JEE Main Online Exam 2019

Questions & Solutions

12th April 2019 | Shift - I

MATHEMATICS

Q.1 The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e \frac{|x^3+1|}{x^2}+C$ (2) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2}+C$ (3) $\log_e \left|\frac{x^3+1}{x}\right|+C$ (4) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|}+C$

Ans. [3]

Sol. $\int \frac{2x^3-1}{x^4+x}dx$

$\Rightarrow \int \frac{(4x^3+1)-(2x^3+2)}{x^4+x}dx$

$\Rightarrow \int \frac{4x^3+1}{x^4+x}dx - 2\int \frac{1}{x}dx$

$x^4 + x = t \Rightarrow (4x^3 + 1)\,dx = dt$

$\Rightarrow \int \frac{dt}{t} - 2\int \frac{1}{x}dx$

$\Rightarrow \ell n|t| - 2\ell nx + C$

$\Rightarrow \ell n\left|\frac{x^4+x}{x^2}\right| + C \Rightarrow \ell n\left|\frac{x^3+1}{x}\right| + C$

Q.2 For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$ If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{7\pi}{12}$ (2) $\tan\frac{11\pi}{12}$ (3) $\tan\frac{\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

Ans. [2]

Sol. $\phi(x) = [(hof)og](x)$

$hof(x) = \frac{1-x}{1+x}$

$(hof)g(x) = \frac{1-\tan x}{1+\tan x}$

$$\phi\left(\frac{\pi}{3}\right) = \frac{1-\tan\frac{\pi}{3}}{1+\tan\left(\frac{\pi}{3}\right)}$$

$$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4}-\frac{\pi}{3}\right) = \tan\left(\frac{11\pi}{12}\right)$$

Q.3 Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio :

(1) 14 : 13 (2) 13 : 11 (3) 5 : 4 (4) 2 : 1

Ans. **[3]**

Sol.

$$\left.\begin{aligned} C_1 &: y^2 = 12x \\ C_2 &: x^2 - \frac{y^2}{8} = 1 \end{aligned}\right] \text{common tangent}$$

$$\left.\begin{aligned} \text{Tangent of } C_1 &: y = mx + \frac{3}{m} \\ C_2 &: y = mx \pm \sqrt{1(m^2)-8} \end{aligned}\right] \Rightarrow \text{Both same}$$

$$\left(\frac{3}{m}\right)^2 = m^2 - 8$$

$9 = m^2\,(m^2 - 8)$

let $m^2 = t$

$\Rightarrow t\,(t-8) = 9$

$\Rightarrow t^2 - 8t - 9 = 0$

$\Rightarrow (t-9)\,(t+1) = 0$

$m^2 = 9$

$\Rightarrow m = -3, 3$

common tangent $y = 3x + 1$

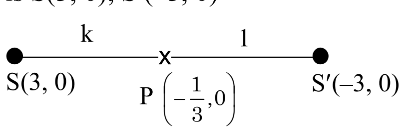
$y = -3x - 1$

P intersection of tangents $P\left(-\frac{1}{3}, 0\right)$

foci of hyperbola $a = 1,\ b = 2\sqrt{2}$

$8 = 1\,(e^2 - 1) \Rightarrow e = 3$

is $S(3, 0),\ S'(-3, 0)$



$$-\frac{1}{3} = \frac{-3k+3}{k+1}$$

$\Rightarrow k = 5 : 4$

Q.4 If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then m·n is equal to

(1) – 1 (2) 1 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

Ans. **[1]**

Sol. $I = \int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx$

$I = \int_0^{\pi/2} \frac{\cos x}{1 + \cos x} = \int_0^{\pi/2} \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 x/2\right) dx$

$I = \left[x - \frac{2}{2}\tan(x/2)\right]_0^{\pi/2}$

$\Rightarrow \left(\frac{\pi}{2} - 1\right) = \frac{(\pi - 2)}{2}$

$m = \frac{1}{2}, n = -2$

$m \cdot n = -1$

Q.5 If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to :

