Part – I

One - Mark Question

MATHEMATICS

1. Three children, each accompanied by a guardian, seek admission in a school. The principal want to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this be done –

(A) 60 (B) 90 (C) 120 (D) 180

Ans. **(B)**

Sol. $\frac{6!}{(2!)^3} = 90$

2. In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has –

(A) No solution (B) Exactly two distinct solutions

(C) Exactly four distinct solutions (D) Infinitely may solutions

Ans. **(D)**

Sol. $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1; x \geq 1$

$\sqrt{(x-1)-2\times 2\sqrt{x-1}+4} + \sqrt{(x-1)-6\sqrt{x-1}+9} = 1$

$\left|\sqrt{x-1}-2\right| + \left|\sqrt{x-1}-3\right| = 1$

Case - I

$\sqrt{x-1}-2+\sqrt{x-1}-3 = 1$ when $x \geq 10$

$2\sqrt{x-1} = 6$

$x = 10$

Case -II

$\sqrt{x-1}-2-\sqrt{x-1}+3 = 1$ when $5 \leq x \leq 10$

Case -III

$-\sqrt{x-1}+2-\sqrt{x-1}+3 = 1$ when $1 \leq x \leq 5$

$2\sqrt{x-1} = 4$

$x = 5$

3. The maximum value M of $3^x + 5^x - 9^x + 15^x - 25^x$, as x varies over reals, satisfies –

(A) $3 < M < 5$ (B) $0 < M < 2$ (C) $9 < M < 25$ (D) $5 < M < 9$

Ans. **(B)**

Sol. $M = a + b - a^2 + ab - b^2$

but $\frac{a^2+b^2}{2} \geq ab \Rightarrow a^2 + b^2 \geq 2ab$ or $-(a^2 + b^2) \leq -2ab$

$M \leq a + b - ab$

So $M < 1 - (a - 1)(b - 1)$

So maximum value of M can be 1.

at x = 0, else where the value of the function is less than 1. So answer must be option (B) but KVPY has mentioned option (C) as answer in its answer key.

4. Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. then |p| =

(A) 0 (B) 3 (C) 5 (D) 17

Ans. **(C)**

Sol. $(x-3)^2 + (y-p)^2 = 9 - 17 + p^2$

Director circle is

$(x-3)^2 + (y-p)^2 = 2(p^2 - 8)$

Passes through (0, 0)

$9 + p^2 = 2p^2 - 16$

$p^2 = 25 \Rightarrow p = \pm 5 \Rightarrow |p| = 5$

5. Let a, b, c, d be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a - c)^2 + b - d$ is –

(A) 0 (B) 5 (C) 30 (D) 36

Ans. **(B)**

Sol. $y = 2x^3 + ax + b$ $\qquad y = 2x^3 + cx + d$

No Solution

$2x^3 + ax + b \neq 2x^3 + cx + d$

$ax + b \neq cx + d$ $\qquad$ for no real x

$(a - c)x \neq d - b$

$x \neq \dfrac{d-b}{a-c}$ $\qquad$ a = c

$(a - c)^2 + (b - d) = 0 + 6 - 1 = 5$

6. Consider the conic $ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = \pi e$. Suppose P is any point on the conic and S_1, S_2 are the foci of the conic, then the maximum value of $(PS_1 + PS_2)$ is –

(A) πe (B) $\sqrt{\pi e}$ (C) $2\sqrt{\pi}$ (D) $2\sqrt{e}$

Ans. **(C)**

Sol. $ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = \pi e$

$e(x^2 - 2ex + e^2) + \pi(y^2 - 2\pi y + \pi^2) = \pi e$

$\dfrac{(x-e)^2}{\pi} + \dfrac{(y-\pi)^2}{e} = 1$

$a^2 = \pi \Rightarrow a = \sqrt{\pi}$ $\qquad$ $\pi > e$

$PS_1 + PS_2 = 2a$ $\qquad$ Major axis is || to axis

$PS_1 + PS_2 = 2\sqrt{\pi}$

7. Let $f(x)=\dfrac{\sin(x-a)+\sin(x+a)}{\cos(x-a)-\cos(x+a)}$, then–

(A) $f(x+2\pi)=f(x)$ but $f(x+\alpha)\neq f(x)$ for any $0<\alpha<2\pi$

(B) f is a strictly increasing function

(C) f is strictly decreasing function

(D) f is a constant function

Ans. **(D)**

Sol. $f(x)=\dfrac{\sin(x-a)+\sin(x+a)}{\cos(x-a)-\cos(x+a)}=\dfrac{2\sin(x).\cos a}{2\sin x.\sin a}=\cot a$

