JEE Main Online Exam 2019

Questions & Solutions

12th April 2019 | Shift - I

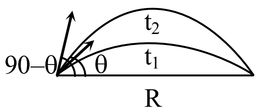
PHYSICS

Q.1 A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is -

(1) 2R/g (2) R/2g (3) R/g (4) R/4g

Ans. **[1]**

Sol.



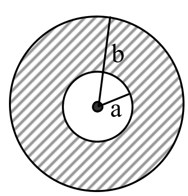
For θ & 90 – θ angle of projection, range will be same

Time of flight for θ : $t_1 = \frac{2u\sin\theta}{g}$

Time of flight for 90 – θ : $t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$

$\Rightarrow t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2u^2}{g}\left(\frac{\sin 2\theta}{g}\right) = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$

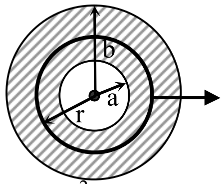
Q.2 A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is:



(1) $\frac{a+b}{3}$ (2) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (3) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (4) $\frac{a+b}{2}$

Ans. **[2]**

Sol.



$dm = \left(\frac{\sigma_0}{r}\right)(2\pi r dr) = 2\pi\sigma_0 dr$

$I = mk^2$: k = radius of gyration

$$\int_a^b (dm)r^2 = k^2 \int_a^b dm$$

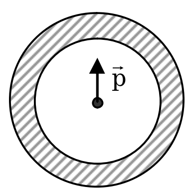
$$\Rightarrow \int_a^b (2\pi\sigma_0 dr)r^2 = k^2 \int_a^b 2\pi\sigma_0 dr$$

$$\Rightarrow 2\pi\sigma_0 \left[\frac{b^3 - a^3}{3}\right] = k^2 2\pi\sigma_0 (b-a)$$

$$\Rightarrow \frac{(b-a)(b^2 + a^2 + ab)}{3} = k^2(b-a)$$

$$k = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

Q.3 Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case;



(1) surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

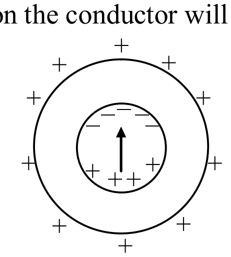
(2) surface charge density on the outer surface depends on $|\vec{p}|$

(3) surface charge density on the inner surface of the shell is zero everywhere

(4) electric field outside the shell is the same as that of a point charge at the centre of the shell

Ans. **[4]**

Sol. The charge distribution at equilibrium on the conductor will be like :



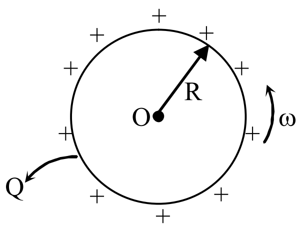
Net charge on the outer surface = Q
Total charge on the inner surface = 0
So for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on the outer surface only.

Q.4 A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

(1) 7×10^{-6} C (2) 4×10^{-5} C (3) 2×10^{-6} C (4) 3×10^{-5} C

Ans. **[4]**

Sol.



$R = 10$ cm $= 10^{-1}$ m

$\omega = 40\,\pi$ rad/s

$B_0 = 3.8 \times 10^{-9}$T

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q}{\left[\frac{2\pi}{\omega}\right]}$$

$$B_0 = \frac{\mu_0 I}{2R} = \frac{\mu_0\left[\frac{Q}{\left(\frac{2\pi}{\omega}\right)}\right]}{2R}$$

$$\Rightarrow B_0 = \frac{\mu_0 Q\omega}{4\pi R}$$

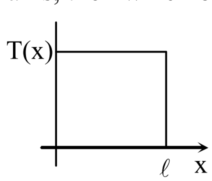
$$\Rightarrow Q = \frac{B_0 4\pi R}{\mu_0 \omega}$$

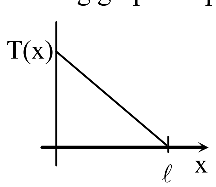
$$= \frac{(3.8\times10^{-9})(4\pi\times10^{-1})}{(4\pi\times10^{-7})(40\pi)}$$

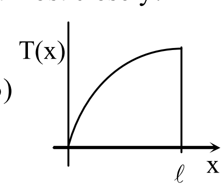
$$= \frac{380\times10^{-2}\times10^{-9}\times10^{-1}}{40\times3.14\times10^{-7}}$$

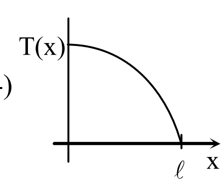
$$= \frac{380}{125.6}\times 10^{-12+7} = 3\times10^{-5}\text{ C}$$

Q.5 A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

(1) 

