \left(\frac{1 + \cos 6\theta}{2}\right) d\theta$

$= \frac{1}{2}\left(\theta + \frac{\sin 6\theta}{6}\right)_{\pi/6}^{5\pi/6}$

$= \frac{\pi}{3}$

Q.12 If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is:

(1) 4 (2) $2\sqrt{2}$ (3) $2\sqrt{5}$ (4) $2\sqrt{7}$

Ans. **[4]**

Sol. $3x + 4y = 12\sqrt{2}$

$y = -\frac{3x}{4} + 3\sqrt{2}$

line is tangent to ellipse

$\therefore\ c^2 = a^2m^2 + b^2$

$(3\sqrt{2})^2 = a^2\left(-\frac{3}{4}\right)^2 + 9$

$18 = \frac{9a^2}{16} + 9$

$2 = \frac{a^2}{16} + 1$

$a^2 = 16$

$\therefore\ e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{9}{16}$

$e = \frac{\sqrt{7}}{4}$

Distance between focii = 2ae

$= 2 \times 4 \times \frac{\sqrt{7}}{4}$

$= 2\sqrt{7}$

Q.13 Let A = $[a_{ij}]$ and B = $[b_{ij}]$ be two 3 × 3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is:

(1) 3 (2) $\frac{1}{9}$ (3) $\frac{1}{81}$ (4) $\frac{1}{3}$

Ans. **[2]**

Sol. $|B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0a_{11} & 3^1a_{21} & 3^2a_{31} \\ 3^1a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{vmatrix}$

$81 = 3^3 . 3^3 . 3^2\ |A|$

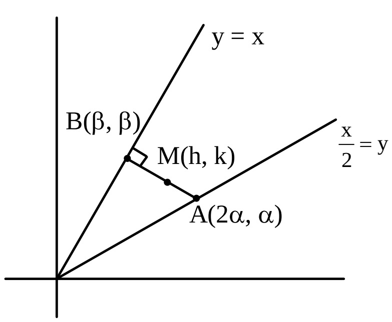
$\Rightarrow |A| = \frac{1}{9}$

Q.14 The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is:

(1) 2x – 3y = 0 (2) 5x – 7y = 0 (3) 7x – 5y = 0 (4) 3x – 2y = 0

Ans. **[2]**

Sol.



Slope AB = $\frac{k-\alpha}{h-2\alpha} = -1$

$\Rightarrow \alpha = \frac{k+k}{3}$ ...... (1)

also $\frac{\beta+2\alpha}{2} = h$, $\frac{\beta+\alpha}{2} = k$

$\alpha = 2h - 2k$ ...... (2)

from (1) & (2)

$\frac{h+k}{3} = 2h - 2k$

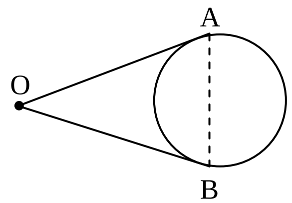
$\Rightarrow 5h = 7k$

$\Rightarrow 5x = 7y$

Q.15 Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to:

(1) $\frac{56}{5}$ (2) $\frac{64}{5}$ (3) $\frac{32}{5}$ (4) $\frac{52}{5}$

Ans. [2]



OA = $\sqrt{S_1}$

$\ell = \sqrt{16} = 4$

Radius = R = $\sqrt{16+4-16} = 2$

Length of AB = $\frac{2RL}{\sqrt{L^2+R^2}} = \frac{2\times4\times2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}}$

$AB^2 = \frac{16\times16}{20} = \frac{64}{5}$

Q.16 The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is:

(1) $\frac{4-\sqrt{5}}{3}$ (2) $\frac{2}{3}$ (3) $\frac{\sqrt{7}-2}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

Ans. [4]

$f(x) = x^3 - 4x^2 + 8x + 11,$ $x \in [0, 1]$

Using LMVT

$f'(c) = \frac{f(1)-f(0)}{1-0}$

$3c^2 - 8c + 8 = \frac{16-11}{1-0}$

$3c^2 - 8c + 3 = 0$

$c = \frac{4-\sqrt{7}}{3} \in (0, 1)$

Q.17 Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is:

(1) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
(2) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$
(3) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
(4) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

Ans. **[3]**

Sol. If $A \subseteq B$ and $B \subseteq D$ then $A \subseteq C$

Contrapositive is

If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

Q.18 If $\frac{3 + i\sin\theta}{4 - i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

(1) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
(2) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
(3) $\tan^{-1}\left(\frac{4}{3}\right)$
(4) $-\tan^{-1}\left(\frac{3}{4}\right)$

Ans. **[2]**

Let $z = \frac{3 + i\sin\theta}{4 - i\cos\theta} \times \frac{(4 + i\cos\theta)}{(4 + i\cos\theta)}$

$= \frac{12 - \sin\theta\cos\theta + i(4\sin\theta + 3\cos\theta)}{16 + \cos^2\theta}$

z is real

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\Rightarrow \tan\theta = \frac{-3}{4}$ [$\because$ θ lies is 2^{nd} quadrant]

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \pi - \tan^{-1}\left(\frac{4}{3}\right)$