8. The value of tan81° – tan63° – tan27° + tan9° is –

(A) 0 (B) 2 (C) 3 (D) 4

Ans. **(D)**

Sol. tan81° – tan63° – tan27° + tan9°

$\tan(90^\circ-9^\circ)-\tan(90^\circ-27^\circ)-\tan 27^\circ+\tan 9^\circ$

$\cot 9^\circ-\cot 27^\circ-\tan 27^\circ+\tan 9^\circ$

By solving we get

= 4

9. The mid- point of the domain of the function $f(x)=\sqrt{4-\sqrt{2x+5}}$ for real x is –

(A) 1/4 (B) 3/2 (C) 2/3 (D) –2/5

Ans. **(B)**

Sol. $f(x)=\sqrt{4-\sqrt{2x+5}}$

$4-\sqrt{2x+5}\geq 0$ $\qquad 2x+5\geq 0$

$\sqrt{2x+5}\leq 4$ $\qquad x\geq -5/2$

$x\leq\dfrac{11}{2}$

$x\in\left[-\dfrac{5}{2},\dfrac{11}{2}\right]$

$\text{mid point}=\dfrac{-5/2+11/2}{2}=\dfrac{3}{2}$

10. Let n be a natural number and let 'a' be a real number. The number of zeros of $x^{2n+1}-(2n+1)x+a=0$ in the interval [–1, 1] is –

(A) 2 if a > 0 (B) 2 if a < 0

(C) At most one for every value of a (D) At least three for every value of a

Ans. **(C)**

Sol. $f(x)=x^{2n+1}-(2n+1)x+a$

$f'(x)=(2n+1)x^{2n}-(2n+1)$

$=(2n+1)(x^{2n}-1)\leq 0$ when $x\in[-1,1]$

$f(x)$ is strictly decreasing in $[-1,1]$

f(x) cut x axis at most one point in given interval

11. Let $f : R \to R$ be the function $f(x) = (x - a_1)(x-a_2)+(x-a_2)(x-a_3) + (x- a_3)(x-a_1)$ with $a_1, a_2, a_3 \in R$. Then $f(x) \geq 0$ if and only if –

(A) At least two of a_1, a_2, a_3 are equal (B) $a_1 = a_2 = a_3$

(C) a_1, a_2, a_3 are all distinct (D) a_1, a_2, a_3, are all positive and distinct

Ans. **(B)**

Sol. Only when $a_1 = a_2 = a_3$

In other cases f(x) will take both positive and negative values

12. The value $\dfrac{\int_0^{\pi/2} (\sin x)^{\sqrt{2}+1} dx}{\int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} dx}$ is –

(A) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (B) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (C) $\dfrac{\sqrt{2}+1}{\sqrt{2}}$ (D) $2-\sqrt{2}$

Ans. **(D)**

Sol. $I_1 = \int_0^{\pi/2} (\sin x)^{\sqrt{2}} . \sin x dx$ $\qquad I_2 = \int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} dx$

$$I_1 = \left((\sin x)^{\sqrt{2}} \int \sin x \, dx \right)_0^{\pi/2} - \int_0^{\pi/2} \left(\sqrt{2} (\sin x)^{\sqrt{2}-1} \cos x \int \sin x \, dx \right)$$

$$= -\left(\cos x (\sin x)^{\sqrt{2}} \right)_0^{\pi/2} + \sqrt{2} \int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} (1 - \sin^2 x) dx$$

$$\frac{I_1}{I_2} = \frac{\sqrt{2}}{1+\sqrt{2}} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = 2 - \sqrt{2}$$

13. The value of $\int_{-2012}^{2012} (\sin(x^3) + x^5 + 1) dx$ is –

(A) 2012 (B) 2013 (C) 0 (D) 4024

Ans. **(D)**

Sol. $\int_{-2012}^{2012} (\sin(x^3) + x^5 + 1) dx = \int_{-2012}^{2012} (\sin(x^3) dx + \int_{-2012}^{2012} x^5 dx + \int_{-2012}^{2012} dx = 4024$

14. Let [x] and {x} be the integer part and fractional part of a real number x respectively. The value of the integral $\int_0^5 [x]\{x\} dx$ is –

(A) 5/2 (B) 5 (C) 34.5 (D) 35.5

Ans. **(B)**

Sol. $\int_0^5 [x]\{x\}dx = \int_0^5 [x](x-[x])dx = \int_0^1 0.dx + \int_1^2 1.(x-1)dx + \int_2^3 2(x-2)dx + \int_3^4 3(x-3)dx + \int_4^5 4(x-4)dx$

$= \left(\frac{(x-1)^2}{2}\right)_1^2 + 2\left(\frac{(x-2)^2}{2}\right)_2^3 + 3\left(\frac{(x-3)^2}{2}\right)_3^4 + 4\left(\frac{(x-4)^2}{2}\right)_4^5$

$= \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2}$

$= 5$

15. Let $S_n = \sum_{k=1}^{n} k$ denote the sum of the first n positive integers. The numbers $S_1, S_2, S_3, \ldots S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is –

(A) 1/2 (B) 49/100 (C) 49/99 (D) 48/99

Ans. **(C)**

Sol. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105 till 98 terms

48 terms are even and 48 terms odd

99^{th} term $= \frac{99 \times 100}{2}$ = even

Total even terms = 48 + 1 = 49

Probability $= \frac{49}{99}$

16. A purse contains 4 copper coins and 3 silver coins. A second purse contains 6 copper coins and 4 silver coins. A purse is chosen randomly and a coin is taken out of it. What is the probability that it is a copper coin

