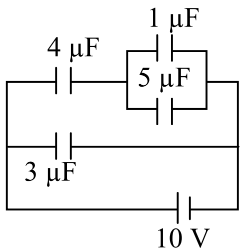
JEE Main Online Exam 2019

Questions & Solutions

12th April 2019 | Shift - II

PHYSICS

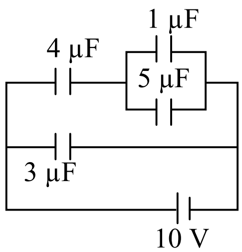
Q.1 In the given circuit, the charge on 4 μF capacitor will be :



(1) 5.4 μC (2) 9.6 μC (3) 13.4 μC (4) 24 μC

Ans. **[4]**

Sol.



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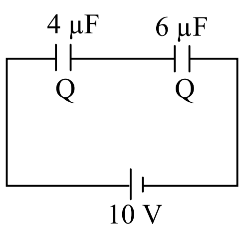


$$10 = \frac{Q}{4} + \frac{Q}{6}$$

$$10 = \frac{Q}{2}\left(\frac{1}{2} + \frac{1}{3}\right) \Rightarrow 10 = \frac{Q}{2}\left(\frac{5}{6}\right) \Rightarrow \boxed{Q = 24\mu C}$$

Q.2 A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ(assume that the particle is at origin t = 0)

(1) $\frac{b^2\tau}{\sqrt{2}}$ (2) $b^2\tau$ (3) $\frac{b^2\tau}{2}$ (4) $\frac{b^2\tau}{4}$

Ans. **[3]**

Sol. $v = b\sqrt{x}$

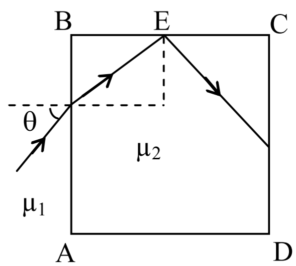
$$\frac{dx}{dt} = bx^{1/2} \Rightarrow \int_0^x x^{-1/2}\,dx = \int_0^\tau b\,dt$$

$\Rightarrow 2\sqrt{x_\tau} = b\tau \Rightarrow \sqrt{x_\tau} = \frac{b\tau}{2}$

$v_\tau = b\frac{b\tau}{2} = \frac{b^2\tau}{2}$

$$\boxed{v_\tau = \frac{b^2\tau}{2}}$$

Q.3 A transparent cube of side, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.

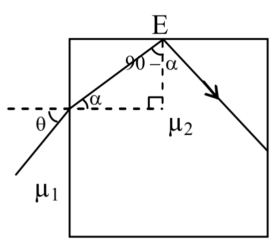


Then θ must satisfy :

(1) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ (2) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ (3) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (4) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

Ans. **[2]**

Sol.



By snell's law

$\mu_1 \sin\theta = \mu_2 \sin\alpha$ … (1)

For TIR

$\mu_2 \sin(90 - \alpha) > \mu_1$

From eq. (1)

$\sin\alpha = \frac{\mu_1}{\mu_2}\sin\theta$

$\cos\alpha = \sqrt{\frac{\mu_2^2 - \mu_1^2\sin^2\theta}{\mu_2}}$

$\cos\alpha = \sqrt{\frac{\mu_2^2}{\mu_2^2} - \frac{\mu_1^2\sin^2\theta}{\mu_2^2}}$

$$\cos\alpha = \sqrt{1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta} \quad \ldots (2)$$

$$\sin(90-\alpha) > \frac{\mu_1}{\mu_2}$$

$$\cos\alpha > \frac{\mu_1}{\mu_2}$$

using eq. (2)

$$\sqrt{1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta} > \frac{\mu_1^2}{\mu_2^2}$$

$$1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta > \frac{\mu_1^2}{\mu_2^2}$$

$$\frac{\mu_1^2}{\mu_2^2}\sin^2\theta < \left(1-\frac{\mu_1^2}{\mu_2^2}\right)$$

$$\Rightarrow \sin^2\theta < \left(\frac{\mu_2^2}{\mu_1^2}-1\right) \Rightarrow \boxed{\theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)}$$

Q.4 Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a from the centre. If A and B do not experience any force, then :

