CAREER POINT

KVPY PAPER-2020 (STREAM SA)

Part – I

One - Mark Questions

Date : 31 / 01 / 2021

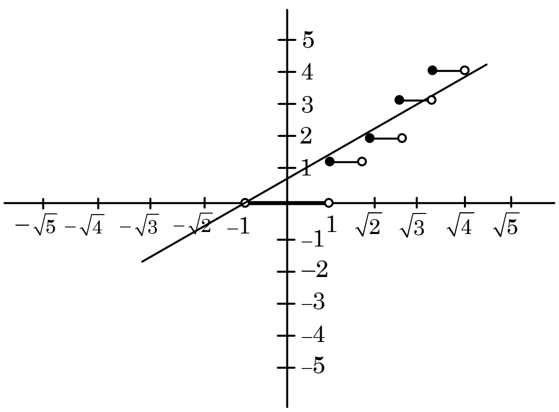
MATHEMATICS

1. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the equation $[x^2] = x + 1$ has

(A) two solutions (B) one solution (C) no solution (D) more than two solutions

Ans. **[C]**

Sol.



From the graph it is clear that the equation has no-solution.

2. Let $p_1(x) = x^3 - 2020x^2 + b_1x + c_1$ and $p_2(x) = x^3 - 2021x^2 + b_2x + c_2$ be polynomials having two common roots α and β. Suppose there exist polynomials $q_1(x)$ and $q_2(x)$ such that $p_1(x)q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2$. Then the correct identity is

(A) $p_1(3) + p_2(1) + 4028 = 0$ (B) $p_1(3) + p_2(1) + 4026 = 0$

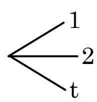
(C) $p_1(2) + p_2(1) + 4028 = 0$ (D) $p_1(1) + p_2(2) + 4028 = 0$

Ans. **[A]**

Sol. $p_1(x)q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2$

$p_1(x) - p_2(x) = x^2 + (b_1 - b_2)x + (c_1 - c_2)$

$\Rightarrow q_1(x) = 1$ & $q_2(x) = -1$

$p_1(x) = x^3 - 2020x^2 + b_1x + c_1$ 

$t + 3 = 2020 \Rightarrow t = 2017$

$p_1(x) = (x - 1)(x - 2)(x - 2017)$

Similarly $p_2(x) = (x - 1)(x - 2)(x - 2018)$

(A) $p_1(3) + p_2(1) + 4028 = 0$

$p_1(3) = -4028$

$p_2(1) = 0$

Hence it is true

3. Suppose p, q, r are positive rational numbers such that $\sqrt{p}+\sqrt{q}+\sqrt{r}$ is also rational. Then

(A) $\sqrt{p}$, $\sqrt{q}$, $\sqrt{r}$ are irrational

(B) $\sqrt{pq}$, $\sqrt{pr}$, $\sqrt{qr}$ are rational, but $\sqrt{p}$, $\sqrt{q}$, $\sqrt{r}$ are irrational

(C) $\sqrt{p}$, $\sqrt{q}$, $\sqrt{r}$ are rational

(D) $\sqrt{pq}$, $\sqrt{pr}$, $\sqrt{qr}$ are irrational

Ans. **[C]**

Sol. $\sqrt{p}+\sqrt{q}+\sqrt{r} \in Q$, p, q, r t $\in$ Q

let $\sqrt{p}+\sqrt{q}+\sqrt{r} = t$

$\sqrt{p}+\sqrt{q} = t - \sqrt{r}$

$p + q + 2\sqrt{pq} = t^2 + r - 2t\sqrt{r}$

$\sqrt{pq}+t\sqrt{r} \in Q = \lambda \quad \lambda \in Q$

$\sqrt{pq} = \lambda - t\sqrt{r}$

$pq = \lambda^2 + t^2r - 2\lambda t\sqrt{r}$

$\Rightarrow \sqrt{r} \in Q$ similarly $\sqrt{p}$ and $\sqrt{q} \in Q$

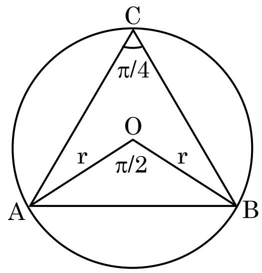
hence $\sqrt{p}$, $\sqrt{q}$, $\sqrt{r} \in Q$

4. Let A, B, C be three points on a circle of radius 1 such that $\angle ACB = \frac{\pi}{4}$. Then the length of the side AB is-

(A) $\sqrt{3}$ (B) $\frac{4}{3}$ (C) $\frac{3}{\sqrt{2}}$ (D) $\sqrt{2}$

Ans. **[D]**

Sol.



Let O be the centre of the circle

In ΔOAB

$AB = \sqrt{2}$ r and r = 1

$AB = \sqrt{2}$ r and r = 1

$\Rightarrow AB = \sqrt{2}$

5. Let x and y be two positive real numbers such that x + y = 1. Then the minimum value of $\frac{1}{x}+\frac{1}{y}$ is-

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

Ans. **[D]**

Sol. $x + y = 1$ and $x, y > 0$

Apply AM ≥ HM

$$\frac{x+y}{2} \geq \frac{2}{\frac{1}{x}+\frac{1}{y}}$$

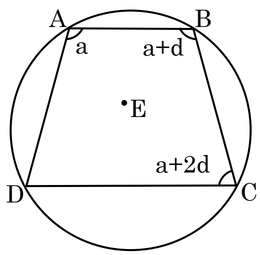
$$\Rightarrow \frac{1}{x}+\frac{1}{y} \geq 4$$

6. Let ABCD be a quadrilateral such that there exists a point E inside the quadrilateral satisfying AE = BE = CE = DE. Suppose ∠DAB, ∠ABC, ∠BCD is an arithmetic progression. Then the median of the set {∠DAB, ∠ABC, ∠BCD} is-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans. **[D]**

Sol.



AE = BE = CE = DE

∠DAB, ∠ABC, ∠BCD → AP

Let ∠DAB = a

∠ABC = a + d

∠BCD = a + 2d

Since AE = BE = CE = DE so ABCD is cyclic quadrilateral

Hence ∠DAB + ∠DCB = 180°

$2a + 2d = 180^o \Rightarrow a + d = 90^o$

so median of {a, a ± d, a + 2d} is a + d = 90°

7. The number of ordered pairs (x, y) of positive integers satisfying $2^x + 3^y = 5^{xy}$ is

(A) 1 (B) 2 (C) 5 (D) infinite

Ans. **[A]**

Sol. $2^x + 3^y = 5^{xy}$

clearly x = y = 1 satisfy the relation

Take x > y

$3^x > 3^y$

$2^x + 3^x > 2^x + 3^y$

$2^x + 3^x > 5^{xy}$

this is false

$5^{xy} > 5^x = (2 + 3)^x > 2^x + 3^x$

Similarly there is no solution for x < y

Hence x = y, hence only x = y = 1 satisfy the given equation ($2^x + 3^x = 5^{x^2}$ is not true for $x \in N - \{1\}$)

8. In the integers from 1 to 2021 are written as a single integer like 123....91011.....20202021, then the 2021st digit (counted from the left) in the resulting number is-

(A) 0 (B) 1 (C) 6 (D) 9

Ans. **[B]**

Sol. 1 2 3 4 5 .... 9 10 11 12 13.... 2020 2021

find the 2021 term

double digit = 90 × 2

$\underbrace{1\,2\,3\,4\,5\,6\,7\,8\,9}_{9\text{-digit}}\quad \underbrace{10\,11\,12\,.........99}_{180\text{-digit}}\quad 100\ 101$

we need 2021[st] digit

Till two digit number we have 189 digit

we need 2021 – 189 = 1832 digit

triple digit = $\frac{1832}{3}$ = 610 × 3 + 2

we take 610 three digit number

100, 101, ..., 709

$\underbrace{1\ 2\ 3\ .....\ 9}_{9\text{ digit}}\ +\ \underbrace{10\ 11\ 12\ 13\ ....\ 99}_{180\text{ digit}}\ +\ \underbrace{100\ 101\ ....\ 709}_{1830\text{ digit}}\ 7\underset{\downarrow}{1} \to 2021^{st}$ digit

Total = 9 + 180 + 1830 = 2019

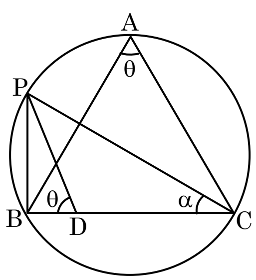
Ans = 1

9. In a triangle ABC, a point D is chosen on BC such that BD : DC = 2 : 5. Let P be a point on the circumcircle ABC such that ∠PDB = ∠BAC. Then PD : PC is

(A) $\sqrt{2} : \sqrt{5}$ (B) 2 : 5 (C) 2 : 7 (D) $\sqrt{2} : \sqrt{7}$

Ans. **[D]**

Sol.



