JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - II

(Memory Based)

MATHEMATICS

Q.1 If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct ?

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(3) d, e, f are in G.P.

(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Ans. **[2]**

Sol. $ax^2 + 2bx + c = 0$ $(b^2 = ac)$

$dx^2 + 2ex + f = 0$

$(af - cd)^2 = (2ae - 2bd)(2bf - 2ec)$

$a^2f^2 + c^2d^2 - 2a + cd = 4aebf - 4ae^2c - 4b^2df + 4bdec$

$a^2f^2 + c^2d^2 + 4b^2e^2 + 2afcd - 4aebf - 4bdec = 0$

$(af + cd - 2be)^2 = 0$

$af + cd = 2be$

$$\frac{af}{b^2} + \frac{cd}{b^2} = \frac{2be}{b^2}$$

$$\frac{af}{ac} + \frac{cd}{ac} = 2\left(\frac{e}{b}\right)$$

$$\frac{f}{c} + \frac{d}{a} = 2\left(\frac{e}{b}\right)$$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in AP.

Q.2 A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is -

(1) $\frac{10}{\sqrt{3}}$

(2) $\frac{100}{3}$

(3) $\frac{100}{\sqrt{3}}$

(4) $\frac{10}{3}$

Ans. **[1]**

Sol. Let unknown observation is x

$$\frac{45 + 54 + 41 + 57 + 43 + x}{6} = 48$$

$x = 48$

$$\sigma^2 = \left(\frac{1}{6}\left(45^2 + 54^2 + 41^2 + 51^2 + 43^2 + 48^2\right)\right) - (43)^2$$

$$\sigma^2 = \frac{1}{6} \times (14024) - (48)^2$$

$$\sigma^2 = \frac{14024 - ((48)^2 \times 6)}{6}$$

$$\sigma^2 = \frac{14024 - 13824}{6}$$

$$\sigma^2 = \frac{200}{6} = \frac{100}{3}$$

$$\sigma^2 = \frac{10}{\sqrt{3}}$$

Q.3 If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q (8, 0, 10), then the distance of R from the origin is -

(1) 6 (2) $\sqrt{53}$ (3) $2\sqrt{14}$ (4) $2\sqrt{21}$

Ans. **[3]**

Sol. Line PQ

$$\frac{x-2}{6} = \frac{y+3}{3} = \frac{3-4}{6} = \lambda$$

Let point R (6λ + 2, 3λ –3, 6λ + 4)

given that 6λ + 2 = 4

point R (4, –2, 6)

Distance between point R and origin = $\sqrt{16+4+36}$ = $\sqrt{56}$ = $2\sqrt{14}$

Q.4 Which one of the following statements is not a tautology ?

(1) $P \to (p \vee q)$ (2) $(p \wedge q) \to (\sim p) \vee q$

(3) $(p \wedge q) \to p$ (4) $(p \vee q) \to (p) \vee (\sim q))$

Ans. **[4]**

Sol.

p	q	$p \vee q$	$\vee q$	$p \vee \sim q$	$(p \vee q) \to (p \vee \sim q)$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	T

Q.5 Given that the slope of the tangent to a curve y = y(x) at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is -

(1) $x \log_e |y| = 2(x - 1)$ (2) $x \log_e |y| = 2(x - 1)$

(3) $x^2 \log_e |y| = -2(x - 1)$ (4) $x \log_e |y| = x - 1$

Ans. **[1]**

Sol. $\frac{dy}{dx} = \frac{2y}{x^2}$

$\frac{dy}{y} = \frac{2}{x^2}dx$

$\log_e |y| = -\frac{2}{x} + c$

process through (1, 1)

$0 = -2 + c \qquad c = 2$

$\log_e |y| = -\frac{2}{x} + 2$

$x \log_e |y| = -2 + 2x$

$x \log_e |y| = 2(x - 1)$

Q.6 If the lengths of the sides of a traingle are in A. P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is -

