



## JEE Main Online Exam 2019

### Questions & Solutions

10<sup>th</sup> April 2019 | Shift - II

#### Mathematics

**Q.1** If  $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$ , then  $a + b$  is equal to :

(1) 1

(2) -4

(3) -7

(4) 5

**Ans.** [3]

**Sol.**  $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$

$$(1)^2 - a(1) + b = 0$$

$$1 - a + b = 0$$

$$a - b = 1 \quad \dots(1)$$

Now

'L' hospital rule

$$2x - a = 5$$

$$2 - a = 5 (\because x = 1)$$

$$a = -3 \quad \dots(2)$$

Put in (1)

$$\therefore b = -4$$

$$a + b = -7$$

**Q.2** The locus of the centres of the circles, which touch the circle,  $x^2 + y^2 = 1$  externally, also touch the y-axis and lie in the first quadrant, is

(1)  $x = \sqrt{1 + 2y}$ ,  $y \geq 0$

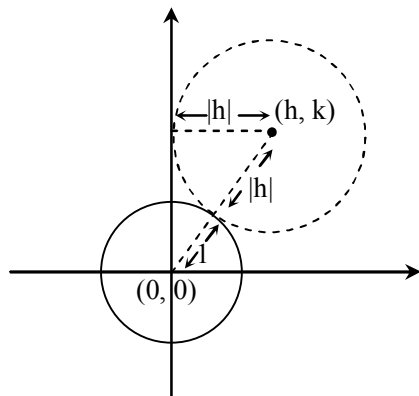
(2)  $y = \sqrt{1 + 2x}$ ,  $x \geq 0$

(3)  $y = \sqrt{1 + 4x}$ ,  $x \geq 0$

(4)  $x = \sqrt{1 + 4y}$ ,  $y \geq 0$

**Ans.** [2]

**Sol.**



$$\sqrt{h^2 + k^2} = 1 + |h|$$

$$h^2 + k^2 = 1 + h^2 + 2|h|$$

$$k^2 = 1 + 2|h|$$

$$y^2 = 1 + 2x$$

**Q.3** The number of real roots of the equation  $5 + |2^x - 1| = 2^x(2^x - 2)$  is  
 (1) 2 (2) 1 (3) 3 (4) 4

**Ans.** [2]

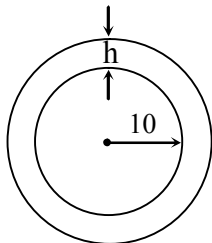
**Sol.**  $2^x \geq 1$   $2^x < 1$   
 $5 + 2^x - 1 = 2^x(2^x - 2)$   $5 + 1 - 2^x = 2^x(2^x - 2)$   
 Let  $2^x = t$   $2^x = t$   
 $5 + t - 1 = t(t - 2)$   $5 + 1 - t = t(t - 2)$   
 $t = 4, -1$  (rejected)  $0 = t^2 - t - 6$   
 $2^x = 4$   $0 = (t - 3)(t - 2)$   
 $x = 2$   $t = 3, -2$   
 only 1 solution  $2^x = 3, 2^x = -2$   
(rejected)

**Q.4** A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of  $50 \text{ cm}^3/\text{min}$ . When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

- (1)  $\frac{5}{6\pi}$  (2)  $\frac{1}{9\pi}$  (3)  $\frac{1}{36\pi}$  (4)  $\frac{1}{18\pi}$

**Ans.** [4]

**Sol.**



$$V = \frac{4}{3}\pi((10 + h)^3 - 10^3)$$

$$\frac{dV}{dt} = 4\pi(10 + h)^2 \frac{dh}{dt}$$

$$-50 = 4\pi(10 + 5)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{-1}{18\pi} \frac{\text{cm}}{\text{min}}$$

**Q.5** The sum  $1 + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + \dots + \frac{1^3 + 2^3 + 3^3 + \dots + 15^3}{1 + 2 + 3 + \dots + 15} - \frac{1}{2}(1 + 2 + 3 + \dots + 15)$  is equal to :  
 (1) 620 (2) 1240 (3) 1860 (4) 660