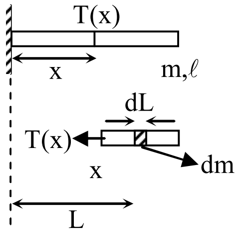
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



$$T(x) = \int_x^{\ell} (dm)\omega^2 L$$

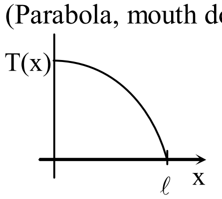
$$= \int_x^{\ell} \left[\frac{m}{\ell} dL\right] \omega^2 L$$

$$= \frac{m}{\ell}\omega^2 \left[\frac{L^2}{2}\right]_x^{\ell}$$

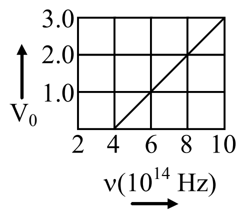
$$T(x) = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

$$T(x) = A - Bx^2$$

(Parabola, mouth down)



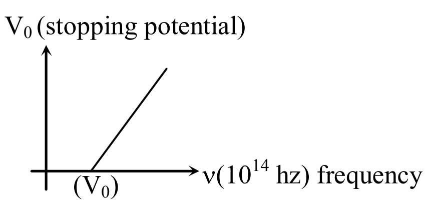
Q.6 The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be: (Given: Planck's constant (h) = 6.63×10^{-34} Js, electron charges e = 1.6×10^{-19} C)



(1) 1.95 eV (2) 2.12 eV (3) 1.82 eV (4) 1.66 eV

Ans. **[4]**

Sol.



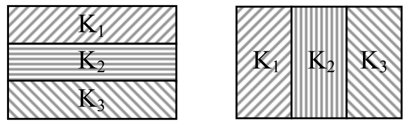
Threshold frequency

Work function = $\phi_0 = h\nu_0$

$$= \frac{(6.63\times10^{-34})(4\times10^{14})}{1.6\times10^{-19}} \text{ eV} = 1.66 \text{ eV}$$

Q.7 Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig.I, and the second one is filled as shown in fig II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)):



(I) (II)

(1) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{K_1K_2K_3}$

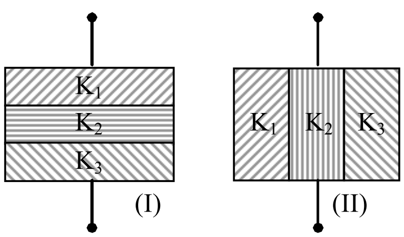
(2) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

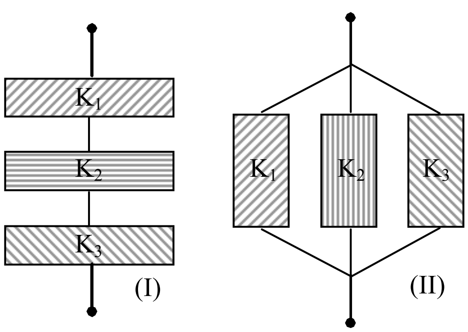
(3) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{9K_1K_2K_3}$

(4) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

Ans. **[4]**

Sol.





$$\frac{1}{C_1} = \frac{1}{k_1\left(\frac{\varepsilon_0 A}{d/3}\right)} + \frac{1}{k_2\left(\frac{\varepsilon_0 A}{d/3}\right)} + \frac{1}{k_3\left(\frac{\varepsilon_0 A}{d/3}\right)}$$

$$\frac{1}{C_1} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]$$

$$C_1 = \frac{3\varepsilon_0 A}{d} \frac{k_1k_2k_3}{(k_1k_2 + k_2k_3 + k_3k_1)}$$

$$C_2 = k_1 \frac{\varepsilon_0 A/3}{d} + k_2 \frac{\varepsilon_0 A/3}{d} + k_3 \frac{\varepsilon_0 A/3}{d} = \frac{\varepsilon_0 A}{3d}[k_1 + k_2 + k_3]$$

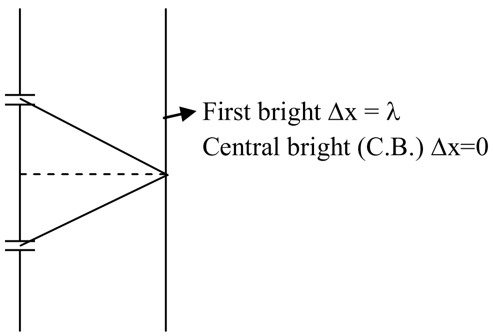
$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_1V^2}{\frac{1}{2}C_2V^2} = \frac{9k_1k_2k_3}{(k_1 + k_2 + k_3)(k_1k_2 + k_2k_3 + k_3k_1)}$$

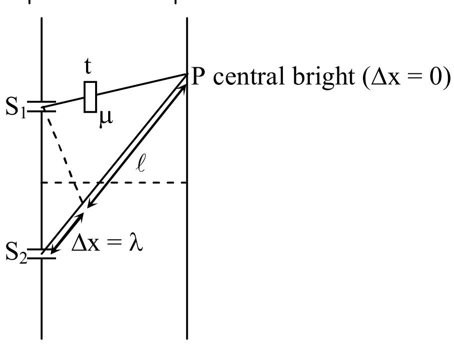
Q.8 In a double slit experiment, when a thin film of thickness t having refractive index μ. is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1) $\frac{\lambda}{2(\mu-1)}$ (2) $\frac{\lambda}{(2\mu-1)}$ (3) $\frac{2\lambda}{(\mu-1)}$ (4) $\frac{\lambda}{(\mu-1)}$