(A) 41/70 (B) 31/70 (C) 27/70 (D) 1/3

Ans. **(A)**

Sol. P_1 : 4 copper coins 3 silver coins

P_2 : 6 copper coins 4 silver coins

E = Event of copper coin

$P(E) = P(P_1). P(E/P_1) + P(P_2). P(E/P_2)$

$= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{10}$

$= \frac{41}{70}$

17. Let H be the orthocenter of an acute - angled triangle ABC and O be its circumcenter. Then $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC}$

(A) is equal to $\overrightarrow{HO}$ (B) is equal to $3\overrightarrow{HO}$

(C) is equal to $2\overrightarrow{HO}$ (D) is not a scalar multiple of $\overrightarrow{HO}$ in general

Ans. **(C)**

Sol. G is centroid

$G = \frac{A+B+C}{3}$

$G = \frac{2O + H}{3}$

$2O + H = 3G$

$\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} = \vec{A} - \vec{H} + \vec{B} - \vec{H} + \vec{C} - \vec{H}$

$= \vec{A} + \vec{B} + \vec{C} - 3\vec{H}$

$= 3\vec{G} - 3\vec{H}$

$= 2\vec{O} + \vec{H} - 3\vec{H}$

$= 2\vec{O} - 2\vec{H}$

$= 2\overrightarrow{HO}$

18. The number of ordered pairs (m, n), where m, n $\in \{1, 2, 3, \ldots\ldots, 50\}$, such that $6^m + 9^n$ is a multiple of 5 is –

(A) 1250 (B) 2500 (C) 625 (D) 500

Ans. **(A)**

Sol. $6^m + 9^n$

$6^1 = 6$ $9^1 = 9$

$6^2 = 6$ $9^2 = 1$

$6^3 = 6$ $9^3 = 9$

$6^4 = 6$ $9^4 = 1$

m can be any value and n will be odd number then sum is multiple of 5

$50 \times 25 = 1250$

19. Suppose $a_1, a_2, a_3, \ldots\ldots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers ?

(A) 0 (B) 1509 (C) 3018 (D) 6036

Ans. **(D)**

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{2012} = 3018$…………………(1)

$\frac{a_1 + a_3}{2} = a_2$

$2a_2 + 2a_4 + 2a_6 + \ldots.. + 2a_{2012} = 6036$

$(a_1 + a_3) + (a_3 + a_5) + (a_5 + a_7) + \ldots.. + (a_{2011} + a_1) = 6036$

$2(a_1 + a_3 + a_5 + \ldots\ldots\ldots + a_{2011}) = 6036$

$a_1 + a_3 + a_5 + \ldots\ldots\ldots + a_{2011} = 3018$……………………………………..(2)

Add (1) and (2)

Sum of all number = 3018 + 3018 = 6036

20. Let S = {1, 2, 3, …..,n} and A = {(a, b) |1≤ a, b ≤ n} = S x S. A subset B of A is said to be a good subset if (x, x) $\in$ B for every x $\in$ S. Then the number of good subsets of A is –

(A) 1 (B) 2^n (C) $2^{n(n-1)}$ (D) 2^{n^2}

Ans. **(B)**

Sol. As total number of elements in set A will be n^2.

Now set B will be

Set B = {(1, 1) , (2, 2) .... (n, n)}

Number of elements in B is 'n'.

So number of good subsets = 2^n

So answer is (B) although KVPY has Mentioned option (C) in its answer key.

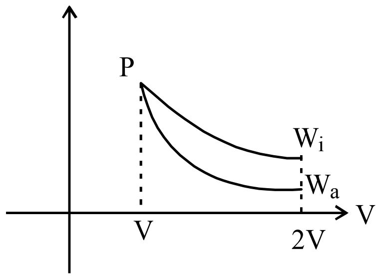
PHYSICS

21. An ideal monatomic gas expands to twice its volume. If the process is isothermal, the magnitude of work done by the gas is W_1. If the process is adiabatic, the magnitude of work done by the gas is W_a . Which of the following is true ?

(A) $W_i = W_a > 0$ (B) $W_i > W_a = 0$ (C) $W_i > W_a > 0$ (D) $W_i > W_a > 0$

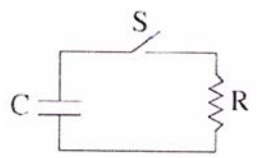
Ans. **(B)**

Sol.