**Ans.** [3]

**Sol.**  $|zw| = 1$

$$|z| |w| = 1$$

$$\text{Let } w = \frac{1}{r} e^{i\theta}$$

$$\text{then } z = r e^{i\left(\theta + \frac{\pi}{2}\right)}$$

$$\bar{z}w = e^{-i\left(\theta + \frac{\pi}{2}\right)} \cdot e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$\& z\bar{w} = e^{i\left(\theta + \frac{\pi}{2}\right)} \cdot e^{-i\theta} = e^{i\pi/2} = i$$

**Q.12** The smallest natural number  $n$ , such that the coefficient of  $x$  in the expansion of  $\left(x^2 + \frac{1}{x^3}\right)^n$  is  ${}^n C_{23}$ , is:

(1) 23

(2) 58

(3) 38

(4) 35

**Ans.** [3]

**Sol.** General term

$$T_{r+1} = {}^n C_r x^{2n-2r} \cdot x^{-3r}$$

$$\therefore 2n - 5r = 1 \Rightarrow 2n = 5r + 1$$

$$\therefore r = \frac{2n-1}{5}$$

$$\therefore \text{Coeff. of } x = {}^n C_{\left(\frac{2n-1}{5}\right)} = {}^n C_{23}$$

$$\therefore \frac{2n-1}{5} = 23 \text{ or } n - \left(\frac{2n-1}{5}\right) = 23$$

$$2n - 1 = 115$$

$$n = 38$$

$$n = 58$$

$$\therefore \text{smallest } n = 38$$

**Q.13** The sum of the real roots of the equation  $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ , is equal to :

(1) -4

(2) 0

(3) 1

(4) 6

**Ans.** [2]

**Sol.** Expand

$$x(-3x \times (x+2) - 2x(x-3)) + (-6)(2(x+2) + 3(x-3)) + (-1)(4x+3(-3x))$$

$$\Rightarrow -5x^3 + 30x - 30 + 5x = 0$$

$$x^3 - 7x + 6 = 0$$

$$\text{Sum of roots} = 0$$

**Q.14** If  $5x + 9 = 0$  is the directrix of the hyperbola  $16x^2 - 9y^2 = 144$ , then its corresponding focus is :

(1)  $\left(\frac{5}{3}, 0\right)$

(2) (5, 0)

(3) (-5, 0)

(4)  $\left(-\frac{5}{3}, 0\right)$

**Ans.** [3]

**Sol.**  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$$a = 3$$

$$b = 4$$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$e^2 = 1 + \frac{16}{9}$$

$$e = \frac{5}{3}$$

$$\therefore \text{focus is } (-ae, 0) = (-5, 0)$$

**Q.15** The area (in sq.units) of the region bounded by the curves  $y = 2^x$  and  $y = |x + 1|$ , in the first quadrant is :

(1)  $\frac{1}{2}$

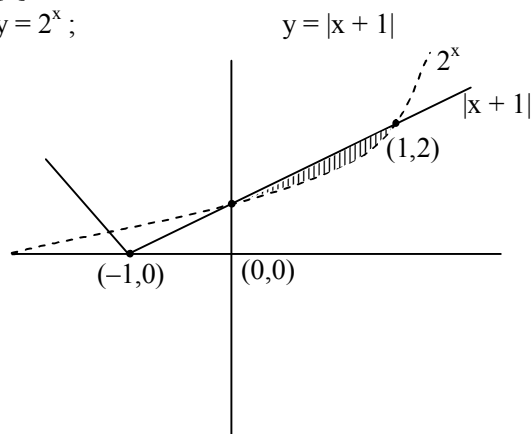
(2)  $\frac{3}{2} - \frac{1}{\log_e 2}$

(3)  $\frac{3}{2}$

(4)  $\log_e 2 + \frac{3}{2}$

**Ans.** [2]

**Sol.**  $y = 2^x$  ;



$$\text{Required Area} = \int_0^1 ((x+1) - 2^x) dx$$

$$= \left( \frac{x^2}{2} + x - \frac{2^x}{\log_e 2} \right)_0^1$$

$$= \frac{3}{2} - \frac{1}{\log_e 2}$$

**Q.16** If the line  $ax + y = c$ , touches both the curves  $x^2 + y^2 = 1$  and  $y^2 = 4\sqrt{2}x$ , then  $|c|$  is equal to :