(1) 5 : 6 : 7 (2) 4 : 5 : 6 (3) 3 : 4 : 5 (4) 5 : 9 : 13

Ans. **[2]**

Sol. Let length of sizes are

$a = A\text{-}D \qquad (D > 0)$

$b = A$

$C = A + D$

sinA, sinB, sinC are in AP

$2 \sin B = \sin A + \sin C$

let $A = 2\theta$

$C = 2\theta$

$B = \pi - (\theta + 2\theta)$

$B = \pi - 3\theta$

$2 \sin (\pi - 3\theta) = \sin \theta + \sin 2\theta$

$2 \sin (3\theta) = \sin \theta + 2 \sin \theta \cos\theta$

$2 (3 - 4 \sin 2\theta) = 1 + 2 \cos \theta$

$2 (4 \cos^2\theta - 1) = 1 + 2 \cos\theta$

$8 \cos^2\theta - 2 \cos \theta - 3 = 0$

$8 \cos^2\theta - 6\cos\theta + 4 \cos\theta - 3 = 0$

$(2 \cos \theta + 1) (4 \cos\theta - 3) = 0$

$\cos q = \frac{-1}{2} \quad \cos\theta = \frac{3}{4}$

Not possible

$\cos \theta = \frac{(A+D)^2 + A^2 (A-D)^2}{2A(A+D)} = \frac{3}{4}$

$\frac{4AD + A^2}{2A(A+D)} = \frac{3}{4}$

$\frac{4AD + A^2}{2A(A+D)} = \frac{3}{4} \; \frac{4D + A^2}{2(A+D)} = \frac{3}{4} \quad \Rightarrow A = 5D$

$a = A - D = 4D$

$b = A = 5D$

$C = A + D = 6D$

$a : b : C : : 4 : 5 : 6$

Q.7 Let f : [–1, 3] → R be defined as

$$f(x)=\begin{cases}|x|+[x] & , \quad -1\le x<1\\ x+|x| & , \quad 1\le x<2\\ x+[x] & , \quad 2\le x\le 3\end{cases}$$

Where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at -

(1) four or more points (2) only three points (3) only two points (4) only one point

Ans. **[2]**

Sol.

$$f(x)=\begin{cases}|x|+[x] & -1\le x<1\\ x+|x| & 1\le x<2\\ x+[x] & 2\le x\le 3\end{cases}$$

f×n is discontinuous

at x = 0, 1, 3

Q.8 The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0 is -

(1) $\vec{r}(\hat{i}-\hat{k})-2=0$ (2) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (3) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$ (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

Ans. **[3]**

Sol. equation of required plane

(x + y + z –1) + 1 (2x + 3y + 4z – 5) =0

x(1 + 2λ) + y (1 + 3λ) + 3 (1 + 4λ) – 5λ – = 0 … (i)

x – y + 3 = 0 … (ii)

Plane (1) & (2) are perpendicular to each other

(1) (1 + 2λ) + (–1) (1 + 3λ) + (1) (1 + 4λ) = 0

1 + 2λ –1 – 3λ + 1 + 4λ = 0

3λ = –1

$\lambda = -\frac{1}{3}$

put in equation (i)

$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$

x – z + 2 = 0

$\vec{r}\cdot(\hat{i}-\hat{k})=-2$

Q.9 The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to

(1) $2-\frac{21}{2^{20}}$ (2) $1-\frac{11}{2^{20}}$ (3) $2-\frac{3}{2^{17}}$ (4) $2-\frac{11}{2^{19}}$

Ans. **[4]**

Sol. $S=\sum_{k=1}^{20}\frac{k}{2k}$

$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+.....+\frac{20}{2^{20}}$ ….(i)