$W_i > W_a > 0$

22. The capacitor of capacitance C in the circuit shown in fully charged initially. Resistance is R.–



After the switch S is closed, the time taken to reduce the stored energy in the capacitor to half its initial value is

(A) $RC/2$ (B) $2RC \ln 2$ (C) $RC \ln 2$ (D) $\frac{RC \ln 2}{2}$

Ans. **(D)**

Sol. Discharging -

$Q = Q_0 e^{-t/RC}$,

$U = \frac{Q^2}{2C} = \frac{Q_0^2}{2C} e^{-2t/RC}$ or $U = U_0 e^{-2t/RC}$

$\frac{U_0}{2} = U_0 e^{-2t/RC} \Rightarrow t = \frac{RC\ell n2}{2}$

23. A liquid drop placed on a horizontal plane has a near spherical shape (slightly flattened due to gravity). Let R be the radius of its largest horizontal section. A small disturbance causes the drop to vibrate with frequency v about its equilibrium shape. By dimensional analysis the ratio $\frac{v}{\sqrt{\sigma/\rho R^3}}$ can be (Here σ is surface tension, ρ is density, g is acceleration due to gravity, and k is arbitrary dimensionless constant)–

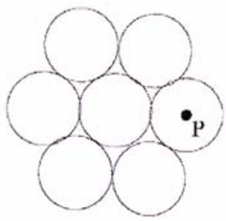
(A) $k\rho g R^2/\sigma$ (B) $k\rho R^2/g\sigma$ (C) $k\rho R^3/g\sigma$ (D) $k\rho/g\sigma$

Ans. **(A)**

Sol. $\frac{v}{\sqrt{\sigma/\rho R^3}}$ is dimensionless

$k\rho g R^2/\sigma$ is also dimensionless

24. Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches its neighbours. Each coin is a thin disc of mass m and radius r. Note that the moment of inertia of an individual coin about an axis passing through centre and perpendicular to the plane of the coin is $mr^2/2$



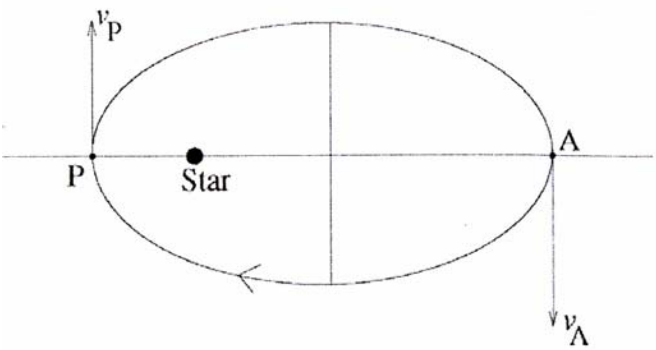
The moment of inertia of the system of seven coins about an axis that passes through the point P (the centre of the coin positioned directly to the right of the central coin) and perpendicular to the plane of the coins is–

(A) $\frac{55}{2}mr^2$ (B) $\frac{127}{2}mr^2$ (C) $\frac{111}{2}mr^2$ (D) $55\,mr^2$

Ans. **(C)**

Sol. By using parallel axis theorem, $I = \frac{111}{2}mr^2$

25. A planet orbits in an elliptical path of eccentricity e around a massive star considered fixed at one of the foci. The point in space where it is closest to the star is denoted by P and the point where it is farthest is denoted by A. Let v_p and v_a be the respective speeds at P and A. Then–



(A) $\frac{v_P}{v_A} = \frac{1+e}{1-e}$ (B) $\frac{v_P}{v_A} = \frac{1+e^2}{1-e}$ (C) $\frac{v_P}{v_A} = 1$ (D) $\frac{v_P}{v_A} = \frac{1+e^2}{1-e^2}$

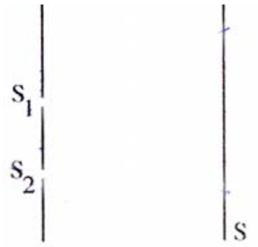
Ans. **(A)**

Sol. Using conservation of angular momentum

$V_p.r_p = V_A.r_A$

$\frac{V_P}{V_A} = \frac{r_A}{r_P} = \frac{a+ae}{a-ae}$

26. In a Young's double slit experiment the intensity of light at each slit is I_0. Interference pattern in observed along a direction parallel to the line S_1 S_2, on screen S.–



The minimum, maximum, and the intensity averaged over the entire screen are respectively

(A) 0, $4I_0$, $2I_0$ (B) 0, $4I_0$, I_0 (C) I_0, $2I_0$, $3I_0/2$ (D) 0, 2_0, I_0

Ans. **(A)**

Sol. $I_{min} = 0$

$I_{max} = 4I_0$

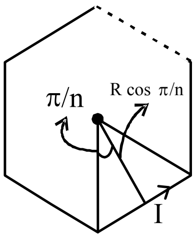
$I_{av} = 2I_0$

27. A loop carrying current I has the shape of a regular polygon of n sides. If R is the distance from the centre to any vertex, then the magnitude of the magnetic induction vector $\vec{B}$ at the centre of the loop is –

(A) $n\frac{\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$ (B) $\frac{\mu_0 I}{2R}$ (C) $n\frac{\mu_0 I}{2\pi R}\tan\frac{2\pi}{n}$ (D) $\frac{\mu_0 I}{\pi R}\tan\frac{\pi}{n}$