(1)  $\frac{1}{\sqrt{2}}$

(2) 2

(3)  $\sqrt{2}$

(4)  $\frac{1}{2}$

**Ans.** [3]

**Sol.** Tangent to the curve  $y^2 = 4\sqrt{2}x$  is  $y = mx + \frac{\sqrt{2}}{m}$

It is tangent to the circle  $x^2 + y^2 = 1$

$$\therefore \left| \frac{\sqrt{2}/m}{\sqrt{1+m^2}} \right| = 1 \Rightarrow m = \pm 1$$

$$\therefore \text{tangent are } y = x + \sqrt{2} \text{ \& } y = -x - \sqrt{2}$$

Compare with  $y = -ax + c$

$$\Rightarrow a = \pm 1 \text{ \& } c = \pm \sqrt{2}$$

**Q.17** The integral  $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \cos \text{ec}^{4/3} x \, dx$  is equal to :

(1)  $3^{5/3} - 3^{1/3}$

(2)  $3^{5/6} - 3^{2/3}$

(3)  $3^{4/3} - 3^{1/3}$

(4)  $3^{7/6} - 3^{5/6}$

**Ans.** [4]

**Sol.**  $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \cos \text{ec}^{4/3} x \, dx$

$$= \int \frac{\sec^2 x}{\tan^{4/3} x} dx$$

Let  $\tan x = t$ ,  $\sec^2 x \, dx = dt$

$$= \int \frac{dt}{t^{4/3}}$$

$$I = -3 (t^{-1/3})$$

$$= \left( -3(\tan x)^{-1/3} \right)_{\pi/6}^{\pi/3}$$

$$= 3 \left( 3^{1/3} - \frac{1}{3^{1/3}} \right)$$

$$= 3^{7/6} - 3^{5/6}$$

**Q.18** Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is :

(1) 180

(2) 170

(3) 190

(4) 210

**Ans.** [2]

**Sol.**  $\frac{{}^{20}C_1 \times {}^{17}C_1}{2} = 170$

**Q.19** Let  $a$ ,  $b$  and  $c$  be in G.P. with common ratio  $r$ , where  $a \neq 0$  and  $0 < r \leq \frac{1}{2}$ . If  $3a$ ,  $7b$  and  $15c$  are the first three terms of an A.P., then the 4<sup>th</sup> term of this A.P. is :

(1)  $a$

(2)  $\frac{7}{3}a$

(3)  $\frac{2}{3}a$

(4)  $5a$

**Ans.** [1]

**Sol.**  $a = a$   
 $b = ar$   
 $c = ar^2$   
 $3a, 7b, 15c \longrightarrow \text{A.P.}$   
 $14b = 3a + 15c$   
 $14(ar) = 3a + 15(ar^2)$   
 $15r^2 - 14r + 3 = 0$   
 $\Rightarrow r = \frac{1}{3}, \frac{3}{5}$  (rejected)  
Common difference =  $7b - 3a$   
 $= 7ar - 3a$   
 $= \frac{7a}{3} - 3a$   
 $= -\frac{2}{3}a$   
 $4^{\text{th}}$  term is  $\Rightarrow 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

**Q.20** If  $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$ , where  $c$  is a constant of integration, then  $g(-1)$  is equal to :

(1)  $-1$                       (2)  $-\frac{5}{2}$                       (3)  $1$                       (4)  $-\frac{1}{2}$

**Ans.** [2]

**Sol.** Let  $x^2 = t$   
 $\Rightarrow \frac{1}{2} \int t^2 e^{-t} dt$   
 $= \frac{1}{2} [-t^2 e^{-t} + \int 2te^{-t} dt]$   
 $= \frac{-t^2 e^{-t}}{2} - te^{-t} - e^{-t}$   
 $= \left( -\frac{x^4}{2} - x^2 - 1 \right) e^{-x^2} + c$   
 $g(x) = -\frac{x^4}{2} - x^2 - 1$   
 $g(-1) = -\frac{1}{2} - 1 - 1$   
 $= -\frac{5}{2}$

**Q.21** Let  $y = y(x)$  be the solution of the differential equation,  $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$ ,  $x \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$ , such that  $y(0) = 1$ . Then :

(1)  $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$                       (2)  $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$   
(3)  $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$                       (4)  $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$



