# JEE Main Online Exam 2019

## Questions & Solutions

### 9th April 2019 | Shift - II

### (Memory Based)

## MATHEMATICS

**Q.1** If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $-\frac{1}{4}$ (4) –4

**Ans.** **[1]**

**Sol.** Given system of equations has non-trival solution

$$\therefore \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow K = \frac{9}{2}$$

so equation are

$$2x + 3y - z = 0 \quad .........(1)$$

$$x + \frac{9}{2}y - 2z = 0 \quad .........(2)$$

$$2x - y + z = 0 \quad .........(3)$$

$(1) - (3) \Rightarrow 4y - 2z = 0$

$$\Rightarrow 2y = z \quad .........(4)$$

$$\Rightarrow \boxed{\frac{y}{z} = \frac{1}{2}}$$

From equation (1) & (4)

$$2x + 3y - 2y = 0$$

$$\Rightarrow 2x + y = 0$$

$$\Rightarrow \boxed{\frac{x}{y} = \frac{-1}{2}}$$

$$\text{or } \boxed{\frac{z}{x} = -4}$$

$$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+K = \frac{1}{2}$$

**Q.2** The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :

(1) (–6, 4) (2) (4, –2) (3) (–4, 6) (4) (6, –2)

**Ans.** **[4]**

**Sol.** circle $x^2 + y^2 = 4$

$\Rightarrow c_1(0, 0)\ ;\ r_1 = 2$

and circle $x^2 + y^2 + 6x + 8y - 24 = 0$

$\Rightarrow c_2\ (-3, -4)\ ;\ r_2 = 7$

$\Rightarrow d = c_1c_2 = 5$

also $d = |r_1 - r_2|$

circles touch externally

equation of common tangent $s_1 - s_2 = 0$

$\Rightarrow 6x + 8y - 20 = 0$

$\Rightarrow 3x + 4y - 10 = 0$

Point (6, –2) satisfy it

**Q.3** Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :

(1) 157 (2) 262 (3) 225 (4) 190

**Ans.** **[4]**

**Sol.** $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow n^2 - 9n - 190 = 0$

$\Rightarrow (n - 19)\ (n + 10) = 0$

$\Rightarrow n = 19$

number of balls = $\frac{19.20}{2} = 190$

**Q.4** If $\int e^{\sec x}\left(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x)\right)dx = e^{\sec x} f(x) + C$, then a possible choice of f(x) is :

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $\sec x + x\tan x - \frac{1}{2}$

(3) $\sec x + \tan x + \frac{1}{2}$ (4) $x\sec x + \tan x + \frac{1}{2}$

**Ans.** **[3]**

**Sol.** $\int e^{\sec x}\ (\sec x \tan x\ f(x) + (\sec x \tan x + \sec^2 x))\ dx$

$= e^{\sec x} f(x) + C$

Diff both side w.r.t x

$\Rightarrow e^{\sec x} (\sec x \tan x\ f(x) + (\sec x \tan x + \sec^2 x))$

$= e^{\sec x} . \sec x \tan x\ f(x) + e^{\sec x} f'(x)$

$\Rightarrow f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec + C$

**Q.5** If the tangent to the parabola $y^2 = x$ at a point $(\alpha, \beta)$, $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then $\alpha$ is equal to :

(1) $\sqrt{2} + 1$ (2) $2\sqrt{2} + 1$ (3) $\sqrt{2} - 1$ (4) $2\sqrt{2} - 1$

**Ans.** **[1]**

**Sol.** Equation of tangent to the parabola $y^2 = x$

at $(\alpha, \beta)$ is T = 0

$$y\beta = \frac{x+\alpha}{2}$$

$$\Rightarrow y\beta = \frac{x+\beta^2}{2} \qquad (\therefore \beta^2 = \alpha)$$

$$\Rightarrow y = \frac{1}{2\beta}x + \frac{\beta}{2}$$

$$\left(m = \frac{2}{2\beta}, c = \frac{\beta}{2}\right)$$

this is also a tangent to ellipre $x^2 + 2y^2 = 1$

$$\therefore C = \pm\sqrt{a^2m^2 + b^2}$$

$$\Rightarrow \frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$$

$$\Rightarrow \frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$$

$$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$$

$$\Rightarrow (\beta - 1)^2 = 2$$

$$\Rightarrow \beta^2 - 1 = \sqrt{2} \quad (\because \beta > 0)$$

$$\Rightarrow \beta^2 = \sqrt{2} + 1$$

$$\alpha = \beta^2 = \sqrt{2} + 1$$

**Q.6** The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in R, x \neq y)$ for which $A^TA = 3I_3$ is :

(1) 2 (2) 4 (3) 3 (4) 6

**Ans.** **[2]**

**Sol.** $(A^T)(A) = 3I_3$

$$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow 8x^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{8}}$$