(1) $\frac{7}{116}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{21}{346}$

Ans. **[3]**

Sol. $\alpha + \beta = \frac{25}{375}$

$\alpha\beta = \frac{-2}{375}$

α & $\beta \in (-1, 1)$, then

$\lim_{n\to\infty} \sum_{r=1}^{n} (\alpha^r + \beta^r)$

$\Rightarrow \alpha + \alpha^2$ ……….infinite + $\beta + \beta^2$........ infinite

$\Rightarrow \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$

$\Rightarrow \frac{\alpha(1-\beta) + \beta(1-\alpha)}{(1-\alpha)(1-\beta)} = \frac{1}{12}$

Q.6 The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 5 (2) 3 (3) 7 (4) 4

Ans. **[1]**

Sol. $1 + \sin^4 x = \cos^2 3x$; $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$

$\Rightarrow$ L.H.S. ≥ 1

R.H.S. ≤ 1

Both satisfy when

L.H.S. = R.H.S. = 1

$\sin^4 x = 0$; $\cos^2 3x = 1$

$x = -2\pi, -\pi, 0, \pi, 2\pi$ total 5 solution

Q.7 The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

(1) a circle of radius 1.

(2) the line through the origin with slope – 1

(3) a circle of radius $\frac{1}{2.}$

(4) the line through the origin with slope 1.

Ans. **[4]**

Sol. $|z - i| = |z - 1|$

$z = x + iy$

$|x + i(y - 1)| = |(x - 1) + iy|$

$\Rightarrow x^2 + (y - 1)^2 = (x - 1)^2 + y^2$

$\Rightarrow y = x$ straight line having slope (+)ve 1,

pass origin (0, 0)

Q.8 If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2},\right)$ at $x = 0$ is equal to :

(1) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (3) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

Ans. **[1]**

Sol. $e^y + xy = e$

at $x = 0$; $e^y + 0 = e$

$y = 1$ $y(0) = 1$

diff. w.r.t. x

$e^y.\ y_1 + xy_1 + y = 0$

$e^{(1)}.y_1 + 0 + 1 = 0$

$y_1 = -1/e$ at $x = 0$

diff again w.r.t. to x

$e^y.y_2 + y_1\ e^y.y_1 + xy_2 + y_1 + y_1 = 0$

at $x = 0$

$\Rightarrow e^{(1)}(y_2) + \left(\frac{-1}{e}\right)^2 e^1 + 0.y_2 + 2\left(\frac{-1}{e}\right) = 0$

$y_2 = 1/e^2 \quad x = 0$

$\left(\frac{-1}{e}, \frac{1}{e^2}\right)$

Q.9 The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) $2^{20} - 1$ (2) 2^{20} (3) 2^{21} (4) $2^{20} + 1$

Ans. **[2]**

Sol. total 31 $\Rightarrow \underbrace{\text{21 distinct \& 10 identical}}_{10}$

$S = {}^{21}C_{10} + {}^{21}C_9(1) + {}^{21}C_8(1) + {}^{21}C_7 + {}^{21}C_6 \ldots\ldots {}^{21}C_0(1)$

$S = {}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} \ldots\ldots {}^{21}C_{21}$

$2S = 2^{21}$

$S = 2^{20}$

Q.10 For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \ldots.. + \left[-\frac{1}{3} - \frac{99}{100}\right]$ is :

(1) – 153 (2) – 135 (3) – 133 (4) – 131

Ans. **[3]**

Sol. $\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \ldots.. \left[-\frac{1}{3} - \frac{66}{100}\right]}_{-1(67\text{ times})} + \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + \left[-\frac{1}{3} - \frac{68}{100}\right] + \ldots.. \left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33\text{ times})}$

$= -67 - 66 = -133$

Q.11 If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(5, 3\sqrt{6})$ (2) $(4, 3\sqrt{3})$ (3) $(4, 3\sqrt{2})$ (4) $(3, 3\sqrt{3})$