$\frac{BD}{DC} = \frac{2}{5}$

∠PDB = ∠BAC = θ

let ∠PCD = α

⇒ ∠DPC = θ – α

∠BAC = ∠BPC = θ

(angle in the same segment)

⇒ BPD = θ – (θ – α) = α

so ΔPCB ~ ΔPDB

$\frac{PC}{DP} = \frac{BC}{PB} = \frac{PB}{BD}$

$\left(\frac{PC}{Dp}\right)^2 = \frac{BC}{PB} \times \frac{PB}{BD} = \frac{BC}{BD}$

$\frac{PC}{DP} = \sqrt{\frac{BC}{BD}} = \sqrt{\frac{7\lambda}{2\lambda}} = \frac{\sqrt{7}}{\sqrt{2}}$

$\frac{DP}{PC} = \frac{\sqrt{2}}{\sqrt{7}}$

10. Let [x] be the greatest less than or equal to x, for a real number x. Then the following sum

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{5^{2020}+1}{5^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

(A) 80 (B) 85 (C) 90 (D) 95

Ans. **[B]**

Sol. $\frac{2^{2020}+1}{2^{2018}+1} = \frac{4.2^{2018}+1}{2^{2018}+1} = \frac{4(2^{2018}+1)-3}{2^{2018}+1}$

$4-\frac{3}{2^{2018}+1} = t,\ 3 < t < 4$

Now $\left[\frac{2^{2020}+1}{2^{2018}+1}\right] = 3$

similarly $\frac{3^{2020}+1}{3^{2018}+1} = \frac{9.3^{2018}+1}{3^{2018}+1} = 9-\frac{8}{3^{2018}+1}$

$\left[\frac{3^{2020}+1}{3^{2018}+1}\right] = 8$

similarly $\left[\frac{n^{2020}+1}{2^{2018}+1}\right] = n^2-1$

$(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1)+(6^2-1)$

$= 3 + 8 + 15 + 24 + 35 = 85$

11. Let r be the remainder when 2021^{2020} is divided by 2020^2. Then r lies between

(A) 0 and 5 (B) 10 and 15 (C) 20 and 100 (D) 107 and 120

Ans. **[A]**

Sol. $(2021)^{2020} = (1+2020)^{2020}$

${}^{2020}C_0 + {}^{2020}C_1 \,.\, 2020 + {}^{2020}C_2 \,.\, 2020^2 + \ldots + {}^{2020}C_{2020} \,.\, 2020^{2020}$

$1 + (2020)^2 + {}^{2020}C_2.2020^2 + \ldots. + {}^{2020}C_{2020}.2020^{2020}$

$1 + (2020)^2 (1 + {}^{2020}C_2 + \ldots. + (2020)^{2018})$

$1 + (2020)^2.\lambda$

Hence $(2021)^{2020} = \lambda(2020)^2 + 1$

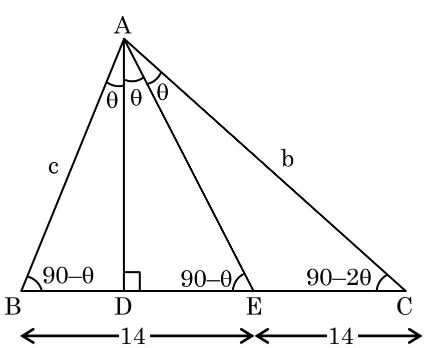
Hence remainder = 1

12. In a triangle ABC, the altitude AD and the median AE divide ∠A into three equal parts. If BC = 28, then the nearest integer to AB + AC is-

(A) 38 (B) 37 (C) 36 (D) 33

Ans. **[A]**

Sol.



ΔABE is isosceles ⇒ BD = DE = 7

$\Delta ADC : \tan(90 - 2\theta) = \frac{AD}{21}$ ...(1)

$\Delta ADE : \tan(90 - \theta) = \frac{AB}{7}$ ...(2)

Divide $\frac{\tan\theta}{\tan 2\theta} = \frac{1}{3} \Rightarrow \frac{1-\tan^2\theta}{2} = \frac{1}{3}$

$1 - \tan 2\theta = \frac{2}{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^o$

$\Delta ABD : \cos(90 - \theta) = \frac{BD}{C} = \sin\theta$

$C = 7\text{cosec}\theta = 14$

$\Delta ADC : \cos(90 - 2\theta) = \frac{DC}{b} = \sin 2\theta$

$b = 21\text{cosec}2\theta = 21\text{cosec}\frac{\pi}{3}$

$b = \frac{42}{\sqrt{3}} = 14\sqrt{3}$

$b + c = 14\sqrt{3} + 14$

$[b + c] = 38$

13. The number of permutations of the letters a_1, a_2, a_3, a_4, a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is-

(A) 96 (B) 78 (C) 60 (D) 42

Ans. **[B]**

Sol.

(When a_1 does not occupy its position) (When a_1 does not occupy its position but a_2 occupy its second position

$$\underset{\substack{a_1\text{can occupy}\\ \text{any position}\\ \text{except }1^{st}}}{4} \times \underset{\substack{\text{Remaining}\\ 4\text{ letter can}\\ \text{be arranged}\\ \text{in 4-position}}}{4!} - \underset{\substack{a_1\text{can occupy}\\ 3\text{-position}\\ \text{except I}^{st}\text{ and II}^{nd}}}{3} \times \underset{\substack{\text{Remaining}\\ \text{three person}\\ \text{arranged in}\\ \text{3-position}}}{3!} = 78$$

14. There are m books in black cover and n books in blue cover, and all books are different. The number of ways these (m + n) books can be arranged on a shelf so that all the book in black cover are put side by side is

(A) m! n! (B) m!(n + 1)! (C) (n + 1)! (D) (m + n)!

Ans. **[B]**

Sol. Put all block cover books together

$A_1A_{12}A_3 \ldots\ldots A_m \rightarrow \alpha$

Total number of books = n + 1

These books be arranged in (n + 1)! ways and m books be arranged m m! ways

No. of way = m! (n + 1)!

15. A 5 digit number $\overline{abcde}$, when multiplies by 9, gives the 5-digit number $\overline{edcba}$. The sum of the digits in the number is-

(A) 18 (B) 27 (C) 36 (D) 45

Ans. **[B]**

Sol. abcde × 9 = edcba

surely a = 1

$\Rightarrow$ 1bcde × 9 = edcb1

9e last digit is 1 $\Rightarrow$ e = 9

$\Rightarrow$ 1bcd9 × 9 = 9dcb1

9 multiply by b $\Rightarrow$ b has to {0, 1} otherwise RHS is a six digit number

C-1 Take b = 0

10cd9 × 9 = 9dc01

9d + 8 = P0 $\rightarrow$ (last digit has to be zero)

$\Rightarrow$ d = 8

10c89 × 9 = 98c01

Now 98c01 is divisible by 9 $\Rightarrow$ sum of digit divisible by 9 $\Rightarrow$ c = 0, 9

takc c = 0, 10089 × 9 = 90801 (rejected)

take c = 9, 1089 × 9 = 9801

a = 1, b = 0, c = 9, d = 8, e = 9

Sum = 27

C-2 take b = 1

11cd9 × 9 = 9dc11

9d + 8 = p1 $\Rightarrow$ d = 7

9 × 11c79 = 97c11

This cannot be true for $c \in \{0, 1, 2, ..., 9\}$

Alternate Solution

$9(a \times 10^4 + b \times 10^3 + c \times 10^2 + b \times 10 + e)$

$= e \times 10^4 + d \times 10^3 + c \times 10^2 + b \times 10 + a$

89999a + 8990b + 800c – 910d – 9991e = 0

for max. value of 'a'

put b = c = 0 and d = e = 9

$a = \frac{98109}{89999} \Rightarrow$ a will be 1

$\therefore$ 89999 + 8990b + 800c – 910d – 9991e = 0

for max. value of b

$\therefore b = \frac{8110}{8990} \Rightarrow$ b will be 0

$\therefore$ 89999 + 800c – 910d – 9991e = 0

for max. value of c

put d = e = 9 $\Rightarrow$ c > 10 (not possible)

put d = e = 8 (not possible)

put d = 9, e = 9 (not possible)

put d = 8, e = 9

$\Rightarrow c = \frac{7200}{800} \Rightarrow c = 9$

$\therefore$ number is 10989

PHYSICS

16. A mouse jumps off from the 15^{th} floor of a high-rise building and lands 12 m from the building. Assume that each floor is of 3m height. The horizontal speed with which the mouse jumps is closest to-

(A) 0 (B) 5 kmph (C) 10 kmph (D) 15 kmph

Ans. **[D]**

Sol. Time of fall = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 45}{10}}$

$t = 3$ sec

horizontal distance = horizontal velocity × time

$12 = v \times 3$

$v = 4$ m/s

$= 4 \times \frac{18}{5}$ km/hr

$v = 14.4$ km/hr

$v \approx 15$ km/hr

17. Consider two wires of same material having their ratio of radii to be 2 : 1. If these two wires are stretched by equal force, the ratio of stress produced in them is-

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Ans. **[A]**

Sol. Stress = $\frac{\text{Force}}{\text{Area}} \propto \frac{1}{\text{Area}}$

Stress $\propto \frac{1}{r^2}$ (Area = πr^2)

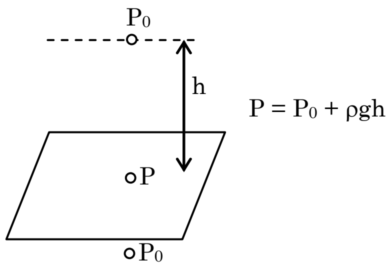
ratio of stress = $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

18. A submarine has a window of area 30×30 cm^2 on its ceiling and is at a depth of 100 m below sea level in a sea. If the pressure inside the submarine is maintained at the sea-level atmosphere pressure, then the force acting on the window is (consider density of sea water = 1.03×10^3 kg/m^3, acceleration due to gravity = 10 m/s^2)

(A) 0.93×10^5 N (B) 0.93×10^3 N (C) 1.86×10^5 N (D) 1.86×10^3 N

Ans. **[A]**

Sol.



P = Pressure on upper surface of window

$= P_0 + \rho g h$

P_{in} = Pressure inside the submarine

$= P_0$

Net force = $(P_0 + \rho gh)A - P_0A$

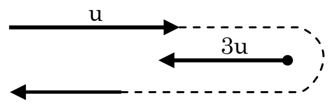
$= \rho gh\, A$

$= 1.03 \times 10^3 \times 10 \times 100 \times 900 \times 10^{-4}$

$= 9.27 \times 10^4$ Newton

$= 0.93 \times 10^5$ Newton

19. A spacecraft which is moving with a speed u relative to the earth in the x-direction, enters the gravitational field of a much more massive planet which is moving with a speed 3u in the negative x-direction. The spacecraft exits following the trajectory as shown below.