$$\Rightarrow 6y^2 = 3 \Rightarrow y = \pm\sqrt{\frac{1}{2}}$$

4 materices are possible

**Q.7** If $f$ : R → R is a differentiable function and f(2) = 6, then $\lim_{x\to 2}\int_{6}^{f(x)}\frac{2t\,dt}{(x-2)}$ is :

(1) 0 (2) $24f'(2)$ (3) $12f'(2)$ (4) $2f'(2)$

**Ans.** **[3]**

**Sol.** $\lim_{x\to 2}\int_{6}^{f(x)}\frac{2tdt}{(x-2)}dx$ {given that f(2) = 6}

$\frac{0}{0}$ form. So we use L-Hospital Rule

$= \lim_{x\to 2}\frac{f'(x).2f(x)}{1}$

$= f'(2)\,.\,2f(2)$

$= 12f'(2)$

**Q.8** If a unit vector $\vec{a}$ makes angles π/3 with $\hat{i}$, π/4 with $\hat{j}$ and θ ∈(0, π) with $\hat{k}$, then a value of θ is :

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

**Ans.** **[4]**

**Sol.** $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1-\frac{3}{4}=\frac{1}{4}$

$\Rightarrow \cos\gamma = \pm\frac{1}{2}$

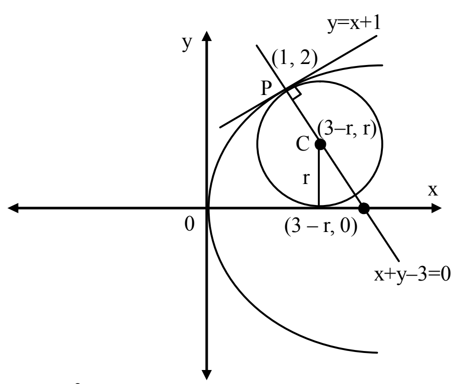
$\Rightarrow \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

**Q.9** The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :

(1) $4\pi(2-\sqrt{2})$ (2) $8\pi(3-2\sqrt{2})$ (3) $4\pi(3+\sqrt{2})$ (4) $8\pi(2-\sqrt{2})$

**Ans.** **[2]**

**Sol.**




equation of tangent to the perabola $y^2 = 4x$ at (1, 2) is

$2y = 4\left(\frac{x+1}{2}\right)$

$\Rightarrow y = x + 1$

equation of normal y = –x + 3

Let center be c(3 – r, r)

Now $PC^2 = r^2$

$\Rightarrow (3 - r - 1)^2 + (r - 2)^2 = r^2$

$\Rightarrow 2(2 - r)^2 = r^2$

$\Rightarrow r^2 - 8r + 8 = 0$

$\Rightarrow r = 4 \pm 2\sqrt{2}$

for $r = 4 + 2\sqrt{2}$ (3 – r < 0) Not possible

So $r = 4 - 2\sqrt{2}$

Area $= \pi r^2 = \pi(16 + 8 - 16\sqrt{2})$

$= 8\pi(3 - 2\sqrt{2})$

**Q.10** Two newspapers A and B are published in a city. It is known that 25 % of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

(1) 13.9 (2) 13.5 (3) 12.8 (4) 13

**Ans.** **[1]**

**Sol.** Let population = 100

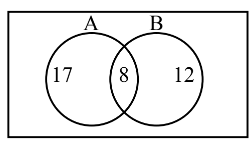
n(A) = 25

n(B) = 20

$n(A \cap B) = 8$

$n(A \cap \bar{B}) = 17$

$n(\bar{A} \cap B) = 12$




Now % of the population who look advertiesement

$= \frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$

= 13.9

**Q.11** The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + .... upto $11^{th}$ term is :

(1) 946 (2) 916 (3) 915 (4) 945

**Ans.** **[1]**

**Sol.** S = 1 + 2 × 3 + 3 × 5 + 4 × 7 + ......+ upto $11^{th}$ terms

nth term of the sevies is

$T_n = n(2n - 1)$

$\Rightarrow S = \sum_{n=1}^{11} Tn = \sum_{n=1}^{11} (2n^2 - n)$

$\Rightarrow S_n = \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

put n = 1

$\Rightarrow S_{11} = \frac{2(11)(12)(23)}{6} - \frac{11(12)}{2}$

$\Rightarrow S_{11} = 946$

**Q.12** If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$

is continous at x = 5, then value of a – b

(1) $\frac{2}{\pi + 5}$ (2) $\frac{-2}{\pi + 5}$ (3) $\frac{2}{\pi - 5}$ (4) $\frac{2}{5 - \pi}$

**Ans.** **[4]**

**Sol.** $f(x) = \begin{cases} a|\pi - x| + 1 & ,x < 5 \\ b|\pi - x| + 3 & ,x > 5 \end{cases}$

cortinous at x = 5

$\therefore$ L.H.L = R.H.L = f(5)

$\Rightarrow b|\pi - 5| + 3 = a|\pi - 5| + 1$

$\Rightarrow -b(\pi - 5) + 3 = -a(5 - \pi) + 1$

$\Rightarrow (a - b)(\pi - 5) = -2$

$\Rightarrow a - b = \frac{-2}{\pi - 5} = \frac{2}{5 - \pi}$

**Q.13** Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5 + 3z}{5(1 - z)}$ then :

(1) $4\,\text{Im}(\omega) > 5$ (2) $5\,\text{Re}(\omega) > 1$ (3) $5\,\text{Im}(\omega) < 1$ (4) $5\,\text{Re}(\omega) > 4$

**Ans.** **[2]**

**Sol.** $\omega = \frac{5 + 32}{5(1 - z)}$

$\Rightarrow 5\omega - 5\omega z = 5 + 3z$

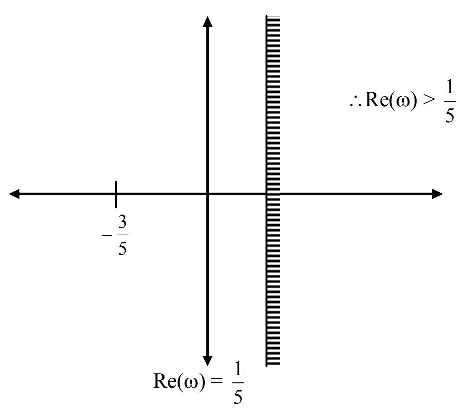
$\Rightarrow z = \frac{5\omega - 5}{3 + 5\omega}$

given $|z| < 1$

$\Rightarrow \left|\frac{5\omega - 5}{3 + 5\omega}\right| < 1$

$\Rightarrow |5\omega - 5| < |3 + 5\omega|$

$\Rightarrow |\omega - 1| < \left|\frac{3}{4} + \omega\right|$



**Q.14** If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim\limits_{x\to 4-} f(x)$ exists but $\lim\limits_{x\to 4+} f(x)$ does not exist.

(2) Both $\lim\limits_{x\to 4-} f(x)$ and $\lim\limits_{x\to 4+} f(x)$ exist but are not equal.

(3) $\lim\limits_{x\to 4+} f(x)$ exists but $\lim\limits_{x\to 4-} f(x)$ does not exist.

(4) $f$ is continuous at x = 4.

**Ans.** **[4]**

**Sol.** $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left\{[x] - \left[\frac{x}{4}\right]\right\} = 4 - 1 = 3$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \left\{[x] - \left[\frac{x}{4}\right]\right\} = 3 - 0 = 3$$

f(4) = 3

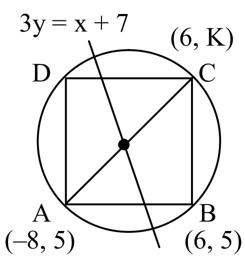
∴ continous at x = 4

**Q.15** A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :

(1) 98 (2) 56 (3) 72 (4) 84

**Ans.** **[4]**

**Sol.**




Let vertex c is (6, K) then center of circle $\left(-1, \frac{5+K}{2}\right)$ it lies on diameter 3y = x + 7

$\Rightarrow 3\left(\frac{5+K}{2}\right) = -1 + 7$

$\Rightarrow K = -1$

So AB = 14 and BC = 6

Area = 14 × 6 = 84

**Q.16** The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :

(1) $(1, 2) \cup (2, \infty)$ (2) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (4) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

**Ans.** **[3]**

**Sol.** $f(x) = \dfrac{1}{4-x^2} + \log_{10}(x^3 - x)$

Let $f_1 = \dfrac{1}{4-x^2}$

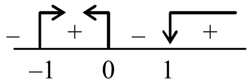
$\Rightarrow 4 - x^2 \neq 0$

$\Rightarrow x \neq \pm 2$

and $f_2 = \log_{10}(x^3 - x)$

$x^3 - x > 0$

$\Rightarrow x(x-1)(x+1) > 0$

$\Rightarrow$ 


$x \in (-1, 0) \cup (1, \infty) - \{2\}$

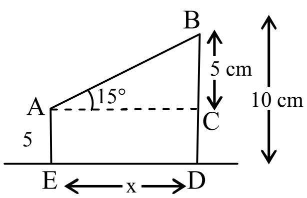
$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

**Q.17** Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes and angle of 15° with the ground. Then the distance (in m) between the poles, is :

(1) $10(\sqrt{3} - 1)$ (2) $5(2 + \sqrt{3})$ (3) $5(\sqrt{3} + 1)$ (4) $\dfrac{5}{2}(2 + \sqrt{3})$

**Ans.** **[2]**

**Sol.**




in $\Delta ABC \Rightarrow \tan 15° = \dfrac{5}{x}$

$\Rightarrow 2 - \sqrt{3} = \dfrac{5}{x}$

$\Rightarrow x = 5(2 + \sqrt{3})$

**Q.18** If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x - (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$ (3) $10\sqrt{5}$ (4) $8\sqrt{5}$

**Ans.** **[4]**

**Sol.** $(m^2 + 1)x^2 - 3x + (m + 1)^2 = 0$

$\Rightarrow \alpha + \beta = \dfrac{3}{m^2 + 1}$

$\alpha\beta = \dfrac{(m+1)^2}{m^2 + 1}$

$\because \alpha + \beta$ is maximum $\therefore m^2 + 1$ is minimum

$\Rightarrow m = 0$

$\therefore \alpha + \beta = 3$ and $\alpha\beta = 1$

$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)|$

$= \left|\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\left\{(\alpha+\beta)^2 - \alpha\beta\right\}\right|$

$= \left|\sqrt{9-4}(9-1)\right|$

$= 8\sqrt{5}$

**Q.19** The value of the integral $\int_0^1 x \cot^{-1}(1-x^2+x^4)\,dx$ is :

(1) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2}-\log_e 2$ (3) $\frac{\pi}{4}-\log_e 2$ (4) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$

**Ans.** **[1]**

**Sol.** $I = \int_0^1 x \cot^{-1}(1-x^2+x^4)\,dx$

$I = \int_0^1 x \tan^{-1}\left(\frac{1}{1-x^2+x^4}\right)dx$

$I = \int_0^1 x \tan^{-1}\left\{\frac{x^2-(x^2-1)}{1+x^2(x^2-1)}\right\}dx$

$I = \int_0^1 x\,\{\tan^{-1}x^2 - \tan^{-1}(x^2-1)\}dx$

let $x^2 = t \Rightarrow 2x\,dx = dt$

$I = \frac{1}{2}\int_0^1 \{\tan^{-1}t - \tan^{-1}(t-1)\}dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt - \frac{1}{2}\int_0^1 \tan^{-1}(t-1)\,dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt - \frac{1}{2}\int_0^1 \tan^{-1}(-t)\,dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt + \frac{1}{2}\int_0^1 \tan^{-1}(t)\,dt$

$= \int_0^1 \tan^{-1}(t)\,dt$

$= (\tan.\tan^{-1})_0^1 - \int_0^1 \frac{t}{1+t^2}\,dt$

$= \left(\frac{\pi}{4}\right) - \frac{1}{2}\left[\log(1+t^2\right]_0^1$

$= \frac{\pi}{4}-\frac{1}{2}\log_e 2$

**Q.20** Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :

(1) $205\sqrt{5}$ (2) $11/\sqrt{5}$ (3) $63\sqrt{5}$ (4) $17/\sqrt{5}$

**Ans.** **[2]**

**Sol.** Equation of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0$

$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + \lambda) - 6 + 5\lambda = 0$

