CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - I

MATHEMATICS

Q.1 Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to :

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ (3) $3\hat{i} - 9\hat{j} - 5\hat{k}$ (4) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

Ans. **[2]**

Sol. $\vec{\beta_1}$ is parallel to $\vec{\alpha}$

$\therefore \vec{\beta_1} = \lambda\vec{\alpha}$

$\Rightarrow \vec{\beta_1} = \lambda(3\hat{i} + \hat{j})$

Given that $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$

$\Rightarrow \vec{\beta_2} = \vec{\beta_1} - \vec{\beta}$

$\Rightarrow \vec{\beta_2} = \lambda(3\hat{i} + \hat{j}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$\Rightarrow \vec{\beta_2} = \hat{i}(3\lambda - 2) + \hat{j}(\lambda + 1) - 3\hat{k}$

Also given that $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$

$\therefore \vec{\beta_2} \cdot \vec{\alpha}$

$\Rightarrow 3(3\lambda - 2) + (\lambda + 1) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

So, $\vec{\beta_1} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$ and $\vec{\beta_2} = \frac{-1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$$\vec{\beta_1} \times \vec{\beta_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3/2 & 1/2 & 0 \\ 1/2 & 3/2 & -3 \end{vmatrix} = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$$

Q.2 Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :

(1) $\left\{\frac{1}{3}, 1\right\}$ (2) $\left\{\frac{1}{3}, -1\right\}$ (3) $\left\{-\frac{1}{3}, -1\right\}$ (4) $\left\{-\frac{1}{3}, 1\right\}$

Ans. [4]

Sol. $y = f(x) = x^3 - x^2 - 2x$

slope of tangent $\frac{dy}{dx} = f'(x) = 3x^2 - 2x - 2$

This tangent is parallel to line segment joining points (1, f(1)) and (–1, f(–1))

$\therefore\ m_1 = m_2$

$\Rightarrow\ 3x^2 - 2x - 2 = \frac{f(-1) - f(1)}{-1-1}$

$\Rightarrow\ 3x^2 - 2x - 2 = \frac{(-1-1+2)-(1-1-2)}{-2}$

$\Rightarrow 3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (3x + 1)(x - 1) = 0$

$\Rightarrow\ x = -\frac{1}{3}, 1$

Q.3 Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(1) π (2) $\frac{13\pi}{6}$ (3) 2π (4) $\frac{5\pi}{3}$

Ans. [3]

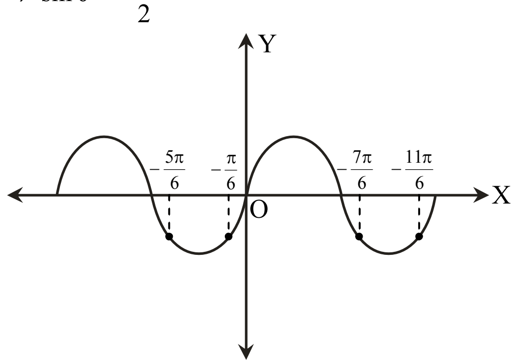
Sol. $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow\ \sin\theta = -\frac{1}{2}$



in $\theta \in [-2\pi, 2\pi]$

$\Rightarrow\ \theta = -\frac{5\pi}{6}, \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum of all roots $= \frac{-5\pi - \pi + 7\pi + 11\pi}{6} = 2\pi$

Q.4 For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is :

(1) $\sim p \vee \sim q$ (2) $p \leftrightarrow q$ (3) $p \wedge q$ (4) $\sim p \wedge \sim q$

Ans. [4]

Sol. $\sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q$

Q.5 The solution of the differential equation $x\dfrac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with $y(1) = 1$, is :

(1) $y = \dfrac{x^3}{5} + \dfrac{1}{5x^2}$ (2) $y = \dfrac{4}{5}x^3 + \dfrac{1}{5x^2}$ (3) $y = \dfrac{3}{4}x^2 + \dfrac{1}{4x^2}$ (4) $y = \dfrac{x^2}{4} + \dfrac{3}{4x^2}$