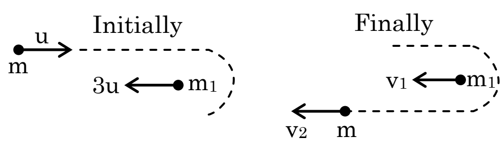


The speed of the spacecraft with respect to the earth a long time after it has escaped the planet's gravity is given by

(A) u (B) 4u (C) 2u (D) 7u

Ans. **[D]**

Sol.



from momentum conservation

$-mu + m_1 3u = m_1v_1 + mv_2$ ....(1)

from energy conservation

$$\frac{1}{2}mu^2 + \frac{1}{2}m_1 9u^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}mu^2 + \frac{1}{2}m_1(3u - v_1)(3u + v_1) = \frac{1}{2}mv_2^2$$

from equation ....(1)

$\Rightarrow m_1(3u - v_1) = m(v_2 + u)$

$$\frac{1}{2}mu^2 + \frac{1}{2}m(v_2 + u)(3u + v_1) = \frac{1}{2}mv_2^2$$

as $m_1 >>> m$, we can assume $v_1 \approx 3u$

$u^2 + (v_2 + u)(6u) = v_2^2 \Rightarrow v_2 = 7u$

20. The earth's magnetic field was flipped by 180º a million years ago. This flip was relatively rapid and took 10^5 years. Then the average change in orientation per year during the flip was closest to,

(A) 1 second (B) 5 seconds (C) 10 seconds (D) 30 seconds

Ans. **[B]**

Sol. 1º = 3600 arc sec

average change in orientation per year

$= \frac{180^\circ}{10^5}$ degree/year

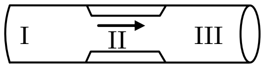
$= \frac{180 \times 3600}{10^5}$ sec/year

$= 1.8 \times 0.36$

$= 6.48$ sec/year

Closest option (B)

21. The platelets are drifting with the blood flowing in a streamline flow through a horizontal artery as shown below :



Artery is contracted in region II. Choose the correct statement.

(A) As the platelets enter a constriction, the platelets gets squeezed closer together in the narrow region and hence the fluid pressure must rise there

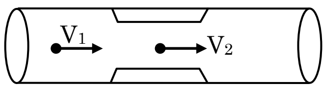
(B) As the platelets enter a constriction, pressure is lower there

(C) The artery's cross section area is smaller in the constriction and thus the pressure must be larger there because pressure equals the force divided by area

(D) Pressure is same in all the parts of the artery

Ans. **[B]**

Sol.



Using equation of continuity

$A_1V_1 = A_2V_2$

where A_1 and A_2 are cross-section area of region-I and region-II.

as $A_2 < A_1$

$\Rightarrow V_2 > V_1$

Using Bernouilli's equation

$P + \frac{1}{2}\rho V^2 = \text{constant}$

as $V_2 > V_1$

$P_2 < P_1$

therefore pressure will be lower at constriction.

22. Which is the following colourful patterns is due to diffraction of light ?

(A) Rainbow

(B) White light dispersed using a prism

(C) Colours observed on compact disc

(D) Blue colour of sky

Ans. **[C]**

Sol. (A) Rainbow occurs because of refraction, reflection and dispersion of light

(B) Dispersion

(C) Due to presence of concentric grrooves in compact disc, light gets diffracted & produced colorful pattern.

(D) Due to scattering of blue color

23. Two balls are projected with the same velocity but with different angles with the horizontal. Their ranges are equal. If the angle of projection of one is 30º and its maximum height is h, then the maximum height of other will be-

(A) 1 h (B) 3 h (C) 6 h (D) 10 h

Ans. **[B]**

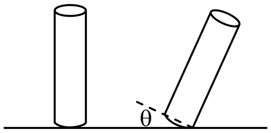
Sol. For the same range, another projection angle will be 90º – 30º = 60º

$$h = \frac{u^2 \sin^2 30^\circ}{2g}$$

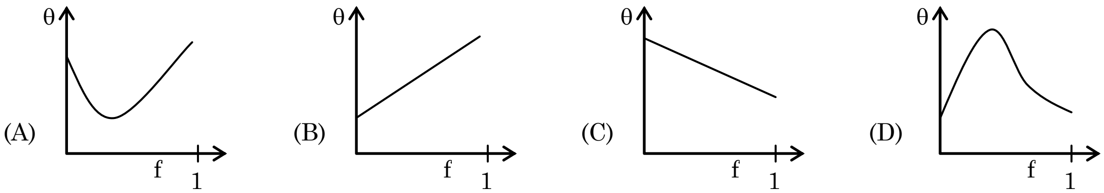
$$h_1 = \frac{u^2 \sin^2 60^\circ}{2g}$$

$$\frac{h_1}{h} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ} = 3 \Rightarrow h_1 = 3h$$

24. Figure below shows a shampoo bottle in a perfect cylindrical shape



In a simple experiment, the stability of the bottle filled with different amount of shampoo volume is observed. The bottle is tilted from one side and then released. Let the angle θ depicts the critical angular displacement resulting in the bottle losing its stability and tipping over. Choose the graph correctly depicting the fraction f of shampoo filled (f = 1 corresponds to completely filled) vs the tipping angle θ.



Ans. **[D]**

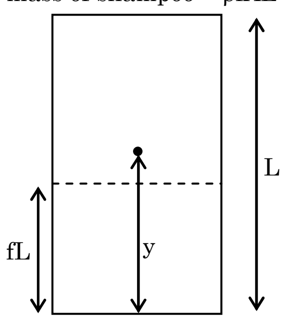
Sol. Mass of bottle = m_0

Length of bottle = L

base Area = A = πr^2

density of shampoo = ρ

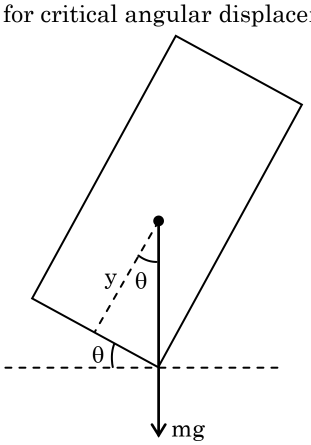
mass of shampoo = ρfAL



Center of mass of system

$$y = \frac{m_0 \frac{L}{2} + (\rho fAL)\left(\frac{fL}{2}\right)}{m_0 + \rho fAL}$$

for critical angular displacement, mg will pass through tilted side.



From the diagram $\tan\theta = \frac{r}{y}$

$$\tan\theta = \frac{r(m_0 + \rho ALf)}{\frac{L}{2}(m_0 + \rho ALf^2)}$$

at f = 0 and f = 1, tipping angle 'θ' will be same.

for very small values of f, we can neglect f^2 terms

$$\Rightarrow \tan\theta = \frac{r}{\frac{L}{2}}\frac{(m_0 + \rho ALf)}{m_0}$$

$$\theta = \tan^{-1}\left(\frac{r}{\frac{L}{2}}\frac{(m_0 + \rho ALf)}{m_0}\right)$$

So if f increases θ will increase.

25. At a height of 10 km above the surface of earth, the value of acceleration due to gravity is the same as that of a particular depth below the surface of earth. Assuming uniform mass density of the earth, the depth is,

(A) 1 km (B) 5 km (C) 10 km (D) 20 km

Ans. **[D]**

Sol. At a height 'h'

$$g_h = g_0\left(1 - \frac{2h}{R}\right)$$

when h (10 km) < R (6400 km)

at a depth 'd'

$$g_d = g_0\left(1 - \frac{d}{R}\right)$$

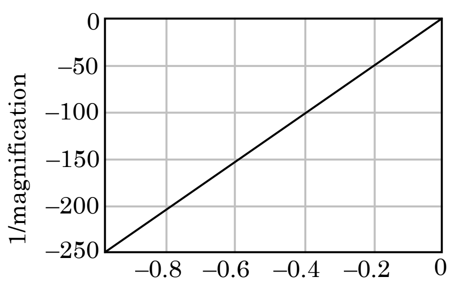
here $g_0 = \frac{GM}{R^2}$

Now $g_h = g_d$

$\Rightarrow d = 2h$

$d = 20$ km

26. The following graph depicts the inverse of magnification versus the distance between the object and lens data for a setup. The focal length of the lens used in the setup is



Distance between the object and the lens (m)

(A) 250 m (B) 0.004 m (C) 125 m (D) 0.002 m

Ans. **[B]**

Sol. Magnification (m) = $+\frac{v}{u}$

From lens formula,

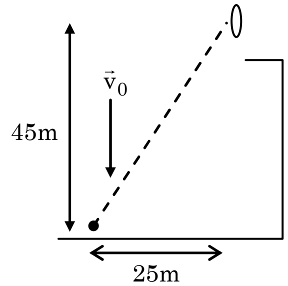
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{u}{v} - 1 = \frac{u}{f} \Rightarrow \frac{u}{v} = \frac{u}{f} + 1$$

$\therefore$ graph between $\left(\frac{u}{v}\right)$ [inverse of magnification] and u will be straight line with slope $\frac{1}{f}$

From graph, slope = 250

$\therefore f = \frac{1}{250}$ m = 0.004 m

27. In a circus, a performer throws an apple towards a hoop held at 45 m height by another performer standing on a high platform (see figure below). The thrower aims for the hoop and throws the apple with a speed of 24 m/s. At the exact moment that the thrower released the apple, the other performer drops the hoop. The hoop falls straight down. At what height above the ground does the apple go through the hoop ?