This plane is $\perp$ to xy - plane

$\therefore 1 + \lambda = 0 \Rightarrow \lambda = -1$

So, equation of plane

$-x - 2y - 11 = 0$

$\Rightarrow x + 2y + 11 = 0$

distance of the point (0, 0, 256) from this plane

$= \left|\frac{0+0+1}{\sqrt{1+4}}\right| = \frac{11}{\sqrt{5}}$

**Q.21** If some three coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is :

(1) 232 (2) 964 (3) 625 (4) 227

**Ans.** **[1]**

**Sol.** given : $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$

$\Rightarrow 15r = 2n - 2r + 2$

$\Rightarrow 17r = 2n + 2$ ......(1)

also given $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70} \Rightarrow \frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 3n - 3r = 14r + 14$

$\Rightarrow 17r = 3n - 14$

Solving (1) & (2)

$n = 16, r = 2$

average of coefficient $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$= \frac{16+120+560}{3}$

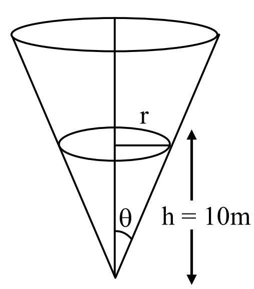
$= 232$

**Q.22** A water tank has the shape of an inverted right cirular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

(1) 1/5 π (2) 1/10 π (3) 1/15 π (4) 2/π

**Ans.** **[1]**

**Sol.** given $\theta = \tan^{-1}\left(\frac{1}{2}\right)$




$\Rightarrow \tan\theta = \frac{1}{2} = \frac{r}{h}$

$\Rightarrow r = \frac{h}{2}$

$v = \frac{1}{3}\pi r^2 h$

$v = \frac{1}{3}\pi \frac{h^3}{4}$

$\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

$S = \frac{\pi}{4}(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$

**Q.23** The value of sin10° sin30° sin50° sin70° is :

(1) $\frac{1}{16}$ (2) $\frac{1}{36}$ (3) $\frac{1}{18}$ (4) $\frac{1}{32}$

**Ans.** **[1]**

**Sol.** = sin10° sin30° sin50° sin70°

= sin30°{sin10° sin(60°–10°) sin(60° + 10°)}

$= \sin 30° \left\{\frac{1}{4}\sin 3(10)°\right\}$

$= \frac{1}{2}\left(\frac{1}{4}\times\frac{1}{2}\right)$

$= \frac{1}{16}$

**Q.24** If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :

(1) F, F, F (2) T, F, F (3) F, T, T (4) T, T, F

**Ans.** **[2]**

**Sol.** $p \Rightarrow (q \vee r)$ is false.

($\because$ T $\Rightarrow$ F = F)

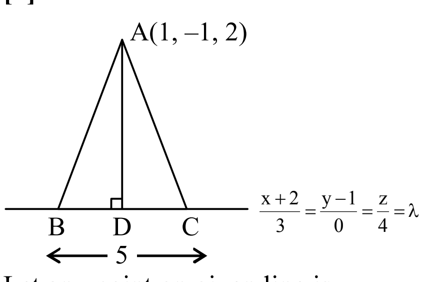
So, P = T, q = F and r = F

**Q.25** The vertices B and C of ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :

(1) $5\sqrt{17}$ (2) $\sqrt{34}$ (3) $2\sqrt{34}$ (4) 6

**Ans.** **[2]**

**Sol.**




Let any point on given line is

D(3λ –2, 1, 4λ)

Now AD ⊥ BC

D.R. of BC $\Rightarrow a_1 = 3, b_1 = 0, c_1 = 4$

D.R. of AD $\Rightarrow a_2 = 3\lambda - 3, b_2 = 2, c_2 = 4\lambda - 2$

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$\Rightarrow 25\lambda = 17$

$\Rightarrow \lambda = \frac{17}{25}$

co-oridnation of point D $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

AD = $\sqrt{\frac{576}{625} + 4 + \frac{324}{25}} = \frac{2}{5}\sqrt{34}$

Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{5}\sqrt{34}$

$= \sqrt{34}$

**Q.26** If $\cos x \frac{dy}{dx} - y\sin x = 6x$, $(0<x< \frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{\pi^2}{4\sqrt{3}}$ (2) $-\frac{\pi^2}{2}$ (3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

**Ans.** **[4]**

**Sol.** $\cos x \frac{dy}{dx} - y\sin x = 6x$

$\Rightarrow \frac{dy}{dx} - y \tan x = 6x \sec x$

I.F = $e^{-\int \tan x dx} = e^{-\log_e \sec x} = \frac{1}{\sec x}$

$\therefore$ solution of equation

$\Rightarrow y.\frac{1}{\sec x} = \int 6x \sec x.\frac{1}{\sec x} dx$

$\Rightarrow \frac{y}{\sec x} = 3x^2 + c$ ........(1)

given $y\left(\frac{\pi}{3}\right) = 0$

So, $0 = \frac{3\pi^2}{9} + C$

$\Rightarrow C = -\frac{\pi^2}{3}$

Now from (1)

$\Rightarrow \frac{y}{\sec x} = 3x^2 - \frac{\pi^2}{3}$

at $x = \frac{\pi}{6}$

$\Rightarrow \frac{\sqrt{3}y}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$

$\Rightarrow y = -\frac{\pi^2}{2\sqrt{3}}$

**Q.27** If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is :

(1) –25 (2) –35 (3) –36 (4) 25

**Ans.** **[1]**

**Sol.** Let the three numbers in A.P. are

$a - d, a, a + d$

given that : $a - d + a + d = 33$

$\Rightarrow a = 11$

and $(a - d)(a)(a + d) = 1155$

$\Rightarrow a(a^2 - d^2) = 1155$

$\Rightarrow 11(121 - d^2) = 1155$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

If $d = 4$ then first term $a - d = 7$

If $d = -4$ then first term $a - d = 15$

$T_{11} = 7 + 10(4) = 47$

or $T_{11} = 15 + 10(-4) = -25$

**Q.28** The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34 x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

(1) 7/2 (2) 9/4 (3) 8/3 (4) 7/3

**Ans.** **[4]**

**Sol.** mean = 42

$\Rightarrow \frac{10+22+26+29+34+x+42+67+70+y}{10} = 45$

$\Rightarrow 420 = 300 + x + y$

$\Rightarrow x + y = 120$ ......(1)

and medium = 35

$\Rightarrow \frac{34+x}{2} = 35$

$\Rightarrow x = 36$

From (1) y = 84

$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

**Q.29** If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is :

(1) $\frac{2}{5}$ (2) $\sqrt{\frac{2}{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{2}}{5}$

**Ans.** **[2]**

**Sol.** Two lines are perpendicular

$\therefore m_1m_2 = -1$

$\Rightarrow \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a + 1)(a^2 - 2a + 2) = 0$

$\therefore$ a = –1

so lines are $L_1 : x - 2y + 1 = 0$

$L_2 : 2x + y - 1 = 0$

Solving there equation we get point of intersetion P $\left(\frac{1}{5}, \frac{3}{5}\right)$

Now distance of P from origin

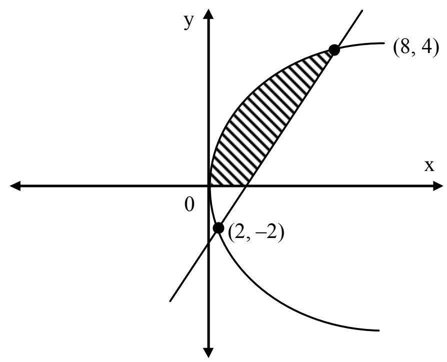
$$OP = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{2}{5}}$$

**Q.30** The area (in sq. units) of the region A = $\{(x, y) : \frac{y^2}{2} \le x \le y + 4\}$ is :

(1) 18 (2) 30 (3) $\frac{53}{3}$ (4) 16

**Ans.** **[1]**

**Sol.**




$y^2 = 2x$ .......(1)

and $x - y - 4 = 0$ .......(2)

solving (1) & (2)

$(x - 4)^2 = 2x$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x = 8_1 + 2$ and $y = 4_1 -2$

$$A = \int_{-2}^{4} \left( y + 4 - \frac{y^2}{2} \right) dy$$

$$A = \left( \frac{y^2}{4} + 4y - \frac{y^3}{3} \right)_{-2}^{4}$$

$$A = \left( 4 + 16 - \frac{64}{6} \right) - \left( 1 - 8 + \frac{8}{6} \right) = 18$$