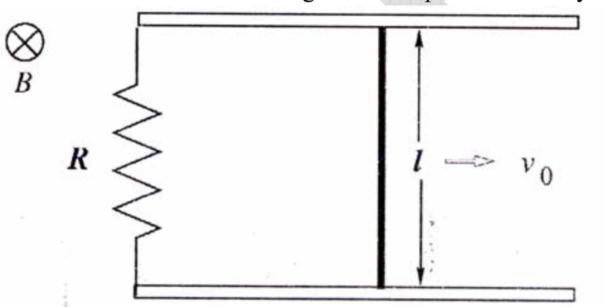
Ans. **(A)**

Sol.



$$B_{net} = n \times B_1 = n.\frac{\mu_0}{4\pi}.\frac{I}{R\cos\frac{\pi}{n}}2\sin\frac{\pi}{n}$$

28. A conducting rod of mass m and length *l* is free to move without friction on two parallel long conducting rails, as shown below. There is a resistance R across the rails. In the entire space around, there is a uniform magnetic field B normal to the plane of the rod and rails. The rod is given an impulsive velocity v_0 –



Finally, the initial energy $\frac{1}{2}mv_0^2$

(A) Will be converted fully into heat energy in the resistor

(B) Will enable rod to continue to move with velocity v_0 since the rails are frictionless.

(C) Will be converted fully into magnetic energy due to induced current

(D) Will be converted into the work done against the magnetic field

Ans. **(A)**

Sol. Due to energy conservation

29. A steady current I flows through a wire of radius r, length L and resistivity ρ. The current produces heat in the wire. The rate of heat loss in a wire is proportional to its surface area. The steady temperature of the wire is independent of–

(A) L (B) r (C) I (D) ρ

Ans. **(A)**

Sol. Concept of fuse wire

30. The ratio of the speed of sound to the average speed of an air molecule at 300 K and 1 atmospheric pressure is close to–

(A) 1 (B) $\sqrt{1/300}$ (C) $\sqrt{300}$ (D) 300

Ans. **(A)**

Sol. $\frac{V_{sound}}{V_{av}} = \sqrt{\frac{\gamma kT}{m} \times \frac{\pi m}{8kT}}$

31. In one model of the electron, the electron of mass m_e is thought to be a uniformly charged shell of radius R and total charge e, whose electrostatic energy E is equivalent to its mass m_e via Einstein's mass energy relation $E = m_e c^2$. In this model, R is approximately ($m_e = 9.1 \times 10^{-31}$ kg, c = 3×10^8 ms^{-1}, $1/4\,\pi\varepsilon_0 = 9 \times 10^9$ Farad m^{-1}, magnitude of the electron charge = 1.6×10^{-19} C) –

(A) 1.4×10^{-15} m (B) 5.3×10^{-11}m (C) 2×10^{-13}m (D) 2.8×10^{-35}m

Ans. **(A)**

Sol. $\frac{e^2}{8\pi\varepsilon_0 R} = m_e c^2$

solving for R

32. A body is executing simple harmonic motion of amplitude a and period T about the equilibrium position x = 0. large numbers of snapshots are taken at random of this body in motion. The probability of the body being found in a very small interval x to x + |dx| is highest at –

(A) $x = \pm a$ (B) $x = \pm a/2$ (C) x = 0 (D) $x = \pm / \sqrt{2}$

Ans. **(A)**

33. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is held at a temperature of 100ºC while the other one is kept 0º C. If the two are brought into contact, then, assuming no heat loss to the environment, the final temperature that they will reach is –

(A) 50º C (B) Less than 50º C (C) More than 50º C (D) 0º C

Ans. **(B)**

Sol. We have, $\Delta\theta = mS\Delta T = CT$ Where C $\rightarrow$ heat capacity

Let final temperature becomes T. then

Heat lost = Heat gain

$C (100 - T) = C' (T - 0)$

or $T = \frac{100}{\left(1+\frac{C'}{C}\right)} < 50$ $(\because C' > C)$

34. A particle is acted upon by a force given by $F = -\alpha x^3 - \beta x^4$ where α and β are positive constants. At the point x = 0, the particle is –

(A) In stable equilibrium (B) In unstable equilibrium (C) In neutral equilibrium (D) Not in equilibrium

Ans. **(B)**

Sol. $F = -\frac{\partial U}{\partial x} = -\alpha x^3 - \beta x^4$

$\left(\frac{\partial U}{\partial x}\right)_{x=0} = 0$ & $\left(\frac{\partial^2 U}{\partial x^2}\right)_{x=0} = 0$

35. The potential energy of a point particle is given by the expression V (x) = –αx + β sin (x/γ). A dimensionless combination of the constant α, β and γ is–

(A) α/βγ (B) $\alpha^2/\beta\gamma$ (C) γ/αβ (D) αγ/β

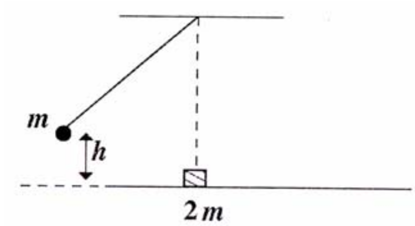
Ans. **(D)**

Sol. $[\alpha] = MLT^{-2}$

$[\beta] = ML^2T^{-2}$

$[\gamma] = L$

36. A ball of mass m suspended from a rigid support by an inextensible massless string is released from a height h above its lowest point. At its lowest point it collides elastically with a block of mass 2m at rest on a frictionless surface. Neglect the dimensions of the ball and the block. After the collision the ball rises to a maximum height of–