Ans. **[2]**

Sol. $f(x)\uparrow : f'(x) \ge 0$; & $f(x_1) \le f(x_2)$, $x \in [0, 3]$

$\Rightarrow f(0) \le f(3)$

$0 \le 3\sqrt{3k - 9}$

$k \ge 3$

$f'(x) = \frac{x(k - 2x)}{2\sqrt{kx - x^2}} + \sqrt{kx - x^2}$

$= \frac{x(k - 2x) + 2(kx - x^2)}{2\sqrt{kx - x^2}} \ge 0$

$= \frac{3kx - 4x^2}{2\sqrt{kx - x^2}} \ge 0$

& $kx - x^2 \ge 0$

$3kx - 4x^2 \geq 0$

$x\,(3k - 4x) \geq 0 \quad x \in [0,3]$

$x\,(4x - 3x) \leq 0$

$k \geq 4$

$k = 4 \Rightarrow$ minimum value of k

$m = 4$

$f(3) = M = 3\sqrt{4(3) - 9} = 3\sqrt{3}$

$M = 3\sqrt{3}$

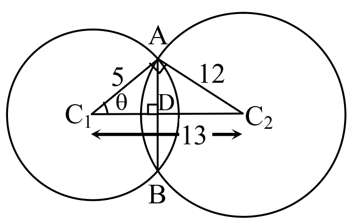
$(m, M) = (4, 3\sqrt{3})$

Q.12 If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{13}{5}$ (2) $\frac{60}{13}$ (3) $\frac{120}{13}$ (4) $\frac{13}{2}$

Ans. **[3]**

Sol.



ΔAC_1C_2

$\tan\theta = \frac{12}{5} \Rightarrow \sin\theta = \frac{12}{13}$ .....(i)

ΔACD:

$\sin\theta = \frac{AB/2}{5}$ .....(ii)

(i) & (ii)

$\Rightarrow \frac{AB}{2.5} = \frac{12}{13}$

$AB = \frac{120}{13}$

Q.13 The coefficient of x^{18} in the product $(1 + x)\,(1 - x)^{10}\,(1 + x + x^2)^9$ is :

(1) 126 (2) – 84 (3) – 126 (4) 84

Ans. **[4]**

Sol. coefficient of x^{18}

$(1 + x)\,(1 - x)^{10}\,(1 + x + x^2)^9$

$\Rightarrow (1 + x)\,(1 - x)\,(1 - x)^9\,(1 + x + x^2)^9$

$\Rightarrow (1 - x^2)\,(1 - x^3)^9$

$\Rightarrow (1 - x^2)\ [{}^9C_r(-1)^r(x^{3r})]$

$\Rightarrow {}^9C_r(-1)^r x^{3r} - {}^9C_r(-1)^r x^{3r+2}$

for x^{18}

$3r = 18$	$3r+2=18$
$r = 6$	r not possible

then coefficient of x^{18} is ${}^9C_6(-1)^6 = 84$

Q.14 Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$, If value of y is 1 when x = 1, then the value of x for which y = 2, is :

(1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$ (3) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (4) $\frac{5}{2} + \frac{1}{\sqrt{e}}$

Ans. **[1]**

Sol. $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int 1/y^2 dy} = e^{-1/y}$

sol. $xe^{-1/y} = \int e^{-1/y}(1/y^3)dy$

$-\frac{1}{y} = t$

$\Rightarrow \frac{1}{y^2}dy = dt$

$xe^t = \int e^t(-t).dt$

$x.e^t = -\left[t.e^t - e^t\right] + c$

$x = -t + 1 + c.e^{-t}$

$x = \frac{1}{y} + 1 + c.\ e^{1/y}$

give y(1) = 1

$1 = 1 + 1 + c.e^{1/1}$

$c = -e^{-1}$

$x = \frac{1}{y} + 1 + (-)e^{-1}.\ e^{1/y}$

at y = 2

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

Q.15 If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then a – b is equal to :