(A) 21 m (B) 22 m (C) 23 m (D) 24 m

Ans. **[B]**

Sol. Velocity of projection = 24 m/s

Distance between point of projection and hoop = $\sqrt{25^2+45^2}$

$\therefore$ Time taken by ball to reach the hoop

$= \frac{\sqrt{25^2+45^2}}{24}$

(Note : We are analysing the motion wrt hoop)

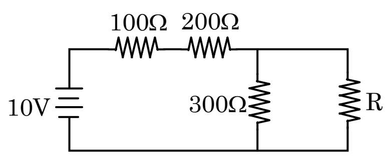
$\therefore$ Distance by which hoop will fall

$= \frac{1}{2}at^2 = \frac{1}{2}\times 10\times\frac{(25^2+45^2)}{24^2}$

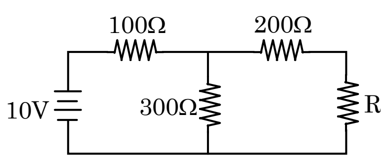
$\therefore$ Height above the ground where apple go through the hoop is given by

$45 - \left[\frac{1}{2}\times 10\times\frac{(25^2+45^2)}{24^2}\right] = 22$ m

28. A student was trying to construct the circuit shown in the figure below maked (a), but ended up constructing the circuit marked (b) Realizing her mistake, she corrected the circuit, but to her surprise, the output voltage (across R) did not change.



(a)



(b)

The value of resistance R is-

(A) 100 Ω (B) 150 Ω (C) 200 Ω (D) 300 Ω

Ans. [A]

Sol. For circuit (a),

$$i_R = \left(\frac{10}{\frac{300R}{300+R}+300}\right) \times \frac{300}{300+R}$$

↑

Current through cell

[Note : 300 Ω and R are in parallel which is in series with 100 & 200 Ω]

$$\therefore \; V_{R_a} = \frac{10 \times 300\, R}{300R + 300^2 + 300R}$$

[V_{R_a} is potential difference across resistance R]

For circuit (b),

$$i_R = \left(\frac{10}{\frac{(200+R)(300)}{200+R+300}+100}\right) \times \frac{300}{300+200+R}$$

↑

Current through cell

[Note : R and 200 Ω are in series which is in parallel with 300 Ω and again the combination is in series with 100 Ω]

$$\therefore \; V_{R_b} = \frac{100 \times 300R}{300 \times 200 + 300R + 100 \times 500 + 100R}$$

[V_{R_b} is potential difference across resistance R]

According to given situation

$V_{R_a} = V_{R_b}$

$\therefore$ 300R + 9 × 10^4 + 300 R = 6 × 10^4 + 400 R + 5 × 10^4

$\Rightarrow$ 200 R = 2 × 10^4 $\Rightarrow$ R = 100 Ω

29. The ratio of gravitational force and electrostatic repulsive force between two electrons is approximately (gravitational constant = 6.7 × 10^{-11} Nm^2/kg^2, mass of an electron = 9.1 × 10^{-31} kg, charge on an electron = 1.6 × 10^{-19} C)

(A) 24 × 10^{-24} (B) 24 × 10^{-36} (C) 24 × 10^{-44} (D) 24 × 10^{-54}

Ans. [C]

Sol. Gravitational force $F_G = \frac{Gm_1m_2}{r^2}$

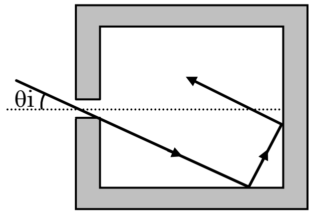
Electrostatic force $F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$$\therefore \frac{F_G}{F_e} = \frac{Gm_1m_2 . 4\pi\epsilon_0}{q_1q_2}$$

$$= \frac{6.7\times10^{-11}\times9.1\times10^{-31}\times9.1\times10^{-31}}{1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^{9}}$$

= 24 × 10^{-44}

30. A monochromatic beam of light enters a square enclosure with mirrored interior surfaces at an angle of incidence θi = (≠0) (see the figure below). For some value (s) of θi, the beam is reflected by every mirrored wall (other than the one with opening) exactly once and exists the enclosure through the same hole. Which of the following statements about this beam is correct ?



(A) The beam will not come out the enclosure for any value of θi

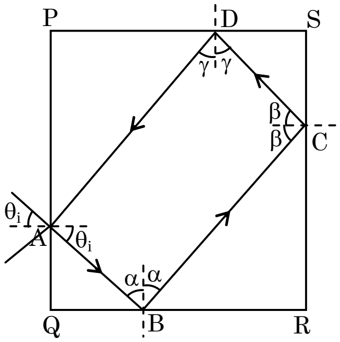
(B) The beam will not come out for more than two values of θi

(C) The beam will not come out only at θi = 45°

(D) The beam will come out for exactly two values of θi

Ans. **[C]**

Sol.



From geometry, $\alpha + \theta_i = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2} - \theta_i$

$\beta + \alpha = \frac{\pi}{2} \Rightarrow \beta = \theta_i$

$\beta + \gamma = \frac{\pi}{2} \Rightarrow \gamma = \frac{\pi}{2} - \theta_i$

Also AD || BC and AB || CD

∴ ABCD is a parallelogram and AB = CD

Also, ΔABQ ≅ ΔCDS

∴ From trigonometry

$\frac{AQ}{QB} = \frac{CR}{BR}$

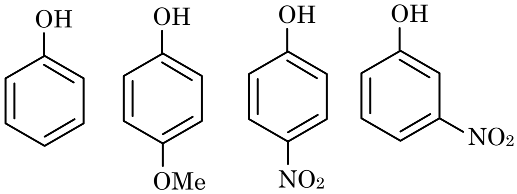
Let length of each side of square by ℓ, AQ = x, and QB = y

$\therefore \frac{x}{y} = \frac{\ell - x}{\ell - y} \Rightarrow x = y$

$\therefore \theta_i = \frac{\pi}{4}$

CHEMISTRY

31. The acidity of



I **II** **III** **IV**

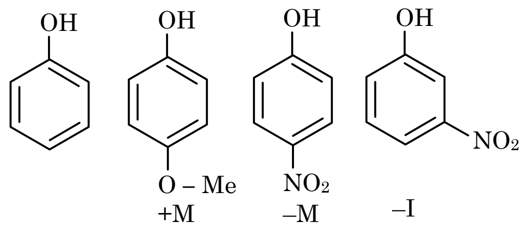
Follows the order

(A) I > II > III > IV (B) IV > III > II > I

(C) III > IV > I > II (D) III > II > IV > I

Ans. **[C]**

Sol.



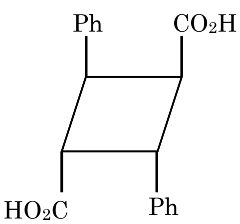
I **II** **III** **IV**

for acidic strength :

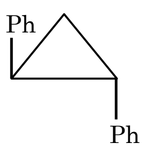
–M > – HC > –I > – > +I > + HC > + M

So III > IV > I > II

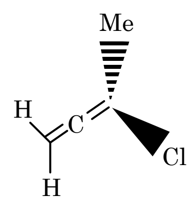
32. Among the following



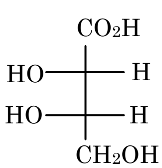
I



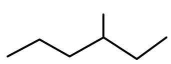
II



III



IV



V

the compounds which can exhibit optical activity are-

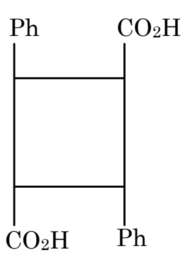
(A) only II, IV and V (B) only IV and V

(C) only I, II and V (D) only I, II and IV

Ans. **[A]**

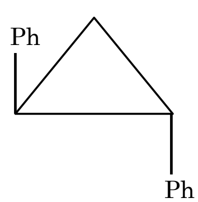
Sol.

(I)



C.O.S. is present so inactive

(II)



C.O.S. $\rightarrow$ X
P.O.S. $\rightarrow$ X
A.A.O.S. $\rightarrow$ X
so active

(III) P.O.S. is present so inactive

(IV) C.O.S and P.O.S. both are not present so active

(V) C.O.S and P.O.S. both are not present so active

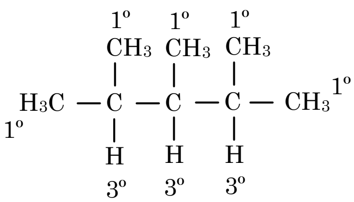
33. A molecule which has 1°, 2° and 3° carbon atoms is-

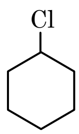
(A) 2,3,4-trimethylpentane (B) chlorocyclohexane

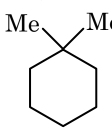
(C) 2,2-dimethylcyclohexane (D) methylcyclohexane

Ans. **[D]**

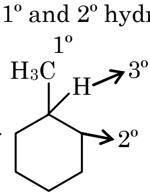
Sol. A $\Rightarrow$ 2,3,4-trimethylpentane only 1° and 3° hydrogen's are present



B $\Rightarrow$  all hydrogen's are 2°

C $\Rightarrow$  1,1-dimethylcyclo hexane

1° and 2° hydrogen's are present

D $\Rightarrow$  1°, 2° & 3° hydrogen's are present

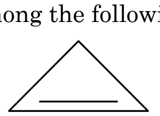
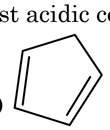
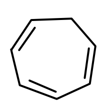
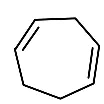
34. The organic compound which can be purified by steam distillation is-

(A) acetone (B) aniline (C) glucose (D) ethanol

Ans. **[B]**

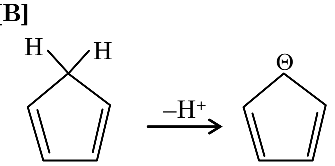
Sol. Aniline is purified by distillation method

35. Among the following, the most acidic compound is

(A)  (B)  (C)  (D) 

Ans. **[B]**

Sol.



Non aromatic Aromatic

36. A closed 10 L vessel contains 1 L water gas (1 : 1 CO : H_2) and 9 L air (20% O_2 by volume) at STP. The contents of the vessel are ignited. The number of moles of CO_2 in the vessel is closest to-

(A) 0.22 (B) 0.022 (C) 0.90 (D) 3.60

Ans. **[B]**

Sol. Water gas (CO : H_2 is 1 : 1) = 1 litre

Air = 9 litre

1 litre water gas at STP $\Rightarrow \frac{1}{22.4}$ moles of gas at STP

No. of moles of CO = $\frac{1}{2} \times \frac{1}{22.4}$ moles.

= No. of moles of CO_2 produced afte ignition

= 0.022

37. A certain metal has a work function of Φ = 2 eV. It is irradiated first with 1 W of 400 nm light and later with 1 W of 800 nm light. Among the following, the correct statement is : [Given Planck constant (h) = 6.626×10^{-34} $m^2\ kgs^{-1}$; Speed of light (c) = $3 \times 10^8\ ms^{-1}$]

(A) Both colors of light give rise to same number of photoelectrons

(B) 400 nm light gives rise to less energetic photoelectrons than 800 nm light

(C) 400 nm light leads to more photoelectrons

(D) 800 nm light leads to more photoelectrons

Ans. **[C]**

Sol. Work function of metal (ϕ) = 2eV

Energy of photon (λ = 400 nm) = $\frac{hc}{\lambda}$ = 3.105 eV

Energy of photon (λ = 800 nm) = $\frac{hc}{\lambda}$ = 1.5525 eV

Hence, photon with λ = 400 nm will emit photoelectrons while photon with λ = 800 nm will not emit photoelectrons.

38. Among the following, the correct statement about the chemical equilibrium is

(A) Equilibrium constant is independent of temperature

(B) Equilibrium constant is independent of temperature

(C) At equilibrium, the forward and the backward reactions stop so that the concentrations of reactants and products are constant

(D) Equilibrium constant is independent of whether you start the reaction with reactants or products

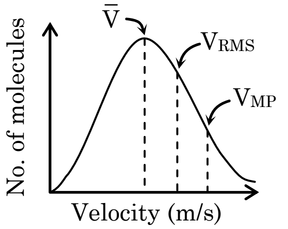
Ans. **[D]**

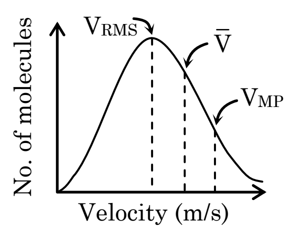
Sol. - Equilibrium constant is dependent on temperature

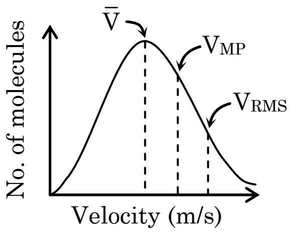
- Equilibrium constant do not tell us about the rate of reaction

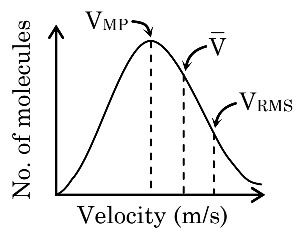
- At equilibrium, the forward and backward reactions do not stop but they have same rate

39. Among the following, the plot that shows the correct marking of most probable velocity (V_{MP}), average velocity ($\vec{V}$), and root mean square velocity (V_{RMS}) is

(A) 

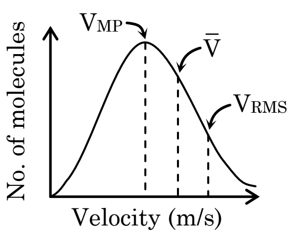
(B) 

(C) 

(D) 

Ans. **[D]**

Sol.

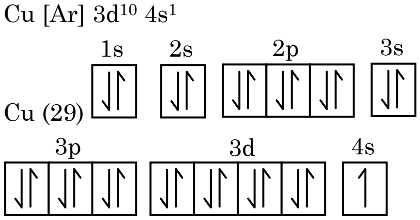


40. The correct set of quantum numbers for the unpaired electron of Cu atom is

(A) $n = 3, \ell = 2, m = -2, s = +1/2$
(B) $n = 3, \ell = 2, m = +2, s = -1/2$
(C) $n = 4, \ell = 0, m = 0, s = +1/2$
(D) $n = 4, \ell = 1, m = +1, s = +1/2$

Ans. **[C]**

Sol. Cu [Ar] $3d^{10}\,4s^1$



The set of quantum numbers for the unpaired e^- of Cu atom is.

$n = 4, \ell = 0, m = 0, s = +\frac{1}{2}$

41. Among the following, the most polar molecule is

(A) $AlCl_3$ (B) CCl_4 (C) $SeCl_6$ (D) $AsCl_3$

Ans. **[D]**

Sol.

$AlCl_3$	non-polar
CCl_4	non-polar
$SeCl_6$	non-polar
$AsCl_3$	polar

42. The covalent characters of $CaCl_2$, $BaCl_2$, $SrCl_2$ and $MgCl_2$ follow the order

(A) $CaCl_2 < BaCl_2 < SrCl_2 < MgCl_2$
(B) $BaCl_2 < SrCl_2 < CaCl_2 < MgCl_2$
(C) $CaCl_2 < BaCl_2 < MgCl_2 < SrCl_2$
(D) $SrCl_2 < MgCl_2 < CaCl_2 < BaCl_2$

Ans. **[B]**

Sol.

$MgCl_2$ $CaCl_2$ $SrCl_2$ $BaCl_2$

⟶ Φ_{M^2} ↓

⟶ Covalent Character ↓

$BaCl_2 < SrCl_2 < CaCl_2 < MgCl_2$ (Covalent character)

43. Among the following, the correct statement is-

(A) 100 has four significant figures

(B) 1.00×10^2 has four significant figures

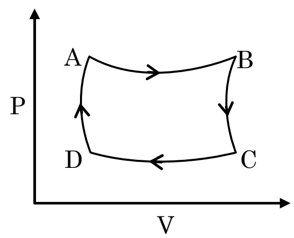
(C) 2.005 has four significant figures

(D) 0.0025 has four significant figures

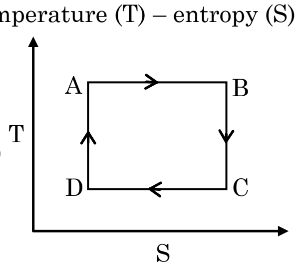
Ans. **[C]**

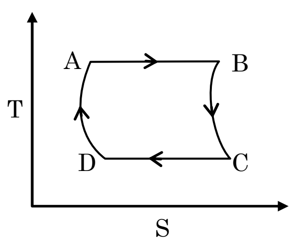
Sol.

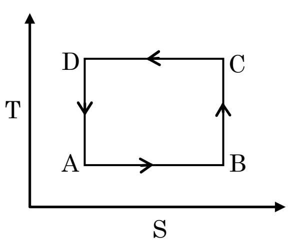
44. A thermodynamic cycle in the pressure (P) – volume (V) plane is given below :

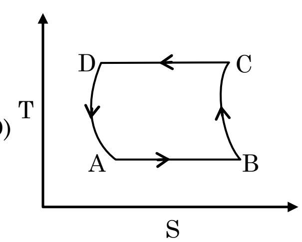


AB and CD are isothermal processes while BC and DA are adiabatic processes. The same cycle in the temperature (T) – entropy (S) plane is-

(A)


(B)


(C)


(D)


Ans. **[A]**

Sol. AB is isothermal reversible expansion process i.e. $\Delta T = 0$ and S increases as there is increase in volume.

BC is adiabatic reversible expansion process ($q_{rev} = 0$) i.e. temperature decreases and $\Delta s = 0$.

CD is isothermal reversible compression process i.e. $\Delta T = 0$ and S decreases as there is decrease in volume.

DA is adiabatic reversible compression process ($q_{rev} = 0$) i.e. temparature decreases and $\Delta S = 0$.

45. The first ionization potential (IP) of the elements Na, Mg, Si, P, Cl and Ar are 5.14, 7.65, 8.15, 10.49, 12.97 and 15.76 eV, respectively. The IP (in eV) of K is closest to-

(A) 13.3 (B) 18.2 (C) 4.3 (D) 6.4

Ans. **[C]**

Sol. The first ionisation potential of K is less than Na.

$\therefore$ The first ionization potential of K is closest to 4.3.

BIOLOGY

46. Which ONE of the following chemicals serves as a substrate for carbonic anhydrase ?

(A) O_2 (B) CO_2 (C) NO_2 (D) CO

Ans. **[B]**

Sol.

47. Which ONE of the following is NOT a function of the small intestine ?

(A) Absorption of end products of digestion

(B) Digestion of proteins

(C) Digestion of lipids

(D) Acidification of ingested food

Ans. **[D]**

Sol.

48. Insulin stimulates the conversion of glucose to

(A) fructose (B) glycogen (C) sucrose (D) starch

Ans. **[B]**

Sol.

49. Which ONE of the following statements about ecosystem energetics is INCORRECT ?

(A) The metabolic requirements of poikilotherms are higher than that of homeotherms

(B) Autotrophs form the base of the food chain in natural ecosystems

(C) In terrestrial ecosystems, most of the primary production is consumed by detritivores and not herbivores

(D) Approximately 10% energy of one trophic level is transferred to the next level

Ans. **[A]**

Sol.

50. Proton motive force is created by pumping protons across the

(A) *trans*-Golgi network (B) endoplasmic reticulum

(C) mitochondrial inner membrane (D) early endosomal membrane

Ans. **[C]**

Sol.

51. Which ONE of the following Mendelian diseases is an example of X-linked recessive disorder ?

(A) Haemophilia (B) Phenylketonuria

(C) Sickle cell anaemia (D) Beta-thalassemia

Ans. **[A]**

Sol.

52. Which ONE of the following pairs gives rise to fruit and seed, respectively, in a typical angiosperm plant ?

(A) Ovule and ovary (B) Ovary and pollen

(C) Pollen and anther (D) Ovary and ovule

Ans. **[D]**

Sol.

53. The concept of vaccinatin arose from Edward Jenner's observation that

(A) injecting inactivated anthrax spores in sheeps protected them from anthrax

(B) injecting humans with tuberculosis-infected lung extracts proteced them from tuberculosis

(C) milk-maids previously infected with cowpox did not contract small pox

(D) injecting inactivated rabies virus in humans protected them from rabies

Ans. **[C]**

Sol.

54. A plant with genotype *AABBCC* is crossed with another plant with *aabbcc* genotype. How many different genotypes of pollens is possible in an F1 plant if these three loci follow independent assortment ?

(A) 8 (B) 4 (C) 2 (D) 1

Ans. **[A]**

Sol.

55. Which ONE of the following sequences of events CORRECTLY represents mitosis ?

(A) Metaphase, telophase, prophase, anaphase

(B) Anaphase, prophase, metaphase, telophase

(C) Prophase, anaphase, metaphase, telophase

(D) Prophase, metaphase, anaphase, telophase

Ans. **[D]**

Sol.

56. The amount of air that is left behind in lungs after expiratory reserve volume has been exhaled is

(A) inspiratory reserve volume (B) tidal volume

(C) residual volume (D) vital capacity

Ans. **[C]**

Sol.

57. Match the species in Column-I with their respective feature of body organisation in Column-II.

	Column-I		Column-II
P.	Mollusca	i.	Pseudocoelom
Q.	Annelida	ii.	Radula
R.	Nematoda	iii.	Radial symmetry
S.	Echinodermata	iv.	Segmentation

Choose the CORRECT combination.

(A) P-ii, Q-i, R-iv, S-iii (B) P-ii, Q-iv, R-i, S-iii

(C) P-iii, Q-iv, R-i, S-ii (D) P-iv, Q-iii, R-ii, S-i

Ans. **[B]**

Sol.

58. Who among the following scientists proposed the theory of natural selection independently of Charles Darwin?

(A) Alfred Russel Wallace (B) Carl Linnaeus

(C) Georges Cuvier (D) Jean-Baptiste Lamarck

Ans. **[A]**

Sol.

59. The maximum concentration of harmful chemical is expected to be found in organisms

(A) at the bottom of a food chain (B) at the middle of a food chain

(C) at the top of a food chain (D) at any level in a food chain

Ans. **[C]**

Sol.

60. The genome of SARS-CoV2 is composed of

(A) double stranded DNA (B) double stranded RNA

(C) single stranded DNA (D) single stranded RNA

Ans. **[D]**

Sol.

Part – II
Two - Mark Questions

MATHEMATICS

61. Let A denote the set of all 4-digit natural numbers with no digit being 0. Let $B \subset A$ consist of all numbers x such that no permutation of the digits of x gives a number that is divisible by 4. Then the probability of drawing a number from B with all even digits is

(A) $\frac{625}{1641}$ (B) $\frac{16}{641}$ (C) $\frac{16}{1641}$ (D) $\frac{1000}{1641}$

Ans. **[C]**

Sol. All even digit numbers in B = {2, 4, 6, 8}

fav : forming a 4 digit nos with all digit is even and not divisible by 4

2222, 6666, 2266,

$\frac{4!}{2!2!} = 6$ cases

(2226), (2666)

$\frac{4!}{3!} = 4$ cases $\frac{4!}{3!} = 4$ cases

Total = 1 + 1 + 6 + 4 + 4 = 16 cases

Total : forming a 4-digit no. from {1, 2, ..., 9} but not divisible by 4

C-1 all 4 digit are add = $5^4 = 625$

C-2 all 4 digit are even = 16

C-3 one even and 3 odd

you can not take 2 or 6 as one of even digit

(12, 16, 32, 36, ...) all are divisible by 4

But you can take 4 or 8 as one of even digit ......

${}^2C_1 \times {}^4C_1 \times 5^3 = 1000$

Take one even digit out of 4 & 8

select one place for 4 & 8 out of 4-places

Total = 1000 + 625 + 16 = 1641

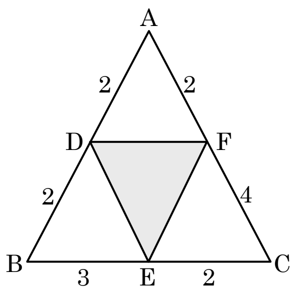
$P = \frac{\text{fav.}}{\text{total}} = \frac{16}{1641}$

62. Let ABC be a triangle such that AB = 4, BC = 5 and CA = 6. Choose points D, E, F on AB, BC, CA respectively, such that AD = 2, BE = 3, CF = 4. Then $\frac{\text{area } \Delta DEF}{\text{area } \Delta ABC}$ is-

(A) $\frac{1}{4}$ (B) $\frac{3}{15}$ (C) $\frac{4}{15}$ (D) $\frac{7}{30}$

Ans. **[C]**

Sol.



$\frac{\text{area }(\Delta DEF)}{\text{area }(\Delta ABC)} =$

Let area (ΔABC) = Δ

$$\frac{\text{area }(\Delta BED)}{\text{area }(\Delta ABC)} = \frac{\frac{1}{2}.6\sin B}{\frac{1}{2}.20\sin B} = \frac{3}{10}$$

$$\frac{\text{area }(\Delta ADF)}{\text{area }(\Delta ABC)} = \frac{\frac{1}{2}.4\sin A}{\frac{1}{2}.24\sin A} = \frac{1}{6}$$

$$\frac{\text{area }(\Delta CEF)}{\text{area }(\Delta ABC)} = \frac{\frac{1}{2}.8\sin C}{\frac{1}{2}.30\sin C} = \frac{4}{15}$$

$$\text{area }(\Delta BED) = \frac{3}{10}\Delta$$

$$\text{area }(\Delta ADF) = \frac{\Delta}{6}$$

$$\text{area }(\Delta CEF) = \frac{4\Delta}{15}$$

$$\text{area }(\Delta DEF) = \Delta - \left(\frac{3}{10}+\frac{1}{6}+\frac{4}{15}\right)\Delta$$

$$= \Delta - \left(\frac{27+15+24}{90}\right)\Delta$$

$$= \Delta - \left(\frac{66}{90}\right)\Delta = \frac{24\Delta}{90}$$

$$\frac{\text{area }(\Delta DEF)}{\text{area }(\Delta ABC)} = \frac{24}{90} = \frac{4}{15}$$

63. The number of ordered pairs (x, y) of integers satisfying $x^3 + y^3 = 65$ is-

(A) 0 (B) 2 (C) 4 (D) 6

Ans. **[B]**

Sol. $x^3 + y^3 = 65$

$(x + y)(x^2 + y^2 - xy) = 65 \times 1$

$= 13 \times 5$

$= 5 \times 13$

$= 1 \times 65$

clearly $x^2 + y^2 - xy > 0$

C-1 : $x + y = 5$ and $x^2 + y^2 - xy = 13$

$x^2 + (5 - x)^2 - x(5 - x) = 13$

$3x^2 - 15x + 12 = 0$

$x^2 - 5x + 4 = 0 \Rightarrow x = 1, 4$

$(x, y) = (1, 4)$ and $(4, 1)$

C-2 : $x + y = 13$ and $x^2 + y^2 - xy = 5$

$x^2 + (13 - x)^2 - x(13 - x) = 5$

$3x^2 - 39x + 164 = 0$, $x \notin I$ (Not possible)

C-3 : $x + y = 1$ and $x^2 + y^2 - xy = 65$

$x^2 + (1 - x)^2 - x(1 - x) = 65$

$3x^2 - 3x - 64 = 0$, $x \notin I$ (Not possible)

C-4 : $x + y = 65$ and $x^2 + y^2 - xy = 65$

No solution

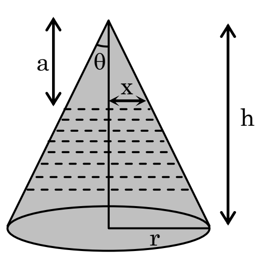
so two ordered pair satisfy the relation

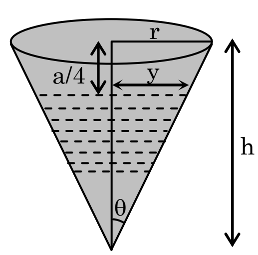
64. A bottle in the shape of a right-circular cone with height h contains some water. When its base is placed on a flat surface, the height of the vertex from the water level is a units. When it is kept upside down, the height of the base from the water level is $\frac{a}{4}$ units. Then the ratio $\frac{h}{a}$ is

(A) $\frac{1+\sqrt{85}}{4}$ (B) $\frac{1+\sqrt{85}}{8}$ (C) $\frac{1+\sqrt{65}}{4}$ (D) $\frac{1+\sqrt{65}}{8}$

Ans. **[B]**

Sol.





$\frac{r}{h} = \frac{x}{a} \Rightarrow x = \frac{ra}{h}$

$\frac{r}{h} = \frac{y}{h - \frac{a}{4}}$

$y = \frac{r}{h}\left(h - \frac{a}{4}\right)$

Equating volume of water in both cases

$$\frac{1}{3}(\pi r^2 h - \pi x^2 a) = \frac{1}{3}\pi y^2\left(h - \frac{a}{4}\right)$$

$$\Rightarrow r^2h - r^2h - \frac{r^2a^2}{h}.a = \frac{r^2}{h^2}\left(h - \frac{a}{4}\right)^2\left(h - \frac{a}{4}\right)$$

$$\Rightarrow \frac{h^2}{a^2} - \frac{h}{4a} - \frac{21}{16} = 0$$

$$\frac{h}{a} = \frac{\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{21}{4}}}{2}$$

$$\frac{h}{a} = \frac{1+\sqrt{85}}{8}$$

65. Consider the following two statements :

I. if n is a composite number, then n divides (n – 1) !

II. There are infinitely many natural numbers n such that $n^3 + 2n^2 + n$ divides n!

Then

(A) I and II are true (B) I and II are false

(C) I is true and II is false (D) I is false and II is true

Ans. **[D]**

Sol. If n is a composite number (Take n = 4)

For n = 4, n does not divide (n – 1) !