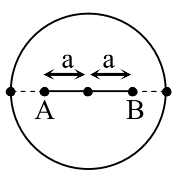
(1) $a = 8^{-1/4}R$ (2) $a = 2^{-1/4}R$ (3) $a = \frac{3R}{2^{1/4}}$ (4) $a = R/\sqrt{3}$

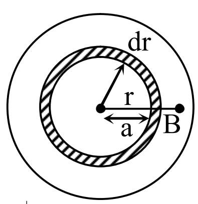
Ans. [1]

Sol. Total charge = 2 Q

Charging density $\rho = kr$

Radius = R





$d(\text{vol.}) = 4\pi r^2 dr$

$dq = \rho(d(\text{vol.}))$

Force on charge at B will be due to charge at A and due to force applied by the charge in sphere

$$\int dq = \int_0^R kq\ 4\pi r^2\, dr$$

$$2Q = k4\pi\int_0^R r^3 dr$$

$$2Q = \frac{k4\pi R^4}{4}$$

$$k = \frac{2Q}{\pi R^4} \quad \ldots(1)$$

$F_{sphere} \longleftarrow \bullet \longrightarrow F_{BA}$

$F_{BA} = F_{sphere}$

Force on charge B due to element

$$dF = \frac{k(dq)Q}{a^2} = \frac{kq(K4\pi r^3)dr}{a^2}$$

$$F = \frac{kQK4\pi}{a^2}\int_0^a r^3 dr = \frac{kQK4\pi a^2}{4}$$

$$F = kQK\pi a^2$$

$F_{BA} = F_{sphere}$

$\Rightarrow \frac{kQ^2}{(2a)^2} = kQ4Ka^2 \Rightarrow$ By replace value of K from (1)

$$\frac{Q^2}{4a^2} = \frac{2Q^2}{\pi R^4}\pi a^2$$

$$\Rightarrow a^4 = \frac{R^4}{8}$$

$$\Rightarrow a = R\, 8^{-1/4}$$

Q.5 A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62ºC, its efficiency is doubled. The temperatures of the source and the sink are, respectively,

(1) 99ºC, 37ºC (2) 124ºC, 62ºC (3) 37ºC, 99ºC (4) 62ºC, 124ºC

Ans. **[3]**

Sol. $\eta = \frac{1}{6}$

$$\frac{1}{6} = 1 - \frac{T_L}{T_H} \quad \ldots (1)$$

$$\frac{1}{3} = 1 - \frac{(T_L - 62)}{T_H} \quad \ldots (2)$$

Solving eq. (1)

$$\Rightarrow \frac{1}{6} = \frac{T_H - T_L}{T_H}$$

$$\Rightarrow T_H = 6T_H - 6T_L$$

$$6T_L = 5T_H$$

$$T_H = \frac{6T_L}{5}$$

Solving eq. (2)

$$\frac{1}{3} = \frac{T_H - (T_L - 62)}{T_H} \Rightarrow T_H = 3T_H - 3T_L + 186$$

$$\Rightarrow 2T_H = 3T_L - 186$$

$$2 \times \frac{6T_L}{5} = 3T_L - 186$$

$$\Rightarrow 12T_L = 15T_L - 930 \Rightarrow 3T_L = 930$$

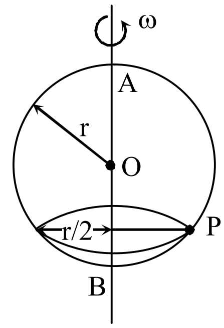
$$T_L = 310 \text{ K}$$

$$T_L = 310 - 273 = 37\ ºC$$

Source temp. is higher & sink temp. is lower

$$T_H = \frac{6T_L}{5} = \frac{6 \times 310}{5} = 372 \text{ K} = 99\ ºC$$

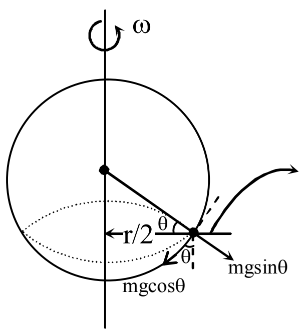
Q.6 A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to -

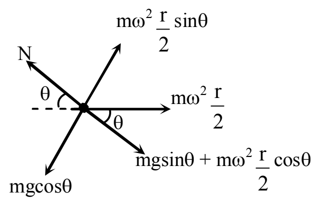


(1) $(g\sqrt{3})/r$ (2) $2g/r$ (3) $\frac{\sqrt{3}g}{2r}$ (4) $2g/(r\sqrt{3})$

Ans. **[4]**

Sol.