(1) $\frac{8}{3}$ (2) 6 (3) $\frac{10}{3}$ (4) $-\frac{2}{3}$

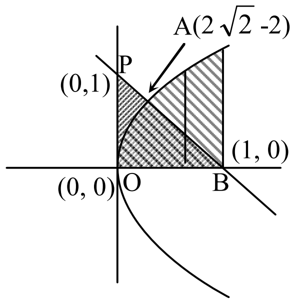
Ans. **[2]**

Sol. $C_1 : y^2 \le 4x$

$C_2 : x + y \le 1$

$x \ge 0$

$y \ge 0$



$\Rightarrow y^2 = 4x$; $y^2 = 4(1-y)$

$y^2 + 4y + 4 = 0$

$y = 2\sqrt{2} - 2, -2\sqrt{2} - 2$

Area : shaded region of curve OAB

A = Area of Δ_{OBP} – Area of region OAP

$\Delta_{OBP} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of OAP = $\int_0^{2\sqrt{2}-2} \frac{y^2}{4} dy + \int_{2\sqrt{2}-2}^{1} (1-y)\, dy$

$= \frac{1}{12}\left[y^3\right]_0^{2\sqrt{2}-2} + \left[y - \frac{y^2}{2}\right]_{2\sqrt{2}-2}^{1}$

$= \frac{1}{12}[(2\sqrt{2}-2)^3] + \left[\left(1 - \frac{1}{2}\right) - \left\{(2\sqrt{2}-2) - \frac{(2\sqrt{2}-2)^2}{2}\right\}\right]$

$= \frac{23}{6} - \frac{8}{3}\sqrt{2}$

$A = \frac{1}{2} - \frac{23}{6} + \frac{8\sqrt{2}}{3}$

$a = \frac{8}{3}, b = -\frac{20}{6}$

$\therefore$ a – b = 6

Q.16 If the data $x_1, x_2, \ldots\ldots x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is :

(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. **[2]**

Sol. $x_1 + x_2 + x_3 + x_4 = 44$

$x_5 + x_6 \ldots\ldots x_{10} = 96$

$\sum x_i = 140$

$\overline{x} = \frac{140}{40} = 14$

$\sum \overline{x}^2 = 2000$

$$SD = \sqrt{\frac{\sum \overline{x}^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{2000}{10} - \left(\frac{140}{10}\right)^2} = \sqrt{200 - 196} = 2$$

Q.17 If A is a symmetric matrix and B is a skew-symmetric matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Ans. **[4]**

Sol. A is symmetric $A^T = A$

B is skew 8 symmetry $B^T = -B$

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \qquad \ldots (i)$$

Transpose

$$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \qquad \ldots .(ii)$$

From (i) + (ii)

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$$

From (i) – (ii)

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\text{then } AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

Q.18 If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4,4) then PQ is equal to :

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{\sqrt{221}}{2}$ (3) $\frac{\sqrt{157}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

Ans. **[4]**

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Normal at P : $(2\cos\theta, \sqrt{3}\sin\theta)$ is $2x\sec\theta - \sqrt{3}\ \text{cosec}\ \theta\ y = 4 = 3 = 1$

Normal at P: $2x\sec\theta - \sqrt{3}\ y\ \text{cosec}\theta = 1$

Slope of normal = $\frac{-2\sec\theta}{-\sqrt{3}\ \text{cosec}\theta} = \frac{2}{\sqrt{3}}\ \frac{\sin\theta}{\cos\theta}$

Normal parallel to $2x + y = 4$

then $\frac{2}{\sqrt{3}}\tan\theta = -2$; $\tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

Point P $\left(-1, \frac{3}{2}\right)$, Q(4, 4)

$PQ = \sqrt{(4+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$

Q.19 Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :

(1) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (3) $4(2\hat{i} + 2\hat{j} + \hat{k})$ (4) $4(2\hat{i} + 2\hat{j} - \hat{k})$

Ans. **[1]**

Sol. $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$

$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$

A vector $\vec{r}$ perpendicular to $(\vec{a} + \vec{b})$ & $(\vec{a} - \vec{b})$ & magnitude 12