Ans. **[4]**

Sol. Normal YDSE without slab





For central bright at the position of first bright

⇒ $S_2P - S_1P = 0$

⇒ $(\lambda + \ell) - (\ell - t + \mu t) = 0$

optical path length

⇒ $\lambda + \ell - \ell + t - \mu t = 0$

⇒ $\lambda = t(\mu - 1)$

⇒ $\boxed{t = \frac{\lambda}{\mu - 1}}$

Q.9 The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1) 0.12 μm (2) 0.38 μm (3) 0.24 μm (4) 0.48 μm

Ans. **[3]**

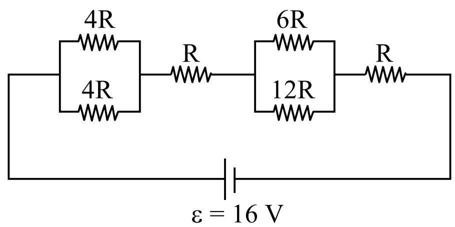
Sol. Numerical aperture of the microscope is given as

$$NA = \frac{0.61\lambda}{d}$$

d = minimum separation between two points to be seen as distinct

$$\Rightarrow d = \frac{0.61\lambda}{NA} = \frac{(0.61)(5000\times10^{-10})}{1.25} = 2.4\times10^{-7}\ m = 0.24\ \mu m$$

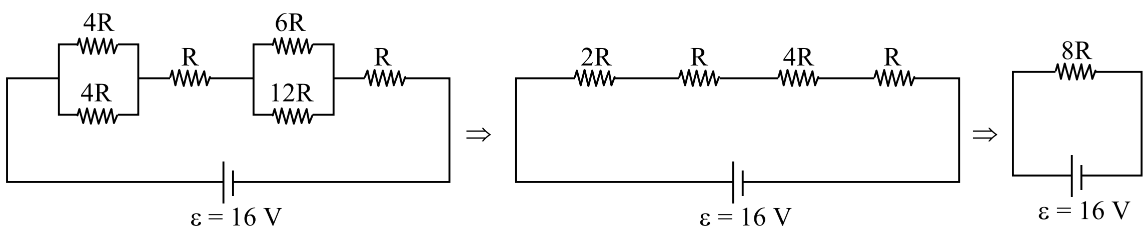
Q.10 The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is:



(1) 16 Ω (2) 1 Ω (3) 8 Ω (4) 6 Ω

Ans. **[3]**

Sol.



$$P = \frac{V^2}{R}$$

$$\Rightarrow 4 = \frac{16\times16}{8R}$$

$$\Rightarrow \boxed{R = 8\Omega}$$

Q.11 The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$) :

(1) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ ms^{-1}$

(2) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(4) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ ms^{-1}$

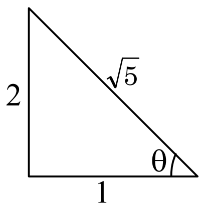
Ans. **[1]**

Sol. $y = x \tan\theta \left(1 - \frac{x}{R}\right)$ .....(i)

Given eq^n of trajectory : $y = 2x - 9x^2 = 2x\left(1 - \frac{9x}{2}\right) = 2x\left(1 - \frac{x}{\left(\frac{2}{9}\right)}\right)$ .... (2)

Comparing equation (1) & (2)

$\tan\theta = 2$ & $R = \frac{2}{9}$



$\cos\theta = \frac{1}{\sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\Rightarrow \frac{u^2 \sin 2\theta}{g} = \frac{2}{9}$

$\Rightarrow \frac{u^2\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]}{10} = \frac{2}{9}$

$\Rightarrow u^2\left[\frac{4}{5}\right] = \frac{2}{9} \times 10$

$\Rightarrow u^2 = \frac{2 \times 10 \times 5}{9 \times 4}$

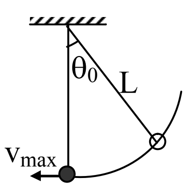
$\Rightarrow u = \frac{10}{6} = \frac{5}{3}$ m/s

Q.12 A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his center of mass moves by a distance $\ell(\ell << L)$, is close to;

(1) $mg\ell\,(1 + \theta_0^2)$ (2) $mg\ell$ (3) $mg\ell\left(1 + \frac{\theta_0^2}{2}\right)$ (4) $mg\ell(1 - \theta_0^2)$

Ans. **[1]**

Sol.



The force acting on the man at the lowest point

$\Rightarrow F = mg + \frac{mv_{max}^2}{L}$

$= mg + \frac{m}{L}(V_{max})^2$

$= mg + \frac{m}{L}[A\omega]^2$

$[T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \omega = \sqrt{\frac{g}{L}}]$

$= mg + \frac{m}{L}[(\theta_0 L)\left(\frac{\sqrt{g}}{L}\right)]^2$

$= mg + mg\,\theta_0^2$

$= mg\,(1 + \theta_0^2)$

Work done = (F) (displacement)

$= [mg(1 + \theta_0^2)][\ell]$

$= mg\ell(1 + \theta_0^2)$

Q.13 A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V= 0 at infinity)

(1) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{-\vec{p}}{4\pi \in_0 d^3}$

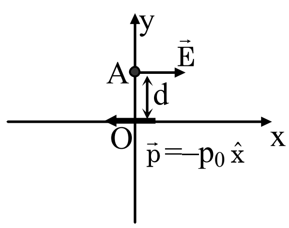
(2) $0, \frac{-\vec{p}}{4\pi \in_0 d^3}$

(3) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{\vec{p}}{4\pi \in_0 d^3}$

(4) $0, \frac{-\vec{p}}{4\pi \in_0 d^3}$

Ans. **[2]**

Sol.