$$m\omega^2 \frac{r}{2}\sin\theta = mg\cos\theta$$

$$\omega^2 = \frac{2g}{r\tan\theta}$$

$$\tan\theta = \frac{\sqrt{r^2 - r^2/4}}{r/2} = \frac{\frac{\sqrt{3r^2}}{4}}{\frac{r}{2}} = \sqrt{3} \Rightarrow \omega^2 = \frac{2g}{\sqrt{3}r}$$

Q.7 A plane electromagnetic wave having a frequency v = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the Electric Field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave ?

(1) $\vec{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\,\hat{j}$

(2) $\vec{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\,\hat{k}$

(3) $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\,\hat{i}$

(4) $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\,\hat{i}$

Ans. [4]

Sol. v = 23.9 GHz

$E_0 = 60$ V/m

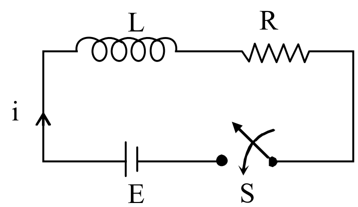
$\because \frac{E_0}{B_0} = C \Rightarrow B_0 = \frac{E_0}{C} = \frac{60}{3\times10^8} = 2\times10^{-7}$

Since the wave is propagating in positive z-direction

So acceptable magnetic field component will be

$\vec{B} = 2\times10^{-7}\sin(0.5\times10^3 z - 1.5\times10^{11}t)\,\hat{i}$

Q.8 Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is :



(1) $\frac{7.3\,EL}{R^2}$ (2) $\frac{2.7\,EL}{R^2}$ (3) $\frac{EL}{7.3\,R^2}$ (4) $\frac{EL}{2.7\,R^2}$

Ans. [4]

Sol. $I = I_{max.}\left(1 - e^{-\frac{Rt}{L}}\right)$ $\qquad I_{max.} = \frac{E}{R}$

$\frac{dq}{dt} = \frac{E}{R}\left(1 - e^{-R\frac{t}{L}}\right)$

$\int_0^{4R} dq = \frac{E}{R}\int_0^{4R}\left(1 - e^{-R\frac{t}{L}}\right)dt$

$Q = \frac{E}{R}\left[L + \frac{L}{R}e^{-R\frac{t}{L}}\right]_0^{L/R}$

$= \frac{E}{R}\left[\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right]$

$Q = \frac{EL}{R^2 e} \Rightarrow Q = \frac{EL}{2.7R^2}$

Q.9 One kg of water, at 20ºC, heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to :

[Specific heat of water = 4200 J/(kg ºC), Latent heat of water = 2260 kJ/kg]

(1) 10 minutes (2) 22 minutes (3) 3 minutes (4) 16 minutes

Ans. **[2]**

Sol. R = 20 Ω $P = \frac{V^2}{R} = \frac{200\times200}{20}$ = 2000 watt

V = 200 V

1 kg water 20ºC → 1 kg water 100ºC

Heat required $Q_1 = ms\Delta T$ = (1)(4200)(80) = 336000

1 kg water 100ºC → 1 kg vapour

Heat required Q_2 = mL = (1) 2260 × 1000 = 2260000

(power) (time) = total heat required

⇒ 2000 × time = 2260000 + 336000

time = 1298 sec.

time = 21.63 mint. ≃ 22 minute

Q.10 Consider an electron in a hydrogen atom revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :

(1) 6.6 Å (2) 3.5 Å (3) 9.7 Å (4) 12.9 Å

Ans. **[3]**

Sol. n = 3 (second exicted state)

$2\pi r_n = n\lambda_{dB}$,

$\Rightarrow \lambda_{dB} = \frac{2\pi r_3}{n} = \frac{2\times3.14\times4.65}{3} \simeq 9.7$ Å

Q.11 In an amplitude modulator circuit, the carrier wave is given by, C(t) = 4 sin(20000 πt) while modulating signal is given by, m(t) = 2 sin (2000 πt). The values of modulation index and lower side band frequency are :