Q.19 In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3$ k, then k is equal to:

(1) 4
(2) $\frac{17}{4}$
(3) $\frac{17}{8}$
(4) $\frac{17}{2}$

Ans. **[3]**

Prob. (at most two machines will be out of service) = $\left(\frac{3}{4}\right)^3.k$

$\Rightarrow {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3.k$

$\Rightarrow \frac{17}{8}.\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3.k$

$\Rightarrow k = \frac{17}{8}$

Q.20 Let y = y(x) be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is:

(1) 2 (2) 2 + e (3) 2 – e (4) –e

Ans. **[3]**

$\frac{dx}{dy} = y^2 - x$

$\frac{dy}{dx} + x = y^2$

I.F. = $e^{\int 1dy} = e^y$

$\Rightarrow x.e^y = \int y^2.e^y dy$

$x.e^y = y^2.e^y - \int 2y.e^y dy$

$xe^y = y^2e^y - 2ye^y + 2e^y + C$

$\because$ y(0) = 1

$\Rightarrow$ C = – e

$\therefore xe^y = y^2e^y - 2ye^y + 2e^y - e$

put y = 0

$\therefore$ x = 0 – 0 + 2 – e

$\Rightarrow$ x = 2 – e

Q.21 Let X = {n ∈ N : 1 ≤ n ≤ 50}. If A = {n ∈ X : n is a multiple of 2} and B = {n ∈ X : n is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is__________.

Ans. **[29]**

Sol. $A = \{2, 4, 6, 8..........50\} \Rightarrow$ 25 elements

$A = \{7, 14, 21, ...49\} \Rightarrow$ 7 elements

$A \cap B = \{14, 28, 42\}$ = 3 elements

Required number of elements = 25 + 7 – 3 = 29

Q.22 If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to_________.

Ans. **[54]**

Sol. Means = 10 = $\frac{3+7+9+12+13+20+x+y}{8}$

16 = x + y ___(1)

Variance $\sigma^2 = 25 = \frac{\sum x_i^2}{8} - (\text{mean})^2$

$25 = \frac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} - 100$

$125 \times 8 = 9 + 49 + 81 + 144 + 169 + 400 + x^2 + y^2$

$x^2 + y^2 = 148$ ___(2)

$(x + y)^2 = x^2 + y^2 + 2xy$

256 = 148 + 2xy

x.y = 54

Q.23 If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by f(x) = $\begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous, then k is equal to________.

Ans. **[5]**

Sol. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0}\left(\frac{1}{x}\ell n(1+3x) - \frac{1}{x}\ell n(1-2x)\right)$

$= \lim\limits_{x\to 0}\left(\frac{3\ell n(1+3x)}{3x} - \frac{2\ell n(1-2x)}{-2x}\right)$

= 3 + 2 = 5

f is continuous $\therefore \lim\limits_{x\to 0} f(x) = f(0)$

$\therefore$ f(0) = 5 = k

Q.24 If the system of linear equations,

x + y + z = 6

x + 2y + 3z = 10

3x + 2y + λz = μ

has more than two solutions, then $\mu - \lambda^2$ is equal to_________.

Ans. **[]**

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 9) + 1(-4) = 0$

$\Rightarrow 2\lambda - 6 - \lambda + 9 - 4 = 0$

$\Rightarrow \lambda = 1$

$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow 6(2\lambda - 6) - 1(10\lambda - 3\mu) + 1(20 - 2\mu) = 0$

$\Rightarrow 12\lambda - 36 - 10\lambda + 3\mu + 20 - 2\mu = 0$

$\Rightarrow 2\lambda + \mu = 16$

$\Rightarrow 2 + \mu = 16$

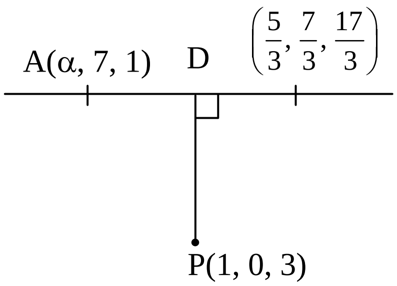
$\Rightarrow \mu = 14$

$\mu - \lambda^2 = 14 - 1 = 13$

Q.25 If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to________.

Ans. **[4]**

Sol. $\Delta \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$



$\overrightarrow{AD}.\overrightarrow{PD} = 0$

$\left(\left(\frac{5}{3}-\alpha\right)\hat{i}+\left(\frac{7}{3}-7\right)\hat{j}+\left(\frac{17}{3}-1\right)\hat{k}\right).\left(\frac{2}{3}\hat{i}+\frac{7}{3}\hat{j}+\frac{8}{3}\hat{k}\right) = 0$

$\left(\frac{5}{3}-\alpha\right)\frac{2}{3}+\frac{7}{3}\times\left(\frac{-14}{3}\right)+\frac{14}{3}\times\frac{8}{3} = 0$

$2(5 - 3\alpha) - 14 \times 7 + 14 \times 8 = 0$

$5 - 3\alpha - 49 + 56 = 0$

$3\alpha = 12$

$\alpha = 4$