**Ans. [2]**

**Sol.**  $\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

I. F. =  $e^{\int \tan x dx} = e^{\ln \sec x} = \sec x$

$y \cdot \sec x = \int (2x + x^2 \tan x) \sec x dx$

$y \sec x = x^2 \sec x + \lambda$

$\Rightarrow y = x^2 + \lambda \cos x$

$y(0) = 0 + \lambda = 1 \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y' = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

**Q.22** The tangent and normal to the ellipse  $3x^2 + 5y^2 = 32$  at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

- (1)  $\frac{14}{3}$                       (2)  $\frac{16}{3}$                       (3)  $\frac{68}{15}$                       (4)  $\frac{34}{15}$

**Ans. [3]**

**Sol.**  $3x^2 + 5y^2 = 32$

$6x + 10yy' = 0$

$y' = \frac{-3x}{5y}$

$y'_{(2,2)} = -\frac{3}{5}$

Tangent  $(y - 2) = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal  $(y - 2) = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area =  $\frac{1}{2}(\text{QR}) \times 2 = \text{QR} = \frac{68}{15}$

**Q.23** If the tangent to the curve  $y = \frac{x}{x^2 - 3}$ ,  $x \in \rho$ , ( $x \neq \pm \sqrt{3}$ ), at a point  $(\alpha, \beta) \neq (0, 0)$  on it is parallel to the line

$2x + 6y - 11 = 0$ , then :

- (1)  $|6\alpha + 2\beta| = 9$                       (2)  $|2\alpha + 6\beta| = 11$                       (3)  $|2\alpha + 6\beta| = 19$                       (4)  $|6\alpha + 2\beta| = 19$

**Ans. [4]**

**Sol.**  $\frac{dy}{dx}\bigg|_{(\alpha,\beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given

$$\frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}$$

$$\Rightarrow \alpha = 0, \pm 3 (\alpha \neq 0)$$

**Q.24** Lines are drawn parallel to the line  $4x - 3y + 2 = 0$ , at a distance  $\frac{3}{5}$  from the origin. Then which one of the following points lies on any of these lines ?

(1)  $\left(\frac{1}{4}, -\frac{1}{3}\right)$

(2)  $\left(-\frac{1}{4}, \frac{2}{3}\right)$

(3)  $\left(\frac{1}{4}, \frac{1}{3}\right)$

(4)  $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

**Ans.** [2]

**Sol.** Line parallel to  $4x - 3y + 2 = 0$

is given as  $4x - 3y + \lambda = 0$

distance from origin is

$$\frac{|\lambda|}{5} = \frac{3}{5}$$

$$\lambda = \pm 3$$

$\therefore$  required lines are  $4x - 3y + 3 = 0$  &  $4x - 3y - 3 = 0$

Now check options

**Q.25** The negation of the Boolean expression  $\sim s \vee (\sim r \wedge s)$  is equivalent to :

(1)  $\sim s \wedge \sim r$

(2)  $r$

(3)  $s \wedge r$

(4)  $s \vee r$

**Ans.** [3]

**Sol.**  $\sim (\sim s \vee (\sim r \wedge s))$

$$s \wedge (r \vee \sim s)$$

$$(s \wedge r) \vee (s \wedge \sim s)$$

$$(s \wedge r) \wedge (s)$$

$$(s \wedge r)$$

**Q.26** If both the mean and the standard deviation of 50 observations  $x_1, x_2, \dots, x_{50}$  are equal to 16, then the mean of  $(x_1 - 4)^2, (x_2 - 4)^2, \dots, (x_{50} - 4)^2$  is :

(1) 400

(2) 480

(3) 380

(4) 525

**Ans.** [1]

**Sol.** Mean( $\mu$ ) =  $\frac{\sum x_i}{50} = 16$

$$\therefore \sum x_i = 16 \times 50$$

$$\text{S.D.}(\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$$

$$\Rightarrow \frac{\sum x_i^2}{50} = 256 \times 2$$

$$\begin{aligned}\text{Required mean} &= \frac{\sum (x_i - 4)^2}{50} \\ &= \frac{\sum x_i^2 + 16 \times 50 - 8 \sum x_i}{50} \\ &= \frac{256 \times 2 + 16 - 8 \times 16}{50} \\ &= 400\end{aligned}$$