A is an equatorial point w.r.t the dipole :

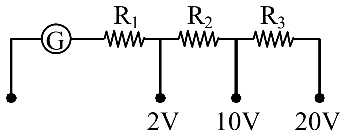
Electric field at A $= \frac{-K\vec{p}}{r^3}$

$= -\left\{\frac{1}{4\pi\varepsilon_0}\frac{(-p_0\hat{x})}{d^3}\right\}$

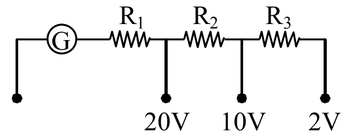
$= \frac{p_0}{4\pi\varepsilon_0 d^3}\hat{x}$

Electric potential at A = 0

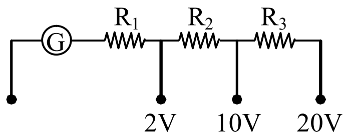
Q.14 A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivitv of 20 μA/division. It is to be converted to a voltmeter with three ranges of 0-2V, 0-10 V and 0-20 V. The appropriate circuit to do so is

(1)


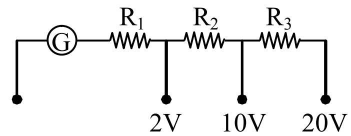
$R_1 = 1900\Omega$
$R_2 = 8000\Omega$
$R_3 = 10000\Omega$

(2)


$R_1 = 19900\Omega$
$R_2 = 9900\Omega$
$R_3 = 1900\Omega$

(3)


$R_1 = 1900\Omega$
$R_2 = 9900\Omega$
$R_3 = 19900\Omega$

(4)


$R_1 = 2000\Omega$
$R_2 = 8000\Omega$
$R_3 = 10000\Omega$

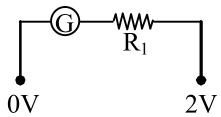
Ans. **[1]**

Sol. sensitivity = 20 μA/div

Total division = 50

⇒ maximum current through galvanometer can be = $I_{max} = (50)\,(20\ \mu A) = 10^{-3}$ A

$R_G = 100\Omega$



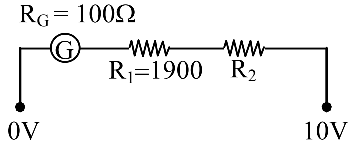
$$I_{max} = \frac{2}{100 + R_1} = 10^{-3}$$

$$\Rightarrow \frac{2}{10^{-3}} = 100 + R_1$$

$\Rightarrow R_1 = 2000 - 100$

$\Rightarrow R_1 = 1900\ \Omega$

$R_G = 100\Omega$



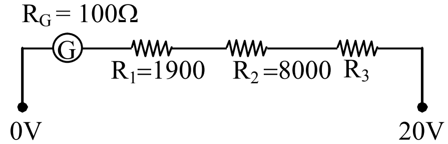
$$I_{max} = \frac{10}{R_G + R_1 + R_2}$$

$$\Rightarrow 10^{-3} = \frac{10}{100 + 1900 + R_2}$$

$$\Rightarrow R_2 + 2000 = \frac{10}{10^{-3}}$$

$\Rightarrow R_2 = 10000 - 2000 = 8000\Omega$

$R_G = 100\Omega$

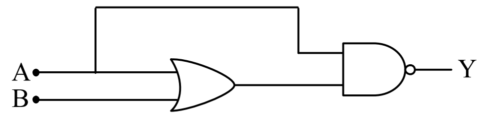
⇒


$$I_{max} = \frac{20}{100 + 1900 + 8000 + R_3}$$

$10000 + R_3 = 20000$

$R_3 = 10000\Omega$

Q.15 The truth table for the circuit given in the fig. is:



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

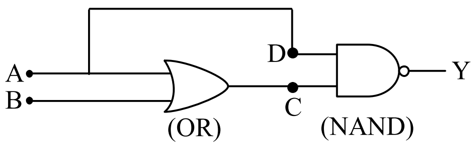
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Ans. **[2]**

Sol.

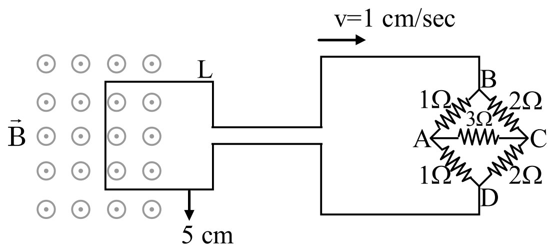


Effectively D $\equiv$ A & C is output of 'OR' gate

Y is output of 'NAND' gate

A	B	C	D	Y
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

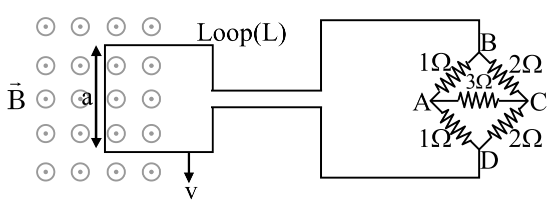
Q.16 The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to :



(1) 115 μA (2) 150 μA (3) 170 μA (4) 60 μA

Ans. **[3]**

Sol.



$V = 1 \text{ cm/s} = 10^{-2} \text{ m/s}$

$R_{loop} = 1.7\ \Omega$

$A = 5 \text{ cm} = 5 \times 10^{-2}$ m equivalent circuit : $R_{total} = R_{loop} + R_{wheatstone}$

$$R_{eq} = \frac{(4)(2)}{4+2} = \frac{8}{6} = \frac{4}{3} = 1.3\ \Omega$$

$R_{Total} = 1.7 + 1.3$

$\Rightarrow R_{Total} = 3\Omega$

Induced emf = $VB\ell$

$$\Rightarrow \text{current} = I = \frac{(VB\ell)}{R_{Total}}$$

$$= \frac{(10^{-2})(1)(5\times10^{-2})}{3}$$

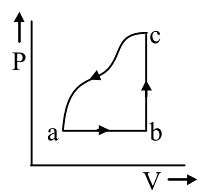
$$= \frac{5}{3}\times10^{-4}$$

$= 1.67 \times 10^{-4}$

$= 167 \times 10^{-6 \text{ A}}$

$\approx 170\ \mu\text{A}$

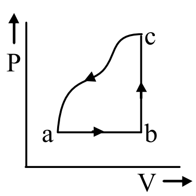
Q.17 A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is:



(1) 140 J (2) 130 J (3) 100 J (4) 120 J

Ans. **[2]**

Sol.



$\Delta U_{ca} = -180\text{J}$

$\Delta U_{ac} = +180\text{J}$ (state function)

$\Delta Q = \Delta U + \Delta W$

$\Delta Q_{a\to c} = \Delta U_{a\to c} + \Delta W_{a\to c}$

$\Delta Q_{a\to b} + \Delta Q_{b\to c} = 180 + \Delta W_{a\to c}$

$250 + 60 = 180 + \Delta W_{a\to c}$

$\Delta W_{a\to c} = 310 - 180 = 130$ J

Q.18 An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation $\hat{s}$, is :

(1) $\hat{s} = \frac{-4\hat{k} + 3\hat{j}}{5}$ (2) $\hat{s} = \frac{4\hat{j} - 3\hat{k}}{5}$ (3) $\hat{s} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$ (4) $\hat{s} = \frac{3\hat{i} - 4\hat{j}}{5}$

Ans. **[3]**

Sol. $\vec{E} = E_0\hat{n}[\omega t + (6y - 8z)]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - (8z - 6y)]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - \left(\frac{8}{10}\hat{k} - \frac{6}{10}\hat{j}\right).10]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - \hat{s}k]$

$\hat{s}$ = direction of propagation

$\hat{s} = \left(\frac{8\hat{k} - 6\hat{j}}{10}\right)$

$= \left(\frac{4\hat{k} - 3\hat{j}}{5}\right)$

$= \frac{-3\hat{j} + 4\hat{k}}{5}$

Q.19 At 40º C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40ºC to 20ºC it regains its original length of 0.2 m. The value of M is close to :

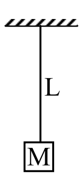
(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; g= 10 ms^{-2})

(1) 1.5 kg (2) 0.5 kg (3) 9 kg (4) 0.9 kg

Ans. **[3]**

Sol. $r = 1 \text{ mm} = 10^{-3}$ m

$L_0 = 0.2$ m



$\Delta L = L_0 \times \Delta T$

$Y = \frac{(F/A)}{(\Delta L/L_0)}$

$\Rightarrow \frac{\Delta L}{L_0} = \frac{F}{AY}$

$\Rightarrow \Delta L = \frac{FL_0}{AY}$

$\Rightarrow L_0\,\alpha\,\Delta T = \frac{FL_0}{AY}$

$\Rightarrow \ \alpha\Delta T = \dfrac{Mg}{AY}$

$\Rightarrow \ M = \dfrac{\alpha\Delta TAY}{g}$

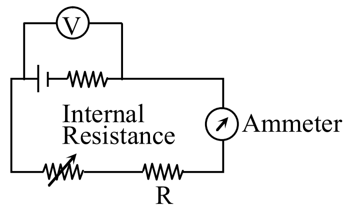
$= \dfrac{(10^{-5})(20)(\pi \times 10^{-6})(10^{11})}{(10)}$

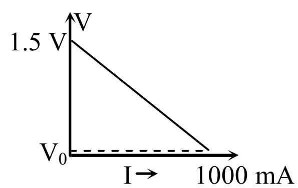
$= 2\pi$ kg

$= 2 \times 3.14$ kg

$= 6.28$ kg (closest to 9)

Q.20 To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained;





If V_0 is almost zero, identify the correct statement :

(1) The value of the resistance R is 1.5 Ω

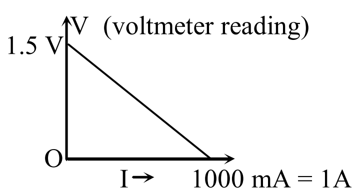
(2) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

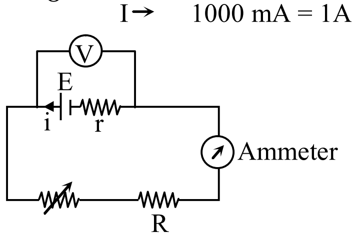
(3) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

Ans. **[2]**

Sol.





When voltmeter reading is zero

$\Rightarrow \ E - ir = 0$

$\Rightarrow \ E - \left(\dfrac{E}{R + r}\right) r = 0$

$\Rightarrow\ 1 - \frac{r}{R+r} = 0$

$\Rightarrow\ R + r - r = 0$

$\Rightarrow\ R = 0$: when voltmeter reading is zero

$\Rightarrow\ R_{eq} = r$ (for circuit)

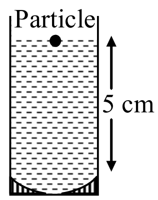
$i = \frac{E}{r}$

$\Rightarrow\ 1000\ mA = \frac{1.5}{r}$ (E = 1.5V from graph)

$\Rightarrow\ 1 = \frac{1.5}{r}$

$\Rightarrow\ r = 1.5\ \Omega$

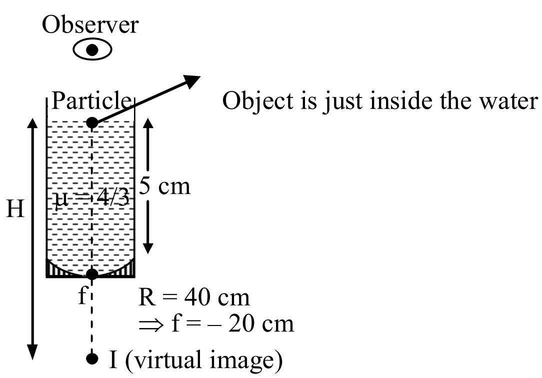
Q.21 A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is dose to: (Refractive index of water = 1.33)



(1) 11.7 cm (2) 6.7 cm (3) 13.4 cm (4) 8.8 cm

Ans. **[4]**

Sol.



$\frac{1}{V} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{V} + \frac{1}{(-5)} = \frac{1}{-20}$

$\Rightarrow \frac{1}{V} = \frac{1}{5} - \frac{1}{20}$

$\Rightarrow \frac{1}{V} = \frac{4-1}{20} = \frac{3}{20}$

$\Rightarrow V = \frac{20}{3}$ cm

$H = 5 + \frac{20}{3} = \frac{35}{3}$ cm

$H_{apperent} = \frac{H}{\mu} = \frac{\left(\frac{35}{3}\right)}{\left(\frac{4}{3}\right)} = \frac{35}{3} \times \frac{3}{4} = \frac{35}{4} = 8.8$ cm

Q.22 Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume ? (R = 8.3 J/mol K)

(1) 21.6 J/mol K (2) 19.7 J/mol K (3) 15.7 J/mol K (4) 17.4 J/mol K

Ans. **[4]**

Sol. $(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$= \frac{(2)\left[\frac{fR}{2}\right]_{mono} + 3\left[\frac{fR}{2}\right]_{diaatomic}}{2+3}$

$= \frac{2\left(\frac{3R}{2}\right) + 3\left(\frac{5R}{2}\right)}{5}$

$= \frac{21R}{10}$

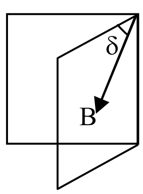
$= \frac{21 \times 8.3}{10} = 17.4$ J/mol-K

Q.23 A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30º. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(1) 1.8 (2) 2.2 (3) 0.7 (4) 3.6

Ans. **[3]**

Sol.



δ = angle of dip

$\tau = \vec{m} \times \vec{B}$

$\Rightarrow \tau = -mB\sin\theta$

$\Rightarrow \tau = -mB\theta$ (small angular displacement)

$\Rightarrow I\alpha = -m(B\cos\delta)\theta$

$\Rightarrow \alpha = -\left(\frac{mB\cos\delta}{I}\right)\theta$

$\omega^2 = \frac{mB\cos\delta}{I}$

$T = 2\pi\sqrt{\frac{I}{mB\cos\delta}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{B_2 \cos\delta_2}{B_1 \cos\delta_1}}$$

$$\frac{\left(\frac{60}{30}\right)}{\left(\frac{60}{40}\right)} = \sqrt{\frac{B_2 \cos\delta_2}{B_1 \cos\delta_1}}$$

$$\Rightarrow \left(\frac{4}{3}\right)^2 = \frac{B_2 \cos\delta_2}{B_1 \cos\delta_1}$$

$$\Rightarrow \frac{16}{9} \frac{\cos\delta_1}{\cos\delta_2} = \frac{B_2}{B_1}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{9}{16} \times \frac{\sqrt{3}\sqrt{2}}{2\times 1}$$

$$= \frac{9\times\sqrt{6}}{32}$$

$$= \frac{9\times 2.44}{32}$$

$$= \frac{22}{32} = 0.7$$

Q.24 Which of the following combinations has the dimension of electrical resistance ($\in_0$ is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

(1) $\sqrt{\frac{\in_0}{\mu_0}}$ (2) $\frac{\in_0}{\mu_0}$ (3) $\frac{\mu_0}{\in_0}$ (4) $\sqrt{\frac{\mu_0}{\in_0}}$

Ans. **[4]**

Sol. $[\varepsilon_0] = (M^{-1} L^{-3} T^4 A^2)$

$[\mu_0] = (M L T^{-2} A^{-2})$

$[R] = (M L^2 T^{-3} A^{-2})$

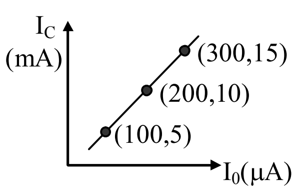
$R = \sqrt{\frac{\mu_0}{\in_0}}$

Q.25 The transfer characteristic curve of a transistor, having input and output resistance 100Ω and 100 kΩ respectively is shown in the figure. The voltage and power gain, are respectively:

(1) 5×10^4, 5×10^5 (2) 5×10^4, 5×10^6 (3) 5×10^4, 2.5×10^6 (4) 2.5×10^4, 2.5×10^6

Ans. **[3]**

Sol.



$B = \frac{I_C}{I_B} = \frac{5\times10^{-3}}{100\times10^{-9}} = 50$

Voltage gain $= A_V = \frac{V_{output}}{V_{input}}$

$= \frac{I_{output}\ R_0}{I_{input}\ R_{in}}$

$= \left(\frac{I_c}{I_b}\right)\left(\frac{R_0}{R_{in}}\right)$

$= (\beta)\left(\frac{R_0}{R_{in}}\right)$

$= (50)\left[\frac{100\times10^3}{100}\right]$

$= 5 \times 10^4$

Power gain $= \beta^2 = \left(\frac{R_0}{R_i}\right)$

$= (50)^2 \times 10^3$

$= 25 \times 10^2 \times 10^3$

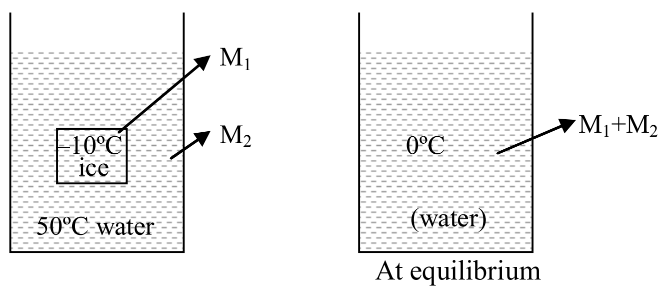
$= 2.5 \times 10^6$

Q.26 When M_1 gram of ice at $-10°C$ (specific heat = 0.5 cal $g^{-1}°C^{-1}$) is added to M_2, gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is :

(1) $\frac{50M_2}{M_1} - 5$ (2) $\frac{50M_2}{M_1}$ (3) $\frac{5M_2}{M_1} - 5$ (4) $\frac{5M_1}{M_2} - 50$

Ans. **[1]**

Sol.



Using energy conservation

$E_{\text{relesed by water}} = E_{\text{used by ice}}$

$\Rightarrow M_2S_W(\Delta T)_{water} = M_1S_{ice}(\Delta T)_{ice} + M_1L_{fusion}$

$\Rightarrow M_2(1)(50) = M_1\left(\frac{1}{2}\right)(10) + M_1L_{fusion}$

$\Rightarrow 50M_2 - 5M_1 = M_1L_{fusion}$

$\Rightarrow L_f = \frac{50M_2 + 5M_1}{M_1}$

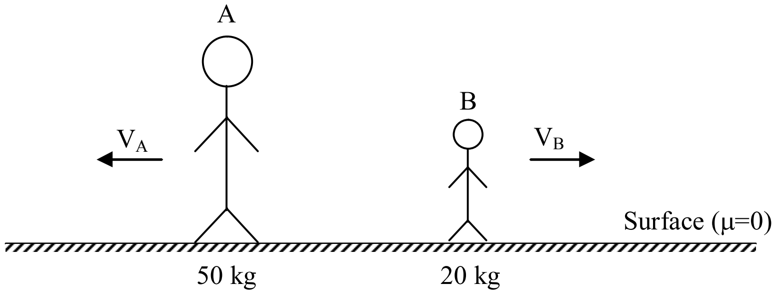
$\Rightarrow L_f = \frac{50M_2}{M_1} - 5$

Q.27 A man (mass = 50 kg) and his son (mass =20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.28 ms^{-1} (2) 0.47 ms^{-1} (3) 0.20 ms^{-1} (4) 0.14 ms^{-1}

Ans. **[3]**

Sol.



$\vec{V}_{B/A} = 0.7\,m/s$

$\Rightarrow \vec{V}_{B/S} - \vec{V}_{A/S} = 0.7$

$\Rightarrow V_B - (-V_A) = 0.7$

$\Rightarrow V_B + V_A = 0.7$

$\Rightarrow V_A + V_B = 0.7$

Momentum conservation ($\vec{F}_{ext} = 0$)

$P_C = P_f$

$\Rightarrow 0 = 20(+V_B) + 50(-V_A)$

$\Rightarrow 2V_B = 5V_A$

$\Rightarrow V_B = \frac{5V_A}{2}$

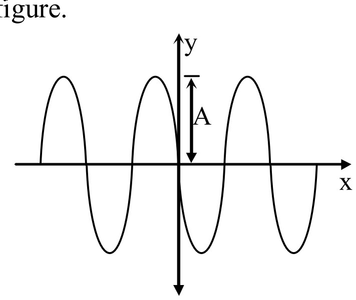
$\Rightarrow V_{A+} \frac{5V_A}{2} = 0.7$

$\Rightarrow \frac{7V_A}{2} = 0.7$

$\Rightarrow V_{A=} 0.7\left(\frac{2}{7}\right)$

$\Rightarrow V_A = 0.2$ m/s

Q.28 A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A\sin(kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure.



For this wave, the phase ϕ is :

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) $-\frac{\pi}{2}$

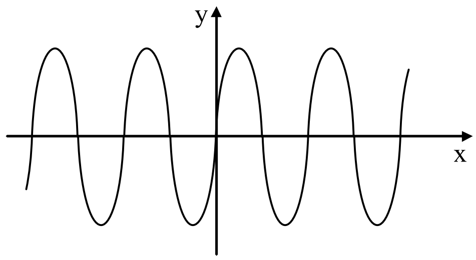
Ans. **[2]**

Sol.

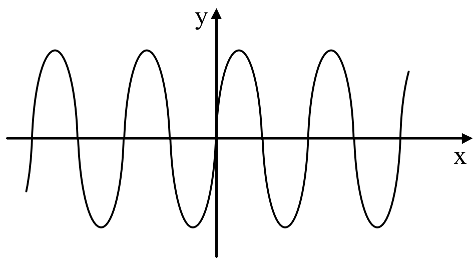
$Y = A \sin (kx - \omega t + \phi)$

At t = 0

$Y = A \sin (kx + \phi)$



Graph of : $y = A \sin(kx)$



Graph of : $y = -A \sin(kx)$

$-A \sin(kx) = A \sin (kx + \phi)$

$\Rightarrow A \sin(kx + \pi) = A \sin (kx + \phi)$

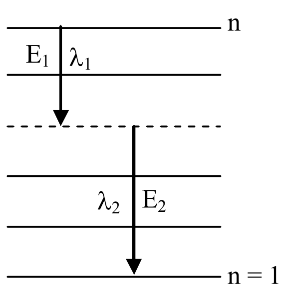
$\Rightarrow \phi = \pi$

Q.29 An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \dfrac{1240 \text{ eV}}{\lambda(\text{in nm})}$.

(1) n = 4 (2) n = 6 (3) n = 5 (4) n = 7

Ans. **[3]**

Sol.



$E = E_1 + E_2$

$$\Rightarrow 13.6\,(Z^2)\left[\frac{1}{1^2} - \frac{1}{n^2}\right] = \frac{1240}{108.5} + \frac{1240}{30.4}$$

$$\Rightarrow 13.6 \times 4\left[1 - \frac{1}{n^2}\right] = 11.43 + 40.79$$

$$\Rightarrow 1 - \frac{1}{n^2} = \frac{52.22}{54.4}$$

$$\Rightarrow \frac{1}{n^2} = 1 - \frac{52.22}{54.4}$$

$\Rightarrow \frac{1}{n^2} = \frac{2.18}{54.4}$

$\Rightarrow n^2 = \frac{54.4}{2.18}$

$\Rightarrow n^2 = 25$

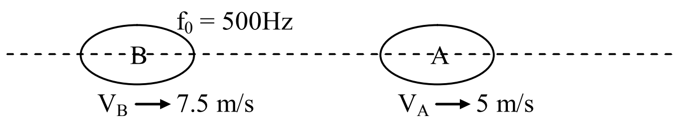
$\Rightarrow n = 5$

Q.30 A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to: (Speed of sound in water =1500 ms^{-1})

(1) 507 Hz (2) 504 Hz (3) 499 Hz (4) 502 Hz

Ans. **[4]**

Sol.



v = speed of sound in water = 1500 m/s

frequency received by A = $f' = \left[\frac{V - V_A}{V - V_B}\right] f_0 = \left[\frac{1500 - 5}{1500 - 7.5}\right] f_0$

frequency received by B = $f'' = \left[\frac{V + V_B}{V + V_A}\right] f' = \left[\frac{1500 + 7.5}{1500 + 5}\right]\left[\frac{1500 - 5}{1500 - 7.5}\right] f$

$f'' = \left(\frac{1500 + 7.5}{1500 - 7.5}\right)\left(\frac{1500 - 5}{1500 + 5}\right)(500)$

$= 502$ Hz