(A) h/3 (B) h/2 (C) h/8 (D) h/9

Ans. **(D)**

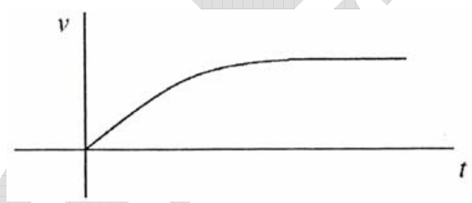
Sol. Let velocity of each blocks are v_1 and v_2 respectively then

$mv_1 + 2mv_2 = mv$ where $v = \sqrt{2gh}$ & $v_2 - v_1 = v$

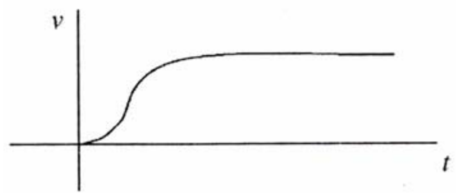
hence from above equations, we have $v_1 = -\frac{v}{3}$

$h \propto v^2$ or $h' = \frac{h}{9}$

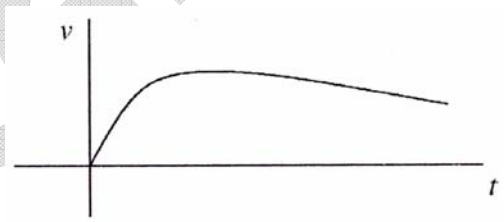
37. A particle released from rest is falling through a thick fluid under gravity. The fluid exerts a resistive force on the particle proportional to the square of its speed. Which one of the following graphs best depicts the variation of its speed v with time t –



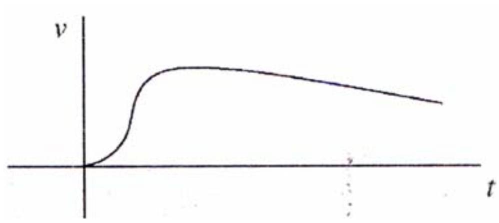
(A)



(B)



(C)



(D)

Ans. **(A)**

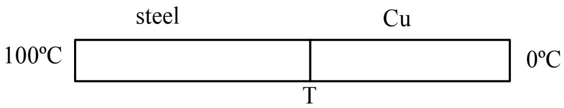
Sol. $mg' - kv^2 = \frac{mdv}{dt}$

38. A cylindrical steel rod of length 0.10 m and thermal conductivity 50 $W.m^{-1} K^{-1}$ is welded end to end to copper rod of thermal conductivity 400 $W.m^{-1}. K^{-1}$ and of the same area of cross section but 0.20 m long. The free end of the steel rod is maintained at 100º C and that of the copper and at 0º C. Assuming that the rods are perfectly insulated from the surrounding, the temperature at the junction of the two rods–

(A) 20º C (B) 30º C (C) 40º C (D) 50º C

Ans. **(A)**

Sol.



$$\frac{50 \times A \times (100 - T)}{0.1} = \frac{400 \times A \times (T - 0)}{0.2}$$

39. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. The half lives of X and Y are 40000 years and 20 years respectively. In a certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20}, the number of Y nuclei present in its is–

(A) 2×10^{17} (B) 2×10^{20} (C) 4×10^{23} (D) 4×10^{20}

Ans. **(A)**

Sol. In radioactive equilibrium

rate of decay of X = rate of decay of Y

$$\lambda_x N_x = \lambda_y N_y, \quad \frac{N_x}{T_x} = \frac{N_y}{T_y}$$

40. An unpolarized beam of light of intensity I_0 passes through two linear polarizers making an angle of 30º with respect to each other. The emergent beam will have an intensity –

(A) $\frac{3I_0}{4}$ (B) $\frac{\sqrt{3}I_0}{4}$ (C) $\frac{3I_0}{8}$ (D) $\frac{I_0}{8}$

Ans. **(C)**

Sol. $$I_0 \xrightarrow[\text{Polarizer}]{\text{First}} I_0/2 \xrightarrow[\text{II polarizer}]{\text{Malus law}} \left(\frac{I_0}{2}\right)\cos^2 30^\circ$$

CHEMISTRY

41. Among the following, the species with the highest bond order is –

(A) O_2 (B) F_2 (C) O_2^+ (D) F_2

Ans. **(C)**

Sol. (A) O_2, B.O = 2

(B) F_2, B.O = 1

(C) O_2^+ B.O = 2.5

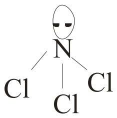
(D) F_2^- B.O = 0.5

42. The moecule with **non-zero** dipole moment is –

(A) BCl_3 (B) $BeCl_2$ (C) CCl_4 (D) NCl_3

Ans. **(D)**

Sol.