$\vec{r} = 12.\hat{n}$

$\therefore\ \hat{n} = \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|}$

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 8[2\hat{i} - 2\hat{j} - \hat{k}]$

$|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = 8.3$

$\hat{n} = \frac{(2\hat{i} - 2\hat{j} - \hat{k})}{3}$

$\vec{r} = 4.\ (2\hat{i} - 2\hat{j} - \hat{k})$

Q.20 If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det(A) + 1 = 0, is :

(1) 2 (2) – 1 (3) 0 (4) 1

Ans. **[4]**

Sol. If B is inverse of A then

AB = I

det(AB) = det(I)

det(A).det(B) = 1

Given det (A) = –1

then det (B) = –1

$$\begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = -1$$

$5(-2-3) - 2\alpha[0-\alpha] + 1[-2\alpha] = -1$

$2\alpha^2 - 2\alpha - 25 = -1$

$2\alpha^2 - 2\alpha - 24 = 0$

$(\alpha - 4)(\alpha + 3) = 0$; $\alpha = 4, -3$

Sum of value of $\alpha = 4 - 3 = 1$

Q.21 A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec, then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

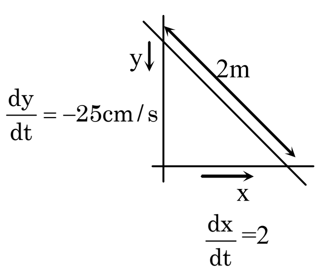
(1) $\frac{25}{3}$ (2) 25 (3) $25\sqrt{3}$ (4) $\frac{25}{\sqrt{3}}$

Ans. **[4]**

Sol. $x^2 + y^2 = 4$

Diff. w.r.t. t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



At y = 1; $x = \sqrt{3}$

then

$$\boxed{\frac{dx}{dt} = \frac{25}{\sqrt{3}} cm/sec}$$

Q.22 The equation $y = \sin x \sin(x+2) - \sin^2(x+1)$ represents a straight line lying in :

(1) first, second and fourth quadrants (2) first, third and fourth quadrants

(3) second and third quadrants only (4) third and fourth quadrants only

Ans. **[4]**

Sol. $y = \sin x \sin(x + 2) - \sin^2(x + 1)$

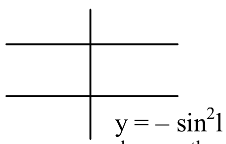
$$y = \frac{1}{2}[2\sin x \sin(x+2)] - \frac{1}{2}[2\sin^2(x+1)]$$

$$y = \frac{1}{2}[\cos(2) - \cos(2x+2)] - \frac{1}{2}[1 - \cos 2(x+1)]$$

$$y = \frac{1}{2}[\cos 2 - 1)]$$

$$y = (-)\frac{1}{2}(2)\sin^2 1$$

$y = -\sin^2 1$



Graph IIIrd & IVth

Q.23 If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively :

(1) T, F, T (2) F, T, T (3) T, T, F (4) T, F, F

Ans. **[3]**

Sol. $p \rightarrow (\sim q \vee r)$ is false

It is true when

$p \rightarrow T$ & $(\sim q \vee r) = $ false

It will true : ~ q false & r false

$\sim q \rightarrow F \mid r \rightarrow F$

$\Rightarrow q \rightarrow T$

Truth value of p, q, r $\Rightarrow$ T, T, F

Q.24 Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 17 (2) 1 (3) 137 (4) 121

Ans. **[3]**

Sol. Given : np = 8 ; nqp = 4

$$q = \frac{1}{2} \Rightarrow p = \frac{1}{2} \Rightarrow n = 16$$

$P(x \le 2) = P(x = 0) + P(x = 1) + (x = 2)$

$$= {}^{16}C_0\left(\frac{1}{2}\right)^{16}\left(\frac{1}{2}\right)^{0} + {}^{16}C_1\left(\frac{1}{2}\right)^{15}\left(\frac{1}{2}\right)^{1} + {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{16}[1 + 16 + 120] = \frac{137}{2^{16}}$$

$\therefore k = 137$

Q.25 Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) – 320 (2) – 380 (3) – 410 (4) – 260

Ans. **[1]**

Sol. S_n = Sum of n terms of an A.P.