$\frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^2}+.....+\frac{20}{2^{21}}$ ….(ii) equation (i) – (ii)

$$\frac{1}{2}S - \left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..........+\frac{1}{2^{20}}\right)-\frac{20}{2^{21}}$$

$$\frac{1}{2}S = \frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-\frac{20}{2^{21}}$$

$$\frac{1}{2}S = 1-\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\frac{1}{2}S = 1-\frac{1}{2^{20}}-\frac{10}{2^{20}}$$

$$\frac{1}{2}S = 1-\frac{11}{2^{20}}$$

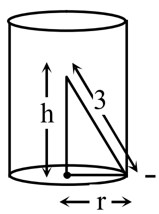
$$S = 2-\frac{11}{2^{19}}$$

Q.10 The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is -

(1) $2\sqrt{3}$ (2) $\sqrt{6}$ (3) $\sqrt{3}$ (4) $\frac{2}{3}\sqrt{3}$

Ans. **[1]**

Sol.



$h^2 + r^2 = 9$

$r^2 = 9 - h^2$

$V = \pi r^2(2h)$

$v = 2\pi h\,(9 - h^2)$

$v = 2\pi\,(9 - 4^3)$

$\frac{dv}{dx} = 0$ $h = \sqrt{3}$

$r = \sqrt{6}$

$V_{max} = \pi r^2(2h)$

$= \pi 6\,(2\sqrt{3})$

$H = 2\sqrt{3} = 2(h)$

Q.11 If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is -

(1) 10 (2) 10^3 (3) 100 (4) 10^4

Ans. **[1, Bonus]**

Sol. $\left(\left(\frac{1}{x^{1+\log_{10}^{x}}}\right)^{\frac{1}{2}} + x^{\frac{1}{12}}\right)^{6}$ $T_4 = 200$ give

$T_4 = {}^6C_3\ (x)^{-\frac{3}{2}(1+\log_{10}^{x})} \cdot \left(x^{\frac{1}{12}}\right)^{3} = 200$

$20 \cdot x^{-\frac{3}{2}(1+\log_{10}^{x})} + \frac{1}{4} = 200$

$x^{-\frac{3}{2}(1+\log_{10}^{x})+\frac{1}{4}} = 10$

log on both side

$\left(-\frac{3}{2}(1+\log_{10}^{x})+\frac{1}{4}\right)\log_{10}^{x} = 1$

let $\log_{10}^{x} = t$

$\left(-\frac{3}{2}(1+t)+\frac{1}{4}\right)t = 1$

$-6(t^2 + t) + \frac{t}{4} = 1$

$-6(t^2 + t) + t = 4$

$-6t^2 - 6t + t = 4$

$6t^2 + 5t + 4 = 0$

$\Delta < 0$

Roots imaginary (Bonus)

Q.12 In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is -

(1) 5 (2) 6 (3) 8 (4) 10

Ans. **[1]**

Sol. Given that $2b - 2a = 10$

$b - a = 5$ ...(i)

given that $2be = 10\sqrt{3}$

$be = 5\sqrt{3}$

$b^2e^2 = 75$

$(b^2 - a^2) = 75$

$(b - a)(b + a) = 75$

$5(b + a) = 75$

$b + a = 15$ … (ii)

from equation (i) & equation (2)

$b = 10$

$a = 5$

length of lotus rectum $= \frac{2a^2}{b}$

$= \frac{2 \times 25}{10} = 5$

Q.13 Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if -

(1) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (2) $r \ge 5\sqrt{\frac{3}{2}}$ (3) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (4) $0 < r \le \sqrt{\frac{3}{2}}$

Ans. **[2]**

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$\vec{a} \times \vec{b} = \hat{i}(2 + x) - \hat{j}(3 - x) + \hat{k}(-5)$

$\vec{a} \times \vec{b} = (2 + x)\hat{i} - +(x - 3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(2 + x)^2 + (x - 3)^2 + 25}$

$= \sqrt{2x^2 - 2x + 38}$

$\ge \sqrt{\frac{75}{2}}$

$\ge 5\sqrt{\frac{3}{2}}$

Q.14 Let S(α) = {(x, y) : $y^2 \le x, \le \alpha$} and A(α) is area of the region S(α). If for a λ, 0 < λ 4, A(λ) : A(4) = 2 : 5, then l equals :