(1) 0.3 and 9 kHz (2) 0.5 and 10 kHz (3) 0.4 and 10 kHz (4) 0.5 and 9 kHz

Ans. **[4]**

Sol. C(t) = 4sin(20000 πt), $A_c = 4$, $\nu_c = 10,000$

m(t) = 2sin(2000πt), $A_m = 2$, $\nu_m = 1000$

Modulation index $\mu = \frac{A_m}{A_c} = \frac{2}{4} = \frac{1}{2} = 0.5$

Lower side bond frequency = $\nu_L - \nu_m$

= 10,000 – 1000

= 9000

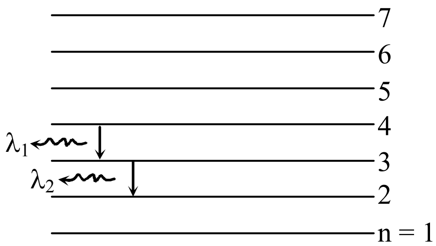
= 9 kHz

Q.12 The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is :

(1) 22/5 (2) 7/5 (3) 9/7 (4) 20/7

Ans. [4]

Sol.



$$\frac{hc}{\lambda_1} = 13.6\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{hc}{\lambda_1} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{7}{9\times16}\right)}{\left(\frac{5}{9\times4}\right)}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{7}{9\times16}\times\frac{9\times4}{5} = \frac{\lambda_2}{\lambda_1} = \frac{7}{20}$$

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{20}{7}}$$

Q.13 A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60º to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly):

(1) 10.67 (2) 5.33 (3) 16.00 (4) 1.80

Ans. [1]

Sol. When unpolarized light of intensity I_0 passes through P_1, P_2 and P_3, let the emergent light from P_1, P_2 and P_3 and I_1, I_2 & I_3. Then from Malus law

$I = I_0\cos^2\theta$

I_0 = Incident Intensity

θ = Angle between pass axes and incident light

So $I_1 = \frac{I_0}{2}$ $\because \langle\cos^2\theta\rangle = \frac{1}{2}$

$$I_2 = \frac{I_0}{2}\cos^2 30º = \frac{3I_0}{8}$$

$$I_3 = \frac{3I_0}{8}\cos^2 60º = \frac{3I_0}{32}$$

So $I = \frac{3I_0}{32}$

$$\frac{I_0}{I} = \frac{32}{3} = 10.67$$

Q.14 A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?

(1) 35 J (2) 40 J (3) 25 J (4) 30 J

Ans. [1]

Sol. W = 10 J at constant pressure

$W = P(V_2 - V_1)$

$= PV_2 - PV_1$

$10 = nR(T_2 - T_1) = nR\Delta T$

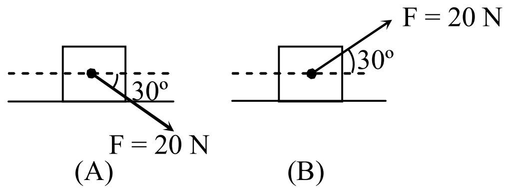
$\Delta Q = \Delta W + \Delta U$

$= 10\text{ J} + \frac{nf}{2} R\Delta T$

$= 10\text{ J} + \frac{5}{2}(10\text{ J})$

$\boxed{\Delta Q = 35\text{ J}}$

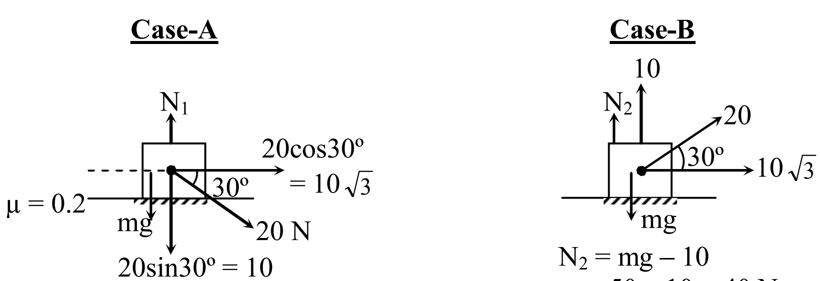
Q.15 A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30º with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the blocks, in case (B) and case (A) will be : ($g = 10\ ms^{-2}$)



(1) 3.2 ms^{-2} (2) 0.8 ms^{-2} (3) 0 ms^{-2} (4) 0.4 ms^{-2}

Ans. [2]

Sol.



Case-A:

$N_1 = mg + 10$

$(fs)_{max} = 0.2 \times (60) = 12$ N

Case-B:

$N_2 = mg - 10$

$= 50 - 10 = 40$ N

$(fs)_{max.} = 0.2 \times 40 = 8$ N

$a_A = \frac{10\sqrt{3} - 12}{5} = \frac{17.32 - 12}{5}$

$a_A = \frac{5.32}{5}$

$a_B = \frac{10\sqrt{3} - 8}{5} = \frac{17.32 - 8}{5}$

$a_B = \frac{9.32}{5}$

difference between acceleration $a_B - a_A = \frac{1}{5}(9.32 - 5.32) = \frac{4}{5}$

$\boxed{\Delta a = 0.8\, m/s^2}$

Q.16 A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0(I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V ($V = GI_0$) by connecting a series resistance R_V to it. Then,

(1) $R_AR_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

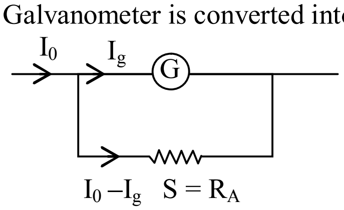
(2) $R_AR_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(3) $R_AR_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

(4) $R_AR_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Ans. **[4]**

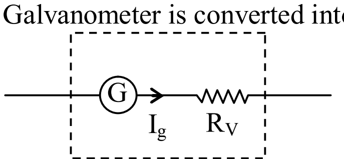
Sol. Galvanometer is converted into ammeter of range 0 to I_0.



$I_gG = (I_0 - I_g) R_A$

$R_A = \frac{I_gG}{(I_0 - I_g)}$ … (1)

Galvanometer is converted into voltmeter of range 0 to V



$V = I_g(G + R_V)$

$GI_0 = I_g(G + R_V)$

$R_V = \frac{GI_0}{I_g} - G$

$R_V = \frac{G(I_0 - I_g)}{I_g}$ … (2)

So from (1) & (2)

$R_A R_V = G^2$

$\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Q.17 A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals :

(1) 1/9 (2) 1/27 (3) 9 (4) 27

Ans. **[3]**

Sol. $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = \frac{R^3}{3^3} \qquad \Rightarrow \boxed{r = \frac{R}{3}}$

$$V_1 = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi R^3 g}{6\pi\eta R}$$

$$V_2 = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi\left(\frac{R}{3}\right)^3 g}{6\pi\eta\left(\frac{R}{3}\right)} = \frac{(\rho_0 - \rho_{liq.})\frac{4}{3}\pi R^3 g \times \frac{1}{27}}{6\pi\eta R \times \frac{1}{3}}$$

$$V_2 = \frac{V_1}{9}$$

Q.18 Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals :

(1) 10.0 m/s (2) 5.5 m/s (3) 15.0 m/s (4) 2.5 m/s

Ans. **[4]**

Sol. S_1 • $\quad$ O $\xrightarrow{u}$ $\quad$ • S_2

As observer goes away from source S_1 so apparent frequency

$\nu_1 = \frac{(v - v_0)}{v}\nu \qquad$ v = speed of round, v_0 = speed of observer

$= \left(\frac{330 - u}{330}\right) \times 660$

$\nu_1 = 2 \times 330 - 2u \qquad \ldots (1)$

As observer goes towards source S_2 so apparent frequency

$\nu_2 = \frac{(v + v_0)}{v}\nu$

$= \left(\frac{330 - u}{330}\right) \times 660$

$\nu_2 = 2 \times 330 + 2u \qquad \ldots (2)$

According to question

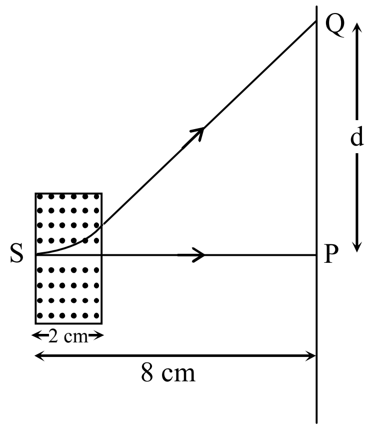
$\nu_2 - \nu_1 = 10$

$4u = 10$

$u = 2.5$ m/s

Q.19 An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}\text{ T})\ \hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is :

(electron's charge = 1.6×10^{-19}C, mass of electron = 9.1×10^{-31} kg)



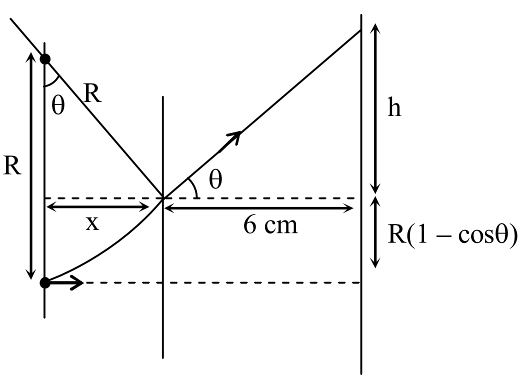
(1) 2.25 cm (2) 12.87 cm (3) 1.22 cm (4) 11.65 cm

Ans. **[2]**

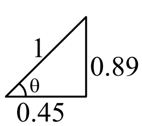
Sol. $R = \dfrac{\sqrt{2mk}}{qB}$

$= \dfrac{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$

R = 2.24 cm



$\sin\theta = \dfrac{x}{R} = \dfrac{2}{2.24} = 0.89$



$\tan\theta = \dfrac{0.89}{0.45} \simeq 1.97 = \dfrac{h}{6}$

h = 11.82 cm

cos θ = 0.45

$d = h + 2(1 - 0.45) = 11.82 + 2(1 - 0.45) \simeq 12.9$ cm

Q.20 Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively, If initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 1 : 8 (3) 8 : 1 (4) 3 : 8

Ans. **[1]**

Sol. $A_1 = \frac{A_0}{2^{60/10}} = \frac{A_0}{2^6} = \frac{A_0}{64}$

$A_2 = \frac{A_0}{2^{60/20}} = \frac{A_0}{2^3} = \frac{A_0}{8}$

No. of decayed nuclei = $A_1' = A_0 - \frac{A_0}{64} = \frac{63}{64}$

$A_2' = A_0 - \frac{A_0}{8} = \frac{7}{8}$

$$\boxed{\text{Ratio} = \frac{63}{64} \times \frac{8}{7} = \frac{9}{8}}$$

Q.21 A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $\lambda_2 = 70$ cm. Then, v is equal to -

(1) 338 ms^{-1} (2) 384 ms^{-1} (3) 379 ms^{-1} (4) 332 ms^{-1}

Ans. **[2]**

Sol. $l_1 = 30$ cm

$l_2 = 70$ cm, $\nu = 480$

$v = 2(l_2 - l_1)\nu$

$= 2(70 - 30) \times 10^{-2} \times 480$

$= 2 \times 40 \times 10^{-2} \times 480$

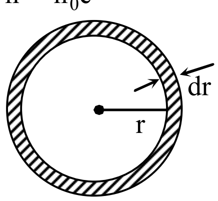
$= 384$ m/sec

Q.22 The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to :

(1) $n_a \alpha^{-3/4}$ (2) $\sqrt{n_0}\,\alpha^{1/2}$ (3) $n_0 \alpha^{1/4}$ (4) $n_0 \alpha^{-3}$

Ans. **[Bonus]**

Sol. $n = n_0 e^{-\alpha r^4}$



taken an element hollow sphere of thickness dr

Vol. of element $dV = (4\pi r^2)dr$

no. of molecules in elementary volume = $n.e^{-\alpha r} 4\pi r^2 dr$

total no. of molecules = $n.\ 4\pi \int_0^\infty r^2 e^{-\alpha r^4} dr = ?$

Note : This equation can't be solved so it should be bonus and full marks should given to students.

Q.23 A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? [Given reference intensity of sound as 10^{-12} W/m^2]

(1) 20 cm (2) 10 cm (3) 40 cm (4) 30 cm

Ans. **[3]**

Sol. $L = 10 \log \frac{I}{I_0}$

$120 = 10 \log \frac{I}{I_0}$

$12 = \log_{10} \frac{I}{10^{-12}}$

$\frac{I}{10^{-12}} = 10^{12}$

$I = 1 = \frac{P}{4\pi r^2}$

$r^2 = \frac{2}{4\pi \times 1}$

$r = \sqrt{\frac{2}{4\pi}}\text{ m} = 100 \times \sqrt{\frac{1}{2\pi}} = 100 \times 0.399 \simeq 39.9 \simeq 40 \text{ cm}$

Q.24 A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to :

(1) $3F/(\pi r2YT)$ (2) $6F/(\pi r^2YT)$ (3) $F/(3\pi r^2YT)$ (4) $9F(\pi r^2YT)$

Ans. **[1]**

Sol. $Y = \frac{\frac{F}{\pi r^2}}{\frac{\Delta\ell}{\ell}} \Rightarrow Y = \frac{F}{\pi r^2} \times \frac{\ell}{\Delta\ell}$

$\Delta\ell = \frac{F\ell}{\pi r^2 Y}$

change in length due to temperature change

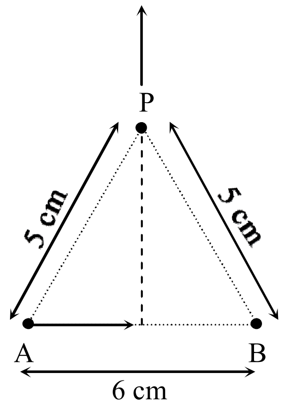
$\Delta\ell = \ell\alpha\Delta T$

$\ell\alpha T = \frac{F.\ell}{\pi r^2 Y}$

$\Rightarrow \alpha = \frac{F}{\pi r^2 YT}$

$\boxed{Y = \frac{3F}{\pi r^2 YT}}$

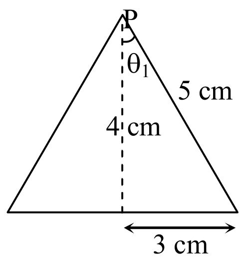
Q.25 Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5A. (See figure) ($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})



(1) 1.5×10^{-5} T (2) 3.0×10^{-5} T (3) 2.0×10^{-5} T (4) 2.5×10^{-5} T

Ans. **[1]**

Sol.



$$B = \frac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

$$= \frac{5}{4\times10^{-2}}\left(\frac{3}{5}+\frac{3}{5}\right)\times10^{-7} \qquad 4\text{ cm} = 4\times10^{-2}\text{ m}$$

$$= \frac{5}{4}\times 2\,\frac{3\times10^{-7}}{5\times10^{-2}}$$

$$\boxed{B = 1.5\times10^{-5}\,T}$$

Q.26 A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constant, k_1 and k_2 will be ;

(1) $\frac{1}{n^2}$ (2) $\frac{1}{n}$ (3) n^2 (4) n

Ans. **[2]**

Sol. l, k = l_1, k_1 + l_2, k_2

given $l_1 = nl_2$

$k_1 = \frac{1/l_1}{l}\times k$ | $k_2 = \frac{1/l_2}{l}\times k$

$k_1 = \frac{k}{l_1 l}$ | $k_2 = \frac{1}{l l_2}k$

$$\frac{k_1}{k_2} = \frac{l_2}{l_1} \Rightarrow \boxed{\frac{k_1}{k_2} = \frac{1}{n}}$$

Q.27 The ratio of the weights of a body on the Earth's surface to that on the surface of a planets is 9 : 4. The mass of the planet is $\frac{1}{9}$ th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(1) $\frac{R}{9}$ (2) $\frac{R}{2}$ (3) $\frac{R}{3}$ (4) $\frac{R}{4}$

Ans. **[2]**

Sol. $\frac{W_e}{W_p} = \frac{9}{4}$ $M_p = \frac{1}{9} M_e$

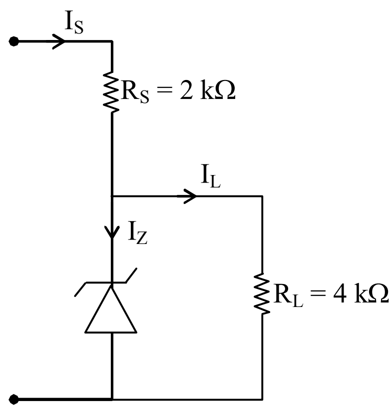
$W_e = m\frac{GM_e}{R^2}$, $W_p = m\frac{GM_e}{R^2}$ $\quad \frac{W_e}{W_p} = \frac{mGM_e}{R^2} \times \frac{R^2}{mGM_p}$

$\frac{W_e}{W_p} = \frac{9R'^2}{R^2}$

$\frac{9}{4} = \frac{9R'^2}{R^2} \Rightarrow R'^2 = \frac{R^2}{4}$

$\boxed{R' = \frac{R}{2}}$

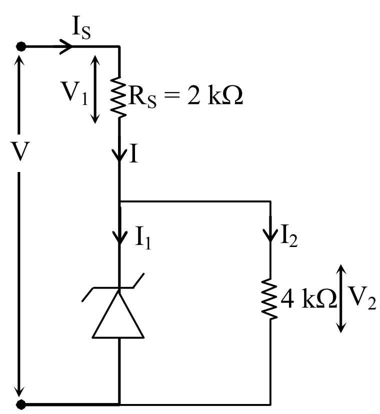
Q.28 Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is maximum Zener current ?



(1) 3.5 mA (2) 1.5 mA (3) 2.5 mA (4) 7.5 mA

Ans. **[1]**

Sol.



Case – I : V = 16 volt

$V_2 = 6$ V then $V_1 = 10$ V

$I = \dfrac{10}{2k\Omega} = 5 \times 10^{-3}$ amp.

$I_2 = \dfrac{6}{4\times1000} = 1.5 \times 10^{-3}$ Amp.

$I_1 = (5 - 1.5) \times 10^{-3}$ Amp.

$= 3.5 \times 10^{-3}$ Amp.

Case – II : V = 10 volt

$V_2 = 6$ V & $V_1 = 4$ vol.

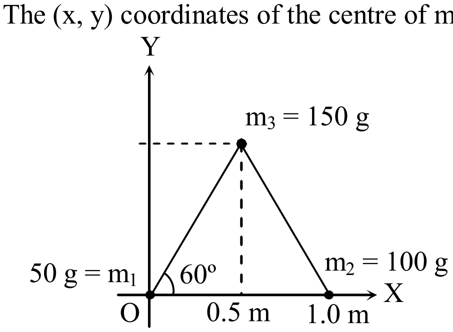
$I = \dfrac{4}{2\times1000} = 2 \times 10^{-3}$ Amp.

$I_2 = \dfrac{4}{4\times1000} = 10^{-3}$ Amp.

$I_1 = (2 - 1) \times 10^{-5} = 10^{-5}$ Amp.

Maximum Zener current is in Case-I that is 3.5×10^{-3} Amp.

Q.29 Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :



(1) $\left(\dfrac{\sqrt{3}}{8}\text{m}, \dfrac{7}{12}\text{m}\right)$ (2) $\left(\dfrac{\sqrt{3}}{4}\text{m}, \dfrac{5}{12}\text{m}\right)$ (3) $\left(\dfrac{7}{12}\text{m}, \dfrac{\sqrt{3}}{8}\text{m}\right)$ (4) $\left(\dfrac{7}{12}\text{m}, \dfrac{\sqrt{3}}{4}\text{m}\right)$

Ans. [4]

Sol.

$m_1 = 50$ g	$m_2 = 100$ g	$m_3 = 150$ g
$x_1 = 0$	$x_2 = 1$m	$x_3 = 0.5$ m
$y_1 = 0$	$y_2 = 0$	$y_3 = \dfrac{\sqrt{3}}{2}$

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (100)10 + (150)(0.5)}{300}$$

$$x_{COM} = \frac{100 + 75}{300} = \frac{175}{300} = \frac{7}{12}\text{ m.}$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (100)(0) + (150)\frac{\sqrt{3}}{2}}{300}$$

$$y_{COM} = \frac{75\sqrt{3}}{300} = \frac{3\sqrt{3}}{12} = \frac{\sqrt{3}}{4}\text{ m}$$

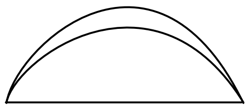
$$\boxed{(x_{COM}, y_{COM}) = \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)}$$

Q.30 Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

(1) $R^2 = h_1h_2$ (2) $R^2 = 16\ h_1h_2$ (3) $R^2 = 4\ h_1h_2$ (4) $R_2 = 2h_1h_2$

Ans. **[2]**

Sol.



Angle of projections must be

$\theta, (90 - \theta)$

$$h_1 = \frac{u^2 \sin^2\theta}{2g},\ h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$h_1h_2 = \frac{u^4\sin^2\theta\cos^2\theta}{4g^2}$$

$$R^2 = \frac{4u^4\sin^2\theta\cos^2\theta}{g^2}$$

$$\boxed{R^2 = 16h_1h_2}$$