$S_4 = 16 = a + 3d$ …..(i)

$S_6 = -48 = a + 5d$ …..(ii)

From (i) & (ii)

$d = -32 \quad a = 112$

$S_{10} = \frac{10}{2}[2.(112) + (10-1)(-32)] = 5[-64]$

$S_{10} = -320$

Q.26 Let f : R → R be a continuously differentiable function such that f(2) = 6 and f(2) = $\frac{1}{48}$. If

$\int_6^{f(x)} 4t^3\,dt = (x-2)g(x)$, then $\lim\limits_{x\to 2} g(x)$ is equal to :

(1) 18 (2) 36 (3) 12 (4) 24

Ans. **[1]**

Sol. $\int_6^{f(x)} 4t^3.dt\ (x-2)g(x)$; f(2) = 6 ; $f'(2) = \frac{1}{48}$

$$g(x) = \frac{\int_6^{f(x)} 4t^3 dt}{(x-2)}$$

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{(x-2)} \text{ at } x = 2\ \frac{0}{0} \text{ form}$$

L-hospital rule

$$\lim_{x\to 2} \frac{4[f(x)]^3.f'(x)}{1}$$

At x = 2, $4[f(2)]^3.f'(2)$

$\Rightarrow 4(6)^3\left(\frac{1}{48}\right) = 18$

Q.27 The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

(1) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$ (2) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$ (3) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$ (4) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Ans. **[4]**

Sol. $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\left[\frac{12}{13}\sqrt{1-\frac{9}{25}} - \frac{3}{5}\sqrt{1-\frac{144}{169}}\right]$

$= \sin^{-1}\left(\frac{33}{65}\right)$

$= \cos^{-1}\left(\frac{56}{65}\right)$

$= \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Q.28 If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

(1) $-\frac{1}{\sqrt{3}}$ (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

Ans. **[3]**

Sol. Volume of parallelopiped

$$\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = 1[1 - 0] - \lambda\,[-\lambda^2] + 1\,[-\lambda]$$

$V = |1 + \lambda^3 - \lambda|$

$\frac{dV}{d\lambda} = 3\lambda^2 - 1 \Rightarrow \frac{dV}{d\lambda} = 0 \Rightarrow \lambda = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

$\frac{d^2V}{d\lambda^2} = 6\lambda$ for minimum

$\frac{d^2V}{d\lambda^2} > 0$ at $\lambda = \frac{1}{\sqrt{3}}$

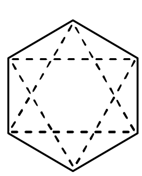
Q.29 If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\frac{1}{10}$ (2) $\frac{3}{10}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

Ans. **[1]**

Sol. Number of total triangle = ${}^6C_3$

Equilateral $\Delta = 2$



Prob. $= \frac{2}{{}^6C_3} = \frac{1}{10}$

Q.30 If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane 2x + 3y – z+13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to :

(1) $2\sqrt{7}$ (2) 14 (3) $2\sqrt{14}$ (4) $\sqrt{14}$

Ans. **[3]**

Sol. $\frac{x-2}{3}=\frac{y+1}{g}=\frac{z-1}{-1}=\lambda$

General point

$(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ = intersect plane

$2x + 3y - z + 13 = 0$ at P then

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$\lambda = -1$

$P(-1, -3, 2)$

Line intersect plane : $3x + y + 4z = 16$ at Q then

$3(3\lambda + 2) + 2\lambda - 1 + 4(-\lambda + 1) = 16 \Rightarrow \lambda = 1$

$Q(5, 1, 0)$ then $PQ = \sqrt{(5+1)^2+(1+3)^2+(0-2)^2}$

$PQ = 2\sqrt{14}$