**Q.27** Let  $a_1, a_2, a_3, \dots$  be an A.P. with  $a_6 = 2$ . Then the common difference of this A.P., which maximises the product  $a_1 a_4 a_5$ , is :

- (1)  $\frac{3}{2}$                       (2)  $\frac{6}{5}$                       (3)  $\frac{8}{5}$                       (4)  $\frac{2}{3}$

**Ans.** [3]

**Sol.** first term = a

Common difference = d

$$\therefore a + 5d = 2$$

$$a_1 \cdot a_4 \cdot a_5 = a(a + 3d)(a + 4d)$$

$$f(d) = (2 - 5d)(2 - 2d)(2 - d)$$

$$f'(d) = 0 \Rightarrow d = \frac{2}{3}, \frac{8}{5}$$

$$f''(d) < 0 \text{ at } d = \frac{8}{5}$$

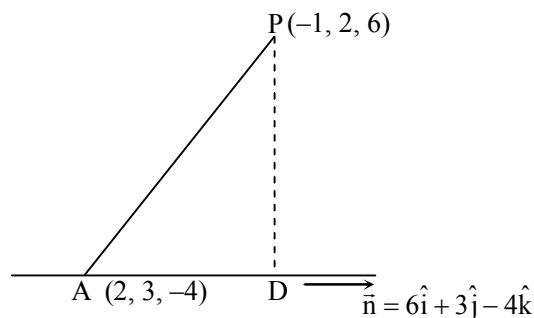
$$\Rightarrow d = \frac{8}{5}$$

**Q.28** The distance of the point having position vector  $-\hat{i} + 2\hat{j} + 6\hat{k}$  from the straight line passing through the point  $(2, 3, -4)$  and parallel to the vector,  $6\hat{i} + 3\hat{j} - 4\hat{k}$  is :

- (1) 6                      (2) 7                      (3)  $2\sqrt{13}$                       (4)  $4\sqrt{3}$

**Ans.** [2]

**Sol.**



$$AD = \frac{|\vec{AP} \cdot \vec{n}|}{|\vec{n}|} = \sqrt{61}$$

$$\begin{aligned}PD &= \sqrt{AP^2 - AD^2} \\ &= \sqrt{110 - 61} \\ &= 7\end{aligned}$$

**Q.29** If  $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$ , where  $-1 \leq x \leq 1$ ,  $-2 \leq y \leq 2$ ,  $x \leq \frac{y}{2}$ , then for all  $x, y$ ,  $4x^2 - 4xy \cos \alpha + y^2$  is equal

to :

- (1)  $4 \sin^2 \alpha$                       (2)  $2 \sin^2 \alpha$                       (3)  $4 \sin^2 \alpha - 2x^2 y^2$                       (4)  $4 \cos^2 \alpha + 2x^2 y^2$

**Ans.** [1]

**Sol.**  $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$

$$\cos \left( \cos^{-1} x - \cos^{-1} \left( \frac{y}{2} \right) \right) = \cos \alpha$$

$$x \frac{y}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\left( \cos \alpha - \frac{xy}{2} \right) = \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}}$$

Squaring both sides

$$x^2 + \frac{y^2}{4} - xy \cos \alpha = 1 - \cos^2 \alpha = \sin^2 \alpha$$

**Q.30** A perpendicular is drawn from a point on the line  $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$  to the plane  $x + y + z = 3$  such that the

foot of the perpendicular Q also lies on the plane  $x - y + z = 3$ . Then the co-ordinates of Q are :

- (1) (4, 0, -1)                      (2) (2, 0, 1)                      (3) (1, 0, 2)                      (4) (-1, 0, 4)

**Ans.** [2]

**Sol.**  $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1} = \lambda$

Let a point P on the line is

$$(2\lambda + 1, -\lambda - 1, \lambda)$$

Foot of  $\perp^r$  Q is given by

$$\frac{x-2\lambda-1}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = -\frac{(2\lambda-3)}{3}$$

$\therefore$  Q lies on  $x + y + z = 3$  &  $x - y + z = 3$

$$\Rightarrow x + z = 3 \text{ \& } y = 0$$

$$\therefore y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda + 3}{3} \Rightarrow \lambda = 0$$

$\therefore$  Q is (2, 0, 1)