(1) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (3) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Ans. **[1]**

Sol. $\frac{A(\lambda)}{A(4)} = \frac{2}{5}$

$\frac{\int_0^{\lambda} \sqrt{x}\, dx}{\int_0^{4} \sqrt{x}\, dx} = \frac{2}{5}$

$\frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$

$\lambda^{3/2} = \frac{2}{5} \times 8$

$\lambda = \left(\frac{16}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{2}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{4}{25}\right)^{1/3}$

Q.15 If f(1) = 1, f'(1) = 3, then the derivative of $f(f(f(x))) + (f(x))^2$ at x = 1 is -

(1) 9 (2) 12 (3) 15 (4) 33

Ans. **[4]**

Sol. f(1) = 1 f ' (1) = 3

$f(f(f(x))) + (f(x))^2$ at x = 1

$f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) f'(x)$

$f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) f'(1)$

$f'(f(f(1))) \cdot f'(1) \cdot f'(1) + 2f'(1)$

$f'(f(1) \cdot f'(1) \cdot f'(1) + 2f'(1)$

$3 \times 3 \times 3 + (2 \times 3)$

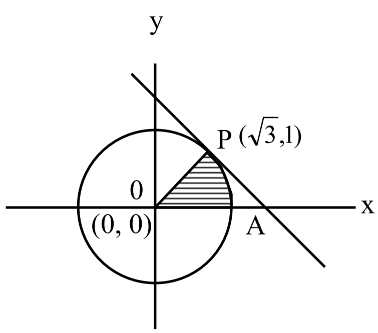
27 + 6 = 33

Q.16 The tangent and the normal lines at the point $(\sqrt{3},1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle, The area of this triangle (in square units) is -

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{1}{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. **[4]**

Sol.



equation of tangent

$\sqrt{3}x + y = 4$

point $A\left(\frac{4}{\sqrt{3}},0\right)$

Area of $\Delta OPA = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$

Q.17 The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is -

(1) 4 (2) 5 (3) 3 (4) 2

Ans. **[1]**

Sol. P (at least one hears) = 1 – [(no one heads) $\geq \frac{90}{100}$

$1-\left(\frac{1}{2}\right)^n \geq \frac{9}{100}$

$\left(\frac{1}{2}\right)^n \leq \frac{10}{100}$ $2^n \geq 10$

least value of n is 4

Q.18 Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x + y) f_2(x - y)$ (2) $2f_1(x + y) f_1(x - y)$ (3) $2f_1(x) f_2(y)$ (4) $2f_1(x) f_1(y)$

Ans. **[4]**

Sol.

$$f(x) = \left(\frac{f(x)+f(-x)}{2}\right) + \left(\frac{f(x)-f(-x)}{2}\right)$$

↓ even ↓ odd

$$f_1(x) = \frac{f(x)+f(-x)}{2}$$

$$f_1(x + y) + f_1(x - y) = \frac{a^{x+y}+a^{-x-y}}{2} + \frac{a^{x-y}+a^{y-x}}{2}$$

$$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^{-x} + a^{-y})\right]$$

$$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y})$$

$$= \frac{1}{2}\left(\frac{a^x+a^{-x}}{2}\right)\left(\frac{a^y+a^{-y}}{2}\right)$$

$= 2f_1(x) f_1(y)$

Q.19 Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals -

(1) $-\frac{1}{7}$ (2) $\frac{1}{3}$ (3) 0 (4) 3

Ans. **[2]**

Sol. equation of line passes through (1, 2) & (–3, 4)

$$(y - 2) = \frac{4-2}{-3-1}(x-1)$$

$$(y - 2) = -\frac{1}{2}(x-1)$$

$2y - 4 = -x + 1$

$x + 2y = 5$ … (i)

$\perp$ line $2x - y = \lambda \rightarrow$ passes through (4, 3)

$2x - y = 5$ …. (2) $\lambda = 5$

Intersection point of line (i) & line (ii) is (3, 1)

$$\frac{k}{h} = \frac{1}{3}$$

Q.20 If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is -

(1) $4x - 3y - 4 = 0$ (2) $3x - 4y - 4 = 0$ (3) $3x - 4y - 1 = 0$ (4) $4x - 3y - 1 = 0$

Ans. **[1]**

Sol. $x - 2y + kz = 1$ …. (i)
$2x + y + z = 2$ … (ii)
$3x - y - kz = 3$ … (iii)
for locus of (x, y)
equation (i) + (iii)
$4x - 3y = 4$
$4x - 3y - 4 = 0$

Q.21 The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant , passes through the point -

(1) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (2) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Ans. **[2]**

Sol. $y^2 = 4x$ … (i)
$x^2 + y^2 = 5$ ….(ii)
for point of intersection
$x^2 + 4x - 5 = 0$
$(x + 5)(x - 1) = 0$
$x = -5$ $x = 1$
not possible $y = \pm 2$
Point in IQ (1, 2)
Tangent at (1, 2)
$$2y = 4\left(\frac{x+1}{2}\right)$$
$y = x + 1$
point $\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on tangent

Q.22 The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is -

(1) 306 (2) 360 (3) 310 (4) 288

Ans. **[3]**

Sol. Given digits are 0, 1, 2, 3, 4, 5
requries four number greater than 4321

5			
1	6	6	6

= 216

4			
	2	6	6

= 72

4	3		
		4	6

= 24

total cae = 22
{substract two case 4320 & 4321}
total = 216 + 72 + 22
= 310

Q.23 The number of integral value of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + 8m) = 0$ has no real root is -

(1) 1 (2) infinitely many (3) 3 (4) 2

Ans. **[2]**

Sol. $4(1 + 3m)^2 - 4 \times (1 + m^2)(1 + 8m) < 0$

$1 + 9m^2 + 6m - (1 + 8m + m^2 + 8m^3) < 0$

$1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3 < 0$

$-8m^3 + 8m^2 - 2m < 0$

$8m^3 - 8m^2 + 2m > 0$

$m(4m^2 - 4m + 2) > 0$

$m[(2m-1)^2 + 1] > 0$

$m > 0$

Q.24 Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If f(x + 5) = g (x), then $\int_0^x f(t)dt$ equals-

(1) $5\int_{x+5}^{5} g(t)dt$ (2) $2\int_{5}^{x+5} g(t)dt$ (3) $\int_{5}^{x+5} g(t)dt$ (4) $\int_{x+5}^{5} g(t)dt$

Ans. **[4]**

Sol. $f(0) = \int_0^0 g(t)dt = 0$

f(0) = 0 f(x) is odd function

give that g(x) is even function

$f(x + 5) = f(-x + 5) = g(x) = g(-x)$

$$I = \int_0^x f(t)dt$$

$z = t + 5$

$$I = \int_5^{x+5} f(3-5)dz$$

$$I = \int_5^{x+5} f(-(5-3))dz$$

$$I = \int_5^{x+5} f(5-3)dz$$

$$I = \int_{X+5}^{5} f(5-z)dz$$

$$I = \int_{X+5}^{5} g(z)dz$$

$$I = \int_{X+5}^{5} g(t)dt$$

Q.25 If $x = \frac{\sqrt{3}}{2} + \frac{i}{2} (i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to -

(1) 0 (2) –1 (3) $(-1 + 2i)^9$ (4) 1

Ans. **[2]**

Sol. $z = \frac{\sqrt{3}}{2} + \frac{p}{2} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$(i + iz + z^5 + i\,z^8)^9$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2} + \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + i\left(\cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6}\right)\right)^3$$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{i}{2} + i\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)^9$$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^9$$

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^9$$

$$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^9$$

$\cos 3\pi + i \sin 3\pi = -1$

Q.26 If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is -

(1) $2x - y - 2 = 0$ (2) $3x - 2y = 0$ (3) $2x - 3y + 10 = 0$ (4) $x - 2y + 8 = 0$

Ans. **[1]**

Sol. Let equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \rightarrow (4, 6)$

$\frac{16}{a^2} - \frac{36}{b^2}$ …. (1)

$a^2 = 1 + \frac{b^2}{a^2}$

$4 = 1 + \frac{b^2}{a^2}$

$b^2 = 3a^2$ …(ii)

from (i) & (ii)

$a^2 = 4$

$b^2 = 12$

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

tangent at (4, 8)

$\frac{4x}{4} - \frac{6y}{12} = 1$

$x - \frac{y}{2} = 1$

$2x - y - 2 = 0$

Q.27 Let $f: R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is qual to -

(1) 1 (2) e^{-1} (3) e (4) e^2

Ans. **[1]**

Sol. $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ $(1)^{\infty}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left[\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right]}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left[\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right]}\left(\frac{0}{0}\right)$

$= e^{\lim_{x\to 0}\left(\frac{f(3+z)-f(x)-f(2-z)+f(z)}{x}\right)}\times\left(\frac{1}{1}\right)$

$= e^{\lim_{x\to 0}\frac{f'(3+x)+f'(2-x)}{1}}$

$= e^{f'(3)+f'(2)} = e^0 = 1$

$= e$

Q.28 If $\int \frac{dx}{x^3(1-x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$

where C is a constant of integration, then the function f(x) is equal to

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$ (3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{6x^3}$

Ans. **[1]**

Sol. $I = \int \frac{dx}{x^3(1+2x^6)^{2/3}}$

$I = \int \frac{x^{-7}dx}{(x^{-b}+1)^{2/3}}$

$x - 6 + 1 = t$

$-6x^{-7}\,dx = dt$

$x^{-7}dx = -\frac{1}{6}dt$

$I = -\frac{1}{6}\int t^{-2/3}dt$

$I = -\frac{1}{6}\times 3(t)^{1/3} + c$

$I = -\frac{1}{2}\left(\frac{1+x^6}{x^6}\right)^{1/3} + c$

$I = -\frac{1}{2}(1+x^6)^{1/3} + c$

$xf(x) = \frac{1}{2x^2}$

$f(x) = -\frac{1}{2x^2}$

Q.29 Let the number 2, b,c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det (A) $\in$ [2, 16], then c lies in the interval -

(1) $(2 + 2^{3/4}, 4)$ (2) [4, 6] (3) $[3, 2 + 2^{3/4}]$ (4) [2, 3)

Ans. **[2]**

Sol. $|A| = (2-b)(b-c)(c-2)$

2, b, c, are in AP.

$b = \frac{2+c}{2}$

Det (A) = $\frac{1}{4}(c-2)^3$

$2 \le \frac{1}{4}(c-2)^3 \le 16$

$8 \le (c-2)^3 \le 64$

$2 \le c - 2 \le 4$

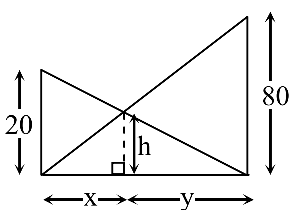
$4 \le c \le 6$

Q.30 Two vertical poles of heights, 20 m and 80 m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is -

(1) 18 (2) 16 (3) 15 (4) 12

Ans. **[2]**

Sol.



$\frac{h}{x} = \frac{80}{x+y}$

$h = \frac{80}{1+(y/x)}$ …. (i)

$h = \frac{80}{1+\frac{h}{20-h}}$

$h = 4(20 - h)$

$h = 80 - 4h$

$5h = 80$

$h = 16$

$\frac{h}{y} = \frac{20}{x+y}$

$h = \frac{20}{\left(\frac{x}{y}\right)+1}$

$\frac{x}{y} + 1 = \frac{20}{h}$

$\frac{x}{y} = \frac{20}{h} - 1$

$\frac{x}{y} = \frac{20-h}{h}$

$\frac{x}{y} = \frac{h}{20-h}$

put in … (i)