Hence Ist statement is false

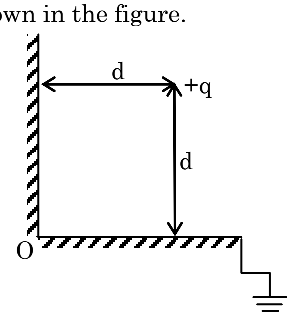
(II) $n^3 + 2n^2 + n = n(n + 1)^2$

$n(n + 1)^2$ divides n!

If n is such that (n + 1) is a prime number (Take n = 6) so $n(n + 1)^2$ does not divide n! but there are infinite values of n(n = 104, 109, 114, ...) for which $n(n + 1)^2$ divide n! but it is not true for every natural numbers.

PHYSICS

66. A charge +q is situated at a distance 'd' away from both the sides of a grounded conducting 'L' shaped sheet as shown in the figure.



The force acting on the charge +q is-

(A) towards O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}(2\sqrt{2}+1)$

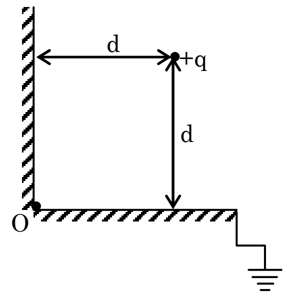
(B) away from O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}(2\sqrt{2}+1)$

(C) towards O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}(2\sqrt{2}-1)$

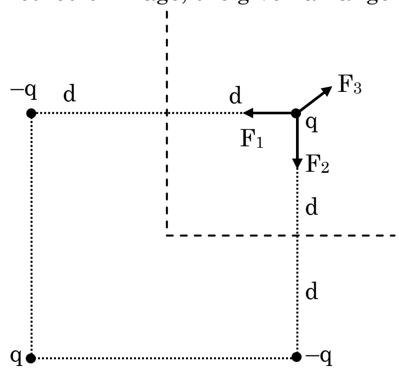
(D) away from O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}(2\sqrt{2}-1)$

Ans. [C]

Sol.



By method of image, the given arrangement is equivalent to



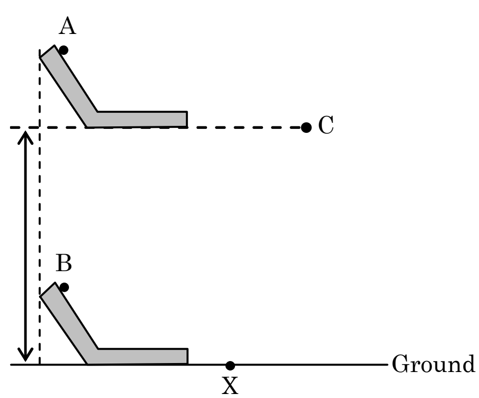
$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2d)^2},\ F_2 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2d)^2}$$

$$F_3 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2\sqrt{2}d)^2}$$

$$\therefore F_{net} = \sqrt{2}\frac{q^2}{16\pi\epsilon_0 d^2} - \frac{q^2}{32\pi\epsilon_0 d^2}$$

$$= \frac{q^2}{32\pi\epsilon_0 d^2}(2\sqrt{2}-1)\ \text{[towards O]}$$

67. Three balls, A, B and C, are released and all reach the point X (shown in the figure). Balls A and B are released from two identical structures, one kept on the ground and the other at height h, from the ground as shown in the figure. They take time t_A and t_B respectively to reach X (time starts after they leave the end of the horizontal portion of the structure). The ball C is released from a point at height h, vertically above X and reaches X in time t_C. Choose the correct statement.



(A) $t_C < t_A < t_C$ (B) $t_C = t_A = t_C$ (C) $t_C = t_A < t_C$ (D) $t_C < t_A = t_C$

Ans. **[B]**

Sol. Work done by gravity on A and B is same.

∴ Horizontal velocity of A = horizontal velocity of B as they leave the horizontal portion of the structure.

∴ $t_A = t_B$ ....(i)

Also vertical velocity of A and vertical velocity of C when released are both zero

∴ They both will cover same vertical distance in same time.

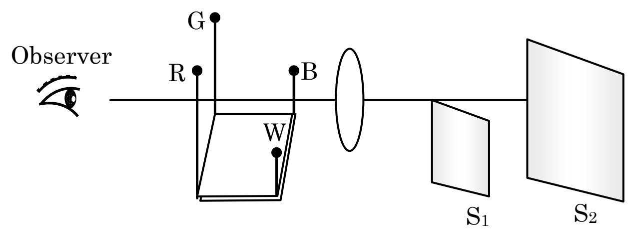
∴ $t_A = t_C$ ....(ii)

From (i) and (ii)

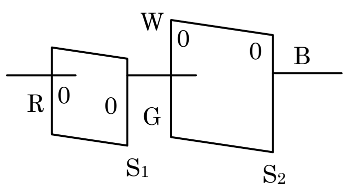
$t_A = t_B = t_C$

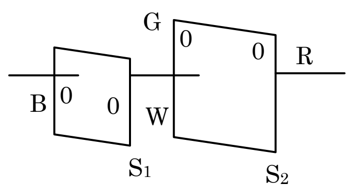
Note : Correction in optio. Option B should be $t_A = t_B = t_C$

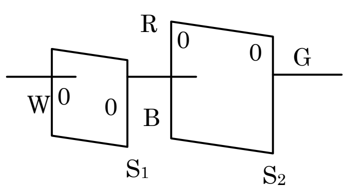
68. Four bulbs, red, green, white and blue (denoted by R, G, W and B respectively) are kept in front of a converging lens (as shown in the figure below). The observer sees that the green and blue bulbs are kept to the left of the principle axis while the red and white bulbs are kept to the right of the principle axis. He also see that the red and green bulbs are above the principle axis while the white and blue bulbs are below the principle axis. The screens S_1 and S_2 are set at appropriate positions for the focusing to view the images.

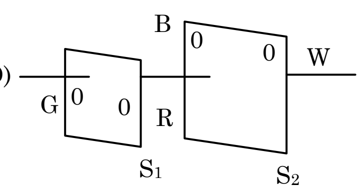


Choose the figure that correctly represents the images are seen by the observer.

(A) 

(B) 

(C) 

(D) 

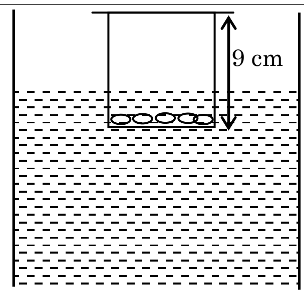
Ans. **[A]**

Sol. Since the images are being made on screen, hence real.

∴ Image will be inverted

Also since blue and white are nearer to lens, hence their real image will be far from lens as compared to red and green.

69. A wide bottom cylindrical massless plastic container of height 9 cm has 40 identical coins inside it and is floating on water with 3 cm inside the water. If we start putting more of such coins on its lid, it is observed that after N coins are put, it equilibrium changes from stable to unstable. Equilibrium in floating is stable if the geometric center of the submerged portion is above the center of mass of the object). The value of N is closest to



(A) 6 (B) 10 (C) 16 (D) 24

Ans. **[B]**

Sol. Let mass of each coin be m.

$\therefore$ Location of center of mass after N coins are kept on lid from bottom of container is

$$\frac{40\,m \times 0 + Nm \times 9}{(40+N)\,m} = \frac{9N}{40+N}\ \text{cm}$$

Also height of submerged portion after keeping N coins on lid will be,

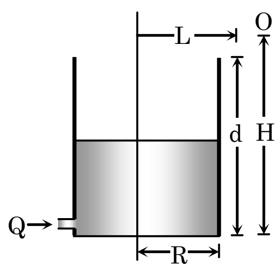
$$\frac{4(40+N)}{40}\ \text{cm}$$

$\therefore$ Equilibrium will just be stable if

$$\frac{3}{40}\,\frac{(40+N)}{2} = \frac{9N}{(40+N)}$$

$\Rightarrow 3N^2 - 480\,N + 4800 = 0 \Rightarrow N = 10.72$

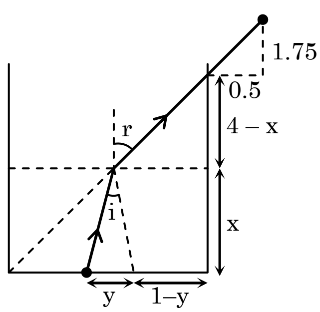
70. A small coin is fixed at the center of the base of an empty cylindrical steel container having radius R = 1m and height d = 4m. At time t = 0 s, the container starts getting filled with water at a flow rate of Q = 0.1 m^3/s without disturbing the coin. Find the approximate time when the coin will first be seen by the observer "O" from the height of H = 5.75 m above and L = 1.5m radially away from the coin as shown in the figure. Refractive index of water is n = 1.33.



(A) 0 s (B) 32 s (C) 63 s (D) 150 s

Ans. **[C]**

Sol.



$$1.33 \sin i = \sin r = \frac{2}{\sqrt{53}} \quad \text{...(i)}$$

Also, $\tan r = \frac{2}{7} = \frac{1-y}{4-x} \Rightarrow y = \frac{2x-1}{7}$ ...(ii)

$\therefore$ From equation (i)

$\frac{1.33y}{\sqrt{y^2+x^2}} = \frac{2}{\sqrt{53}} \Rightarrow (1.33)^2\, 53y^2 = 4(x^2 + y^2)$

$\Rightarrow 89.7517\, y^2 = 4x^2 \Rightarrow y = \frac{2x}{\sqrt{89.7517}}$ ...(iii)

From equation (ii) & (iii),

$\frac{2x-1}{7} = \frac{2x}{\sqrt{89.7517}} \Rightarrow 14x = (2x-1)\, 9.47$

$\therefore x = 1.92$

$\therefore$ volume of water filled = $\pi R^2 x = (3.14 \times 1^2 \times 1.92)\ m^3$

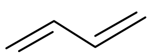
$\therefore$ Qt = 6.0288 [Q is volume flow rate]

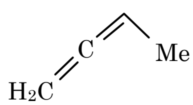
$\therefore$ t = 60.288 sec

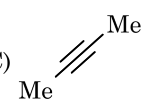
so option (C) is the nearest value.

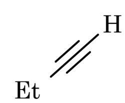
CHEMISTRY

71. A hydrocarbon X with molecular formula C_4H_6 decolorizes bromine water and forms a white precipitate in ethanolic $AgNO_3$ solution Treatment of X with $HgCl_2$ in aqueous H_2SO_4 produces a compound, which gives a yellow precipitate when treated with I_2 and NaOH. The structure of X is-

(A) 

(B) 

(C) 

(D) 

Ans. **[D]**

Sol.

$Et - C \equiv CH \xrightarrow{Br_2/H_2O}$ Decolourize brown colour of Br_2

(X)

$\downarrow$ Ethanolic $AgNO_3$

$Et - C \equiv \overset{\ominus}{C}\, Ag^+$

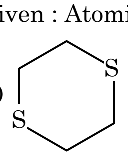
White P.P.T.

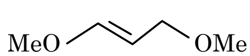
$\underset{[X]}{Et - C \equiv CH} \xrightarrow[aq.H_2SO_4]{HgCl_2} Et - \overset{O}{\overset{||}{C}} - CH_3$

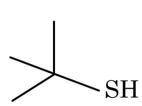
$\downarrow$ NaOH+I_2

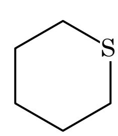
$Et - COO^-Na^+ + CHI_3$

72. 0.102 g of an organic compound X was oxidized with fuming nitric acid. The resulting solution, after reaction with an excess of aqueous $BaCl_2$, produced 0.233 g of $BaSO_4$ as a precipitate. Compound X is likely to be : [Given : Atomic wt. Ba = 137]

(A) 

(B) 

(C) 

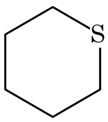
(D) 

Ans. **[D]**

Sol. 0.233 gm $BaSO_4$ has 1 millimole $BaSO_4$ and hence has 1 millimole S

$\therefore$ organic compound (X) also has 1 millimole S % of S in 0.102 gm of organic compound (X)

$= \frac{0.032}{0.102}\times 100 = 31.37\ \%$



102 gm of this organic compound has 32 gm S

This has same % of S.

73. The specific heat of a certain substance is 0.86 J g^{-1} K^{-1}. Assuming ideal solution behavior, the energy required (in J) to heat 10 g of 1 molal of its aqueous solution from 300 K to 310 K is closest to [Given : molar mass of the substance = 58 g mol^{-1}; specific heat of water = 4.2 J g^{-1} K^{-1}]

(A) 401.7 (B) 424.7 (C) 420.0 (D) 86.0

Ans. **[A]**

Sol. Specific heat capacity of substance

= 0.86 J g^{-1} K^{-1}

1 molal aqueous solution

$\Rightarrow$ 1000 gm water has 58 gm solute (total mass of solution = 1058 gm)

If we take 10 gm solution it would have

water = $\frac{1000}{1058}\times 10$ gm

substance = $\frac{58}{1058}\times 10$ gm

Heat required = $\frac{1000}{1058}\times 10\times 4.2\times 10$ (for water) = 396.975

+

$\frac{58}{1058}\times 10\times 0.86\times 10$ (for substance) = 4.715

= 396.975 + 4.715 = 401.69 $\simeq$ 401.7

74. Strength of a H_2O_2 solution is labelled as 1.79 N. Its strength can also be expressed as closest to-

(A) 20 volume (B) 5 volume (C) 10 volume (D) 15 volume

Ans. **KVPY Given [C]**

Sol. $2H_2O_{2(aq.)} \rightarrow 2H_2O(\ell) + O_2(g)$

It is a common trend that n-factor (H_2O_2) is taken as '2'

By the definition of volume strength of H_2O_2 if we consume 1 ℓ of 1 N H_2O_2 in the above equation we are using 1 gram equivalent of H_2O_2 = 0.5 moles of H_2O_2 (by using n-factor = 2)

This will produce $\frac{1}{4}$ moles of O_2 gas at N.T.P.

$\equiv \frac{1}{4}\times 22.4 = 5.6\ \ell$ of O_2 gas

i.e. 1 ℓ, 1 N H_2O_2 solution gives 5.6 ℓ O_2 at N.T.P.

Hence 1 N = 5.6 'vol.' H_2O_2 solution

In the given question it is 1.76 N H_2O_2 solution

Hence volume strength = 5.6 × 1.79 $\simeq$ 10 volumes.

By Career Point Ans. (A)

$2H_2O_{2(aq.)} \rightarrow 2H_2O(\ell) + O_2(g)$

The above equation is a classic example of disproportionation of H_2O_2, hence we should take n-factor of $H_2O_2 = 1$

1 ℓ, 1 N H_2O_2 solution ≡ 1 gm equivalent of H_2O_2

≡ 1 mole of H_2O_2

This will produce $\frac{1}{2}$ moles of O_2 gas at N.T.P.

$= \frac{1}{2} \times 22.4 = 11.2\ \ell$ of O_2 gas

i.e., 1ℓ, 1 N H_2O_2 solution gives 11.2 ℓ of O_2 gas at N.T.P.

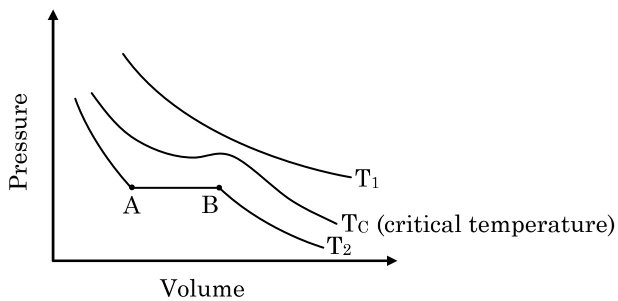
Hence 1 N ≡ 11.2 'vol' H_2O_2 solution

In the given question it is 1.79 N H_2O_2 solution

Hence, volume strength = $1.79 \times 11.2 \simeq 20$ volumes

Suggested Answer is option (A)

75. The isotherms of a gas are shown below :



Among the following,

(i) At T_1, the gas cannot be liquefied

(ii) At point B, liquid starts to appear at T_2

(iii) T_C is the highest temperature at which the gas can be liquefied

(iv) At point A, a small increase in pressure condenses the whole system to a liquid

The correct statements are :

(A) only (i) and (ii)
(B) only (i), (iii) and (iv)
(C) only (ii), (iii) and (iv)
(D) (i), (ii), (iii) and (iv)

Ans. **[D]**

Sol. Since $T_1 > T_C$, the gas cannot be liquefied at T_1 T_C is the highest temperature at which the gas can be liquefied.

At temperature T_2, liquid starts to appear at point B, however a small increase in pressure at point A condenses the whole system to liquid.

BIOLOGY

76. Anthropocene refers to the geological age during which

(A) the earliest hominids radiated from their ancestral forms

(B) human activity significantly influenced climate and environment

(C) arthropod radiation was highest

(D) arthropod radiation significantly influenced climate and environment

Ans. **[B]**

Sol.

77. Match the vitamins listed in Column-I with the diseases caused due to their deficiency in Column II.

Column-I	Column-II
P. Vitamin A	i. Pellegra
Q. Vitamin B_2	ii. Rickets
R. Vitamin D	iii. Ariboflavinosis
S. Vitamin B_{12}	iv. Pernicious anaemia

Choose the CORRECT combination

(A) P-iv; Q-ii; R-iii; S-v
(B) P-i; Q-ii; R-iv; S-iii
(C) P-iv; Q-iii; R-ii; S-v
(D) P-iii; Q-iv; R-v; S-i

Ans. **[C]**

Sol.

78. An adult mammal with 50kg body weight has the following functional parameters of its lungs.

Inspiratory reserve volume = 40 ml/kg body weight

Expiratory reserve volume = 15 ml/kg body weigth

Vital capacity = 60 ml/kg body weight

Breathing rate = 20/min

The volume (in litre) of air that its lungs displace in 24 hours is-

(A) 72,000 (B) 7,200 (C) 3,600 (D) 1,200

Ans. **[B]**

Sol.

79. In a breed of dog, long-haired phenotype is recessive to short-hair. In a litter, one pup is short-haired and its sibling is long-haired. Consider following possible phenotypes of the parents.

i. Both parents are short-haired

ii. Both parents are long-haired

iii. One parent is short-haired, and one is long-haired

Choose the CORRECT combination of the possible parential phenotypes.

(A) i only (B) ii only (C) iii only (D) i or iii

Ans. **[D]**

Sol.

80. In medical diagonostics for a disease, *sensitivity* (denoted by a) of a test refers to the probability that a test result is positive for person with the disease, whereas *specificity* (denoted by b) refers to the probability that person without the disease tests negative. A diagnostic test for COVID-19 has the values of $a = 0.99$ and $b = 0.99$. If the prevalence of COVID-19 in a population is estimated to be 10%, what is the probability that a randomly chosen person tests positive for COVID-19 ?

(A) 0.099 (B) 0.10 (C) 0.108 (D) 0.11

Ans. **[C]**

Sol.