$\mu_R \neq 0$

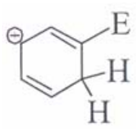
43. For a one-electron atom, the set of allowed quantum numbers is –

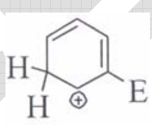
(A) $n = 1, l = 0, m_l = 0, m_s = +½$ (B) $n = 1, l = 1, m_l = 0, m_s = +½$

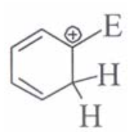
(C) $n = 1, l = 0, m_l = -1, m_s = -½$ (D) $n = 1, l = 1, m_l = 1, m_s = -½$

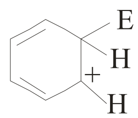
Ans. **(A)**

44. In the reaction benzene with an electrophile E^+, the structure of the intermediate σ– complex can be represented as

(A) 

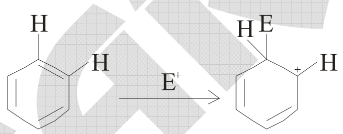
(B) 

(C) 

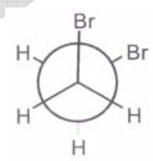
(D) 

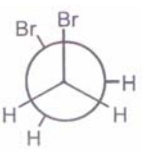
Ans. **(D)**

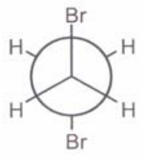
Sol.

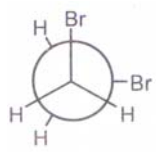


45. The most stable conformation of 2, 3-dibromobutane is –

(A) 

(B) 

(C) 

(D) 

Ans. **(C)**

46. Typical electronic energy gaps in molecules are about 1.0 eV. In terms of temperature, the gap is closed to –

(A) 10^2 K (B) 10^4 K (C) 10^3 K (D) 10^5 K

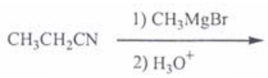
Ans. **(B)**

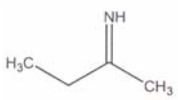
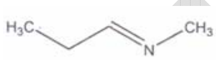
Sol. $\frac{3}{2}KT = 1.6 \times 10^{-19}$

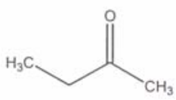
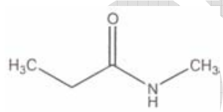
$\frac{3}{2} \times 1.38 \times 10^{-23}\ T = 1.6 \times 10^{-19}$

$T = 10^4$ K

47. The major final product in the following reaction is –



(A)  (B) 

(C)  (D) 

Ans. **(C)**

Sol.

$$H_3C-CH_2-\underset{\displaystyle}{\overset{\overset{\displaystyle N-MgBr}{\|}}{C}}-CH_3 \xrightarrow{H_3O^+} H_3C-CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$$

48. A zero-order reaction, A → Product, with an initial concentration $[A]_0$ has a half-life of 0.2 s. If one starts with the concentration $2[A]_0$, then the half-life is –

(A) 0.1 s (B) 0.4 s (C) 0.2 s (D) 0.8 s

Ans. **(B)**

Sol. $t_{1/2} \propto \frac{1}{a^{n-1}}$

for zero order reaction n = 0

so $t_{1/2} \propto a$

so

$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{a_1}{a_2}$

$\frac{.2}{(t_{1/2})_2} = \frac{[A_0]}{2[A]_0}$

$t_{1/2} = .4\,\text{sec}$

49. The isoelectronic pair of ions is –

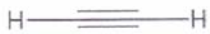
(A) Sc^{2+} and V^{3+} (B) Mn^{3+} and Fe^{2+} (C) Mn^{2+} and Fe^{3+} (D) Ni^{3+} and Fe^{2+}

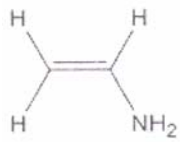
Ans. **(C)**

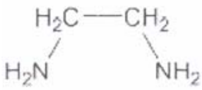
Sol. $Mn^{+2} = 23\ e^-$

$Fe^{+3} = 23\ e^-$

50. The major product in the following reaction is –

(A) 

(B) 

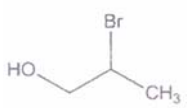
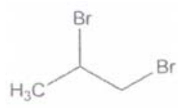
(C) 

(D) 

Ans. **(A)**

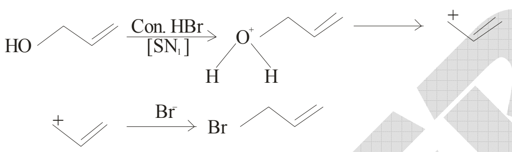
Sol. $CH_2 = CHBr \xrightarrow[-HBr]{NaNH_2} CH \equiv CH$

51. The major product of the following reaction is –

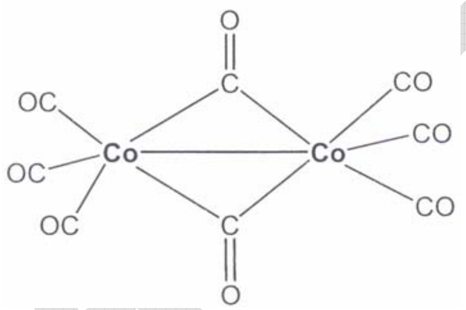
(A) 
(B) 
(C) 
(D) 

Ans. **(B)**

Sol.