Ans. **[4]**

Sol. $x\dfrac{dy}{dx} + 2y = x^2 \quad (x \neq 0)$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{2y}{x} = x$

I.F. $= e^{\int \frac{2}{x}dx} = e^{\log_e x^2} = x^2$

$\therefore$ Solution is

$\Rightarrow yx^2 = \int x^2 \cdot x\, dx$

$\Rightarrow yx^2 = \dfrac{x^4}{4} + C$

at $y(1) = 1 \Rightarrow 1 = \dfrac{1}{4} + C$

$\Rightarrow C = \dfrac{3}{4}$

$\Rightarrow yx^2 = \dfrac{x^4}{4} + \dfrac{3}{4}$

$\Rightarrow y = \dfrac{x^2}{4} + \dfrac{3}{4x^2}$

Q.6 If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

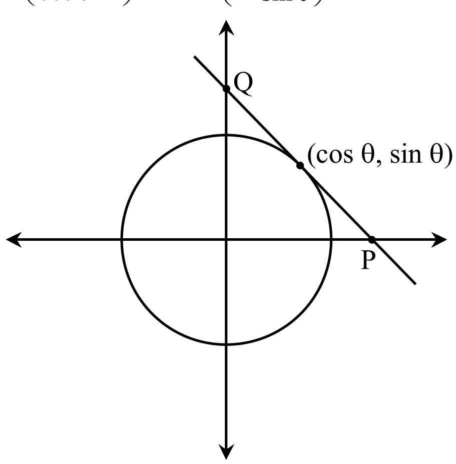
(1) $x^2 + y^2 - 2x^2y^2 = 0$ (2) $x^2 + y^2 - 4x^2y^2 = 0$ (3) $x^2 + y^2 - 16x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

Ans. **[2]**

Sol. Let the equation of tangent is $x \cos\theta + y \sin\theta = 1$

co-ordinates of P and Q are

$P\left(\dfrac{1}{\cos\theta}, 0\right)$ and $Q\left(0, \dfrac{1}{\sin\theta}\right)$



Let mid point of P and Q is (h, k)

so, $h = \dfrac{\dfrac{1}{\cos\theta} + 0}{2}$ and $k = \dfrac{0 + \dfrac{1}{\sin\theta}}{2}$

$\Rightarrow \cos\theta = \dfrac{1}{2h}$ and $\sin\theta = \dfrac{1}{2k}$

squaring and adding we get

$$\frac{1}{4h^2} + \frac{1}{4k^2} = 1$$

$\therefore$ locus $\dfrac{1}{4x^2} + \dfrac{1}{4y^2} = 1$

$\Rightarrow x^2 + y^2 - 4x^2 y^2 = 0$

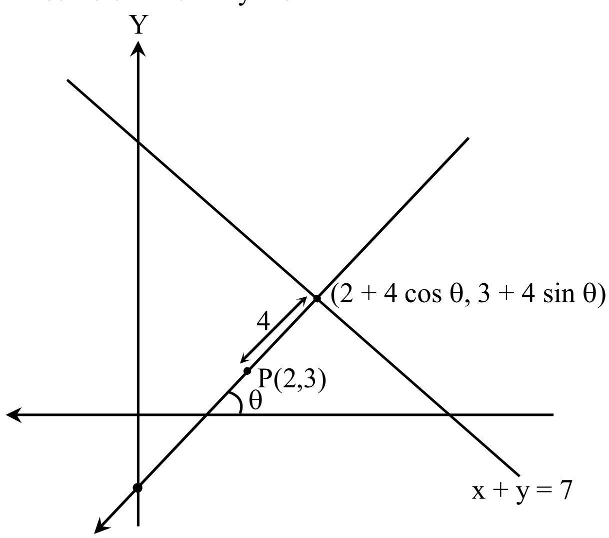
Q.7 Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is :

(1) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\dfrac{\sqrt{7}-1}{\sqrt{7}+1}$ (3) $\dfrac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\dfrac{1-\sqrt{5}}{1+\sqrt{5}}$

Ans. **[3]**

Sol. Let any point on the line is P($2 \pm 4\cos\theta$, $3 \pm 4\sin\theta$)

it also lie on line x + y = 7



$\therefore (2 \pm 4\cos\theta) + (3 \pm 4\sin\theta) = 7$

$\Rightarrow (\sin\theta + \cos\theta) = \pm\dfrac{1}{2}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = \dfrac{1}{4}$

$\Rightarrow 1 + \sin 2\theta = \dfrac{1}{4}$

$\Rightarrow \sin 2\theta = -\dfrac{3}{4}$

$\Rightarrow \dfrac{2\tan\theta}{1+\tan^2\theta} = -\dfrac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \dfrac{-8 \pm 2\sqrt{7}}{6} = \dfrac{8 - 2\sqrt{7}}{6} = \dfrac{(1-\sqrt{7})^2}{1-7} = \dfrac{1-\sqrt{7}}{1+\sqrt{7}}$

Q.8 If f(x) is a non-zero polynomial of degree four, having local extreme points at x = – 1, 0, 1; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly :

(1) four rational numbers.
(2) two irrational and two rational numbers.
(3) two irrational and one rational number.
(4) four irrational numbers.

Ans. **[3]**

Sol. Four degree polynomial function f(x) have local extreme points at x = –1, 0, 1

$\therefore f'(x) = \lambda(x+1)(x-0)(x-1) = \lambda(x^3 - x)$

$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + K$

Now, f(x) = f(0)

$\Rightarrow \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + K = K$

$\Rightarrow \frac{x^4}{4} - \frac{x^2}{2} = 0$

$\Rightarrow x = 0, \pm\sqrt{2}$

Two irrational and one rational number.

Q.9 A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m + n = 68
(2) m = n = 78
(3) m = n = 68
(4) n = m – 8

Ans. **[2]**

Sol. Given : (8 males, 5 females)

Committee to be selected = 11 members

m = no. of ways the committee is formed with at least 6 males.

$\Rightarrow$ (6M, 5F) or (7M, 4F) or (8M, 3F)

$= {}^8C_6 \times {}^5C_5 + {}^8C_7 \times {}^5C_4 + {}^8C_8 \times {}^5C_3 = 78$

n = no. of ways the committee is formed with atleast 3 female

$\Rightarrow$ (8M, 3F) or (7M, 4F) or (6M, 5F)

$= {}^8C_8 \times {}^5C_3 + {}^8C_7 \times {}^5C_4 + {}^8C_6 \times {}^5C_5$

= 10 + 40 + 28 = 78

$\Rightarrow$ m = n = 78

Q.10 The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\,dx$ is equal to : (Here C is a constant of integration)

(1) $-3\tan^{-1/3}x + C$
(2) $-\frac{3}{4}\tan^{-4/3}x + C$
(3) $3\tan^{-1/3}x + C$
(4) $-3\cot^{-1/3}x + C$

Ans. **[1]**

Sol. $I = \int \sec^{2/3} x \operatorname{cosec}^{4/3} x\,dx$

$I = \int \frac{dx}{(\sin x)^{4/3}(\cos x)^{2/3}} dx$

$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} \cdot \cos^2 x}$

$$I = \int \frac{\sec^2 x \; dx}{(\tan x)^{4/3}}$$

put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$I = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + C$$

$$I = \frac{-3}{(\tan x)^{1/3}} + C$$

Q.11 Let p, q $\in$ R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $p^2 - 4q + 12 = 0$ (2) $q^2 - 4p - 16 = 0$ (3) $q^2 + 4p + 14 = 0$ (4) $p^2 - 4q - 12 = 0$

Ans. **[4]**

Sol. If one root of equation $x^2 + px + q = 0$ is $2-\sqrt{3}$

then other root will be $2+\sqrt{3}$

$\therefore$ equation $x^2 - 4x + 1 = 0$

$\Rightarrow p = -4$ and $q = 1$

$\Rightarrow p^2 - 4q - 12 = 0$

Q.12 Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) {10, 15} (2) {5, 10, 15} (3) {10} (4) {5, 10, 15, 20}

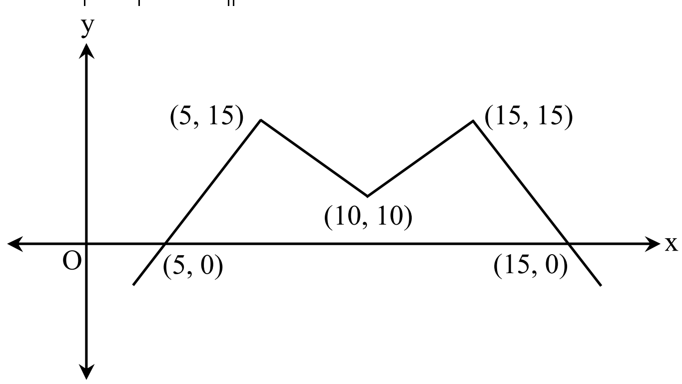
Ans. **[2]**

Sol. $f(x) = 15 - |x - 10|$

$g(x) = f[f(x)] = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



$\therefore$ g(x) is not differentiable at x = 5, 10, 15

Q.13 Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is :

(1) 2 (2) 3 (3) 16 (4) 4

Ans. **[2]**

Sol. Given $f(1) = 2$ and $f(x + y) = f(x) \cdot f(y)$

at $x = 1, y = 1 \Rightarrow f(2) = f(1) \cdot f(1) = 2^2$

$x = 2, y = 1 \Rightarrow f(3) = f(2) \cdot f(1) = 2^3$

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$f(n) = 2^n$

Now $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$

$\Rightarrow f(a+1) + f(a+2) + \ldots.. + f(a+10) = 16(2^{10}-1)$

$\Rightarrow 2^{a+1} + 2^{a+2} + \ldots\ldots + 2^{a+10} = 16(2^{10}-1)$

$\Rightarrow 2^a [2^1 + 2^2 + \ldots\ldots + 2^{10}] = 16(2^{10}-1)$

$\Rightarrow 2^a\left[\frac{2(2^{10}-1)}{2-1}\right] = 16(2^{10}-1)$

$\Rightarrow 2^{a+1} = 16$

$\Rightarrow a = 3$

Q.14 If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where $k > 0$, then k is equal to :

(1) $\sqrt{6}$ (2) $2\sqrt{\frac{10}{3}}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $2\sqrt{6}$

Ans. **[4]**

Sol. $S.D. = \sqrt{\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2}$

Now mean $\bar{x} = \frac{-1+0+1+k}{4} = \frac{k}{4}$

Given that S.D. = $\sqrt{5}$

$\Rightarrow \sqrt{5} = \sqrt{\frac{1}{4}(1+0+1+k^2) - \frac{k^2}{16}}$

$\Rightarrow 5 = \frac{2+k^2}{4} - \frac{k^2}{16}$

$\Rightarrow 80 = 8 + 4k^2 - k^2$

$\Rightarrow 3k^2 = 72 \quad \Rightarrow k^2 = 24$

$\Rightarrow k = \pm 2\sqrt{6}$

$k = 2\sqrt{6} \; (\because k > 0)$

Q.15 If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(1) $\frac{3}{\sqrt{5}}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{\sqrt{15}}{2}$

Ans. **[2]**

Sol. Equation of normal of hyperbola in slope form is $y = mx \pm \frac{m(a^2+b^2)}{\sqrt{a^2-b^2m^2}}$

$\therefore \; 7\sqrt{3} = \frac{42m}{\sqrt{24-18m^2}}$

$\Rightarrow 72 - 54\ m^2 = 36\ m^2$

$\Rightarrow 72 = 90\ m^2$

$\Rightarrow\ m^2 = \frac{72}{90} = \frac{4}{5}$

$\Rightarrow\ m = \pm\frac{2}{\sqrt{5}}$

$m = \frac{2}{\sqrt{5}}$

Q.16 If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is :

(1) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\13 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

Ans. **[4]**

Sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+\ldots.+n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow 1 + 2 + 3 + \ldots\ldots + (n-1) = 78$

$\Rightarrow \frac{n(n-2)}{2} = 78$

$\Rightarrow n = 13$

Now inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ i.e. $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$ is $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ Ans.

Q.17 If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by $f(x) = \begin{cases}\frac{\sqrt{2}\cos x - 1}{\cot x - 1} & x \neq \frac{\pi}{4}\\ k, & x = \frac{\pi}{4}\end{cases}$ is continuous, then k is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) 1 (3) $\frac{1}{2}$ (4) 2

Ans. **[3]**

Sol. $\lim_{x\to\frac{\pi}{4}}\left(\frac{\sqrt{2}\cos x - 1}{\cot x - 1}\right) = f\left(\frac{\pi}{4}\right)$

$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$

using L-Hospital Rule

$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}(-\sin x)}{-\operatorname{cosec}^2 x} = k$

$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = k$

$\Rightarrow k = \frac{1}{2}$ Ans.

Q.18 Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(1) $\frac{25}{192}$ (2) $\frac{25}{32}$ (3) $\frac{7}{32}$ (4) $\frac{1}{192}$

Ans. **[2]**

Sol. Let four persons are A, B, C, D.

Probability of Hitting target = 1 – (None of four person Hit the target)

$= 1 - P(\overline{A})\cdot P(\overline{B})\cdot P(\overline{C})\cdot P(\overline{D})$

$= 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{7}{8}$

$= \frac{25}{32}$ Ans.

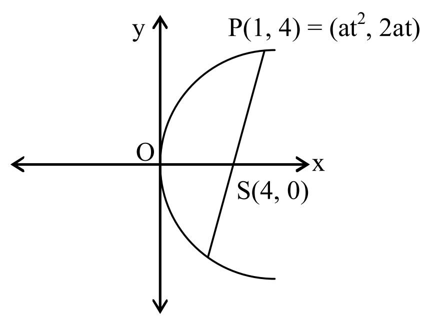
Q.19 If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

(1) 24 (2) 20 (3) 25 (4) 22

Ans. **[3]**

Sol. Parabola $y^2 = 16x$

$\{4a = 16 \Rightarrow a = 4\}$



One end $(at^2, 2at) = (1, 4)$

$\Rightarrow 2at = 4$

$\Rightarrow 2(4)\,t = 4$

$\Rightarrow t = 1/2$

Length of focal chord $= a\left(t + \frac{1}{t}\right)^2 = 4\left(\frac{1}{2} + 2\right)^2 = 25$ Ans.

Q.20 Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to :

(1) $y(y^2 - 3)$ (2) y^3 (3) $y(y^2 - 1)$ (4) $y^3 - 1$

Ans. **[2]**

Sol. Roots of eq^n. $x^2 + x + 1 = 0$ are α and β

$\alpha, \beta = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

$\Rightarrow \alpha = \omega, \beta = \omega^2$ (complex cube root of unity)

$$\Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = \begin{vmatrix} y & y & y \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix} \quad (\because\ 1 + \omega + \omega^2 = 0)$$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$\Delta = y(y^2)$

$\Delta = y^3$ Ans.

Q.21 All the points in the set $S = \left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\}$ $(i = \sqrt{-1})$ lie on a :

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is –1.

(4) circle whose radius is $\sqrt{2}$.

Ans. **[1]**

Sol. Let $\frac{\alpha+i}{\alpha-i} = z$

$\Rightarrow \left|\frac{\alpha+i}{\alpha-i}\right| = |z|$

$\Rightarrow |z| = 1$

$\Rightarrow$ Circle of radius = 1

Q.22 The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is :

(1) $\frac{\pi-2}{8}$

(2) $\frac{\pi-1}{2}$

(3) $\frac{\pi-1}{4}$

(4) $\frac{\pi-2}{4}$

Ans. **[3]**

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ …(1)

$$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos x + \sin x} dx$ …(2)

Adding (1) & (2) we get

$$\Rightarrow 2I = \int_0^{\pi/2} \left(\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\right) dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)}{(\sin x + \cos x)} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} (1 - \sin x \cos x) dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin 2x\right) dx$$

$$\Rightarrow 2I = \left(x + \frac{\cos 2x}{4}\right)_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \left(\frac{\pi}{2} - \frac{1}{4}\right) - \left(\frac{1}{4}\right) = \frac{\pi}{2} - \frac{1}{2}$$

$$\Rightarrow I = \left(\frac{\pi - 1}{4}\right)$$

Q.23 The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is :

(1) $\frac{3}{2}(1 + \cos 20^\circ)$ (2) 3/2 (3) 3/4 (4) $\frac{3}{4} + \cos 20^\circ$

Ans. **[3]**

Sol. $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$

$= \frac{1}{2}[2\cos^2 10^\circ - 2\cos 10^\circ \cos 50^\circ + 2\cos^2 50^\circ]$

$= \frac{1}{2}[(1 + \cos 20^\circ) - (\cos 60^\circ + \cos 40^\circ) + (1 + \cos 100^\circ)]$

$= \frac{1}{2}[2 - \cos 60^\circ + \cos 20^\circ + (\cos 100^\circ - \cos 40^\circ)]$

$= \frac{1}{2}[2 - \frac{1}{2} + \cos 20^\circ + 2\sin 70^\circ \sin(-30^\circ)]$

$= \frac{1}{2}[\frac{3}{2} + \cos 20^\circ - \sin 70^\circ]$

$= \frac{1}{2}[\frac{3}{2} + \cos 20^\circ - \sin(90^\circ - 20^\circ]$

$= \frac{3}{4}$ Ans.

Q.24 Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :

(1) (50, 50 + 46A) (2) (50, 50 + 45A) (3) (A, 50 + 45A) (4) (A, 50 + 46A)

Ans. **[4]**

Sol. $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$T_n = 50n + \left(\frac{n(n-7)}{2}\right)A - 50(n-1) - \left(\frac{(n-1)(n-8)}{2}\right)A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n-4)$
Now, $d = T_n - T_{n-1}$
$= 50 + A(n-4) - 50 - A(n-5)$
$= A$
and $T_{50} = 50 + 46A$
$(d, A_{50}) = (A, 50 + 46A)$ Ans.

Q.25 If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :

(1) $\sqrt{5}/2$ (2) 7/2 (3) $2\sqrt{5}$ (4) 9/2

Ans. **[4]**

Sol. Line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4} = k$ (say)
any point on this line $P(2k + 1, 3k - 1, 4k + 2)$
This point P lies on plane $x + 2y + 3z = 15$
$\therefore (2k + 1) + 2(3k - 1) + 3(4k + 2) = 15$
$\Rightarrow 20k + 5 = 15$
$\Rightarrow 20k = 10$
$\Rightarrow k = 1/2 \therefore P\left(2, \frac{1}{2}, 4\right)$
Distance of P from origin is
$= \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$ Ans.

Q.26 A plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point :

(1) $(\sqrt{2}, -1, 4)$ (2) $(-\sqrt{2}, 1, -4)$ (3) $(-\sqrt{2}, -1, -4)$ (4) $(\sqrt{2}, 1, 4)$

Ans. **[4]**

Sol. Let $ax + by + cz = 1$ be the eq^n. of plane
it passed through $(0, -1, 0)$ and $(0, 0, 1)$
$\Rightarrow b = -1$ and $c = 1$
other plane is $y - z + 5 = 0$
Given that angle b/w them is $\frac{\pi}{4}$

$\cos\theta = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right|$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{a^2+1+1}\sqrt{0+1+1}}$
$\Rightarrow a^2 + 2 = 4$
$\Rightarrow a = \pm\sqrt{2}$
$\therefore eq^n$. of plane $\pm\sqrt{2}\,x - y + z = 1$
Now for –ve sign
$-\sqrt{2}(\sqrt{2}) - 1 + 4 = 1$
$\therefore (\sqrt{2}, 1, 4)$ satisfy the eq^n. of plane.

Q.27 If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve?

(1) $(2, -2)$ (2) $(-2, 2)$ (3) $(-2, 1)$ (4) $(2, -1)$

Ans. **[1]**

Sol. $y = x^3 + ax - b$

$(1, -5)$ lies on curve

$\therefore -5 = 1 + a - b$

$\Rightarrow a - b = -6$ …(1)

$\frac{dy}{dx} = 3x^2 + a$

Slope of tangent at $(1, -5)$

$\Rightarrow \frac{dy}{dx} = 3 + a$

This tangent is perpendicular to $-x + y + 4 = 0$

$\therefore (3 + a)(1) = -1$

$\Rightarrow a = -4$ …(2)

By (1) & (2) $a = -4$, $b = 2$

So, eq^n. of curve $y = x^3 - 4x - 2$

$(2, -2)$ lies on this curve

Q.28 If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is :

(1) 8^2 (2) 8^3 (3) 8 (4) 8^{-2}

Ans. **[1]**

Sol. $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ $(x > 0)$

$\Rightarrow T_4 = 20 \times 8^7$

$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3 \left(x^{\log_8 x}\right)^3 = 20 \times 8^7$

$\Rightarrow \frac{160}{x^3} x^{3\log_8 x} = 20 \times 8^7$

$\Rightarrow x^{3\log_8 x - 3} = 8^6$

$\Rightarrow x^{\log_2 x - 3} = 8^6 = 2^{18}$

$\Rightarrow \log_2\left(x^{\log_2 x - 3}\right) = \log_2 2^{18}$

$\Rightarrow (\log_2 x - 3)(\log_2 x) = 18$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow t = 6, -3$

$\Rightarrow \log_2 x = 6$ $\Rightarrow x = 2^6 = 8^2$

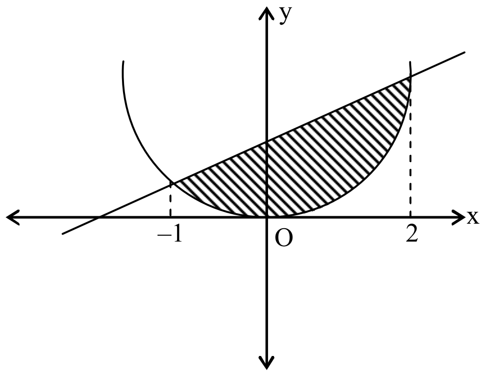
$\Rightarrow \log_2 x = -3$ $\Rightarrow x = 2^{-3} = 1/8$

Q.29 The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is :

(1) $\frac{9}{2}$ (2) $\frac{31}{6}$ (3) $\frac{10}{3}$ (4) $\frac{13}{6}$

Ans. **[1]**

Sol.



$x^2 \le y \le x + 2$

$x^2 = y;\ y = x + 2$

$\Rightarrow x^2 = x + 2$

$\Rightarrow x = 2, -1$

So, area = $\int_{-1}^{2} \{(x+2) - x^2\}dx = \frac{9}{2}$

Q.30 If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to :

(1) $R - \{-1\}$ (2) $R - [-1, 0)$ (3) $R - (-1, 0)$ (4) $[0, \infty)$

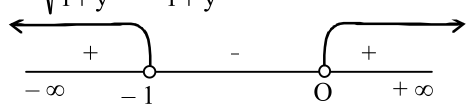
Ans. **[2]**

Sol. $y = \frac{x^2}{1-x^2}$

$\Rightarrow y - x^2y = x^2$

$\Rightarrow x^2 = \frac{y}{1+y}$

$\Rightarrow x = \sqrt{\frac{y}{1+y}} \Rightarrow \frac{y}{1+y} \ge 0$

$\Rightarrow$ 

Range of y is $R - [-1, 0)$

For surjective function codomain = Range

$\therefore$ A is $R - [-1, 0)$