52. The oxidation state of cobalt in the following molecule is –



(A) 3 (B) 1 (C) 2 (D) 0

Ans. **(D)**

53. The pK_a of a weak acid is 5.85. The concentrations of the acid and its conjugate base are equal at a pH of –

(A) 6.85 (B) 5.85 (C) 4.85 (D) 7.85

Ans. **(B)**

Sol.

$$pH = pKa + \log\frac{[\text{Conjugate base}]}{[\text{Acid}]}$$

$\because [\text{Conjugate base}] = [\text{Acid}]$

$pH = pKa = 5.85$

54. For a tetrahedral complex $[MCl_4]^{2-}$, the spin-only magnetic moment is 3.83 B.M. The element **M** is –

(A) Co (B) Cu (C) Mn (D) Fe

Ans. **(A)**

Sol. $[MCl_4]^{2-}$ Tetrahedral = sp^3 hybridisation

M^{+2}

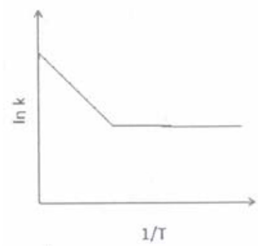
$\because \mu = \sqrt{n(n+2)}\text{B.M.} = 3.83$

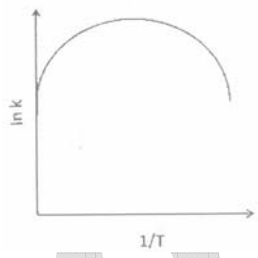
$n = 3$

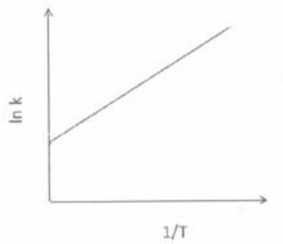
Means configuration of $M^{+2} = 3d^7$

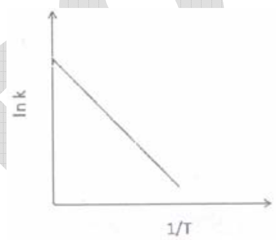
so, $M = 3d^7 4s^2 = {}_{27}Co$

55. Among the following graphs showing variation of rate (k) with temperature (T) for a reaction, the one that exhibits arrhenius behavior over the entire temperature range is –

(A) 

(B) 

(C) 

(D) 

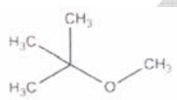
Ans. **(D)**

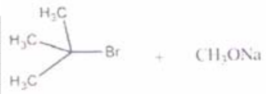
Sol.

$$K = Ae^{-\frac{E_a}{RT}}$$

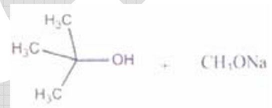
$$\ln K = \ln A - \frac{E_a}{RT}$$

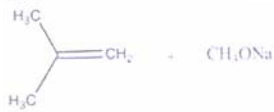
56. The reaction that gives the following molecule as the major product is –



(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol.

$$H_3C-\overset{\overset{\large H_3C}{|}}{\underset{\underset{\large CH_3}{|}}{C}}-ONa \xrightarrow{CH_3Br} H_3C-\overset{\overset{\large CH_3}{|}}{\underset{\underset{\large CH_3}{|}}{C}}-OCH_3$$

57. The C–O bond length in CO, CO_2 and CO_3^{2-} follows the order –

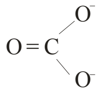
(A) $CO < CO_2 < CO_3^{2-}$ (B) $CO_2 < CO_3^{2-} < CO$ (C) $CO > CO_2 > CO_3^{2-}$ (D) $CO_3^{2-} < CO_2 < CO$

Ans. **(A)**

Sol. $\text{Bond Length} \propto \dfrac{1}{\text{Bond Order}}$

CO, B.O. = 3

O = C = O, B.O. = 2



B.O. = 1.33

58. The equilibrium constant for the following reactions are K_1 and K_2, respectively.

$2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g)$

$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$

Then the equilibrium constant for the reaction

$2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g)$ is –

(A) K_1K_2 (B) $K_1K_2^2$ (C) $K_1^2K_2^2$ (D) $K_1^2K_2$

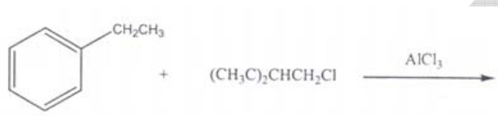
Ans. **(B)**

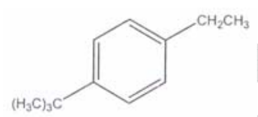
Sol. $2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g) \quad K_1$

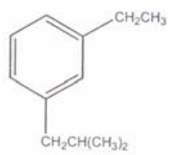
$2PCl_3(g) + 2Cl_2(g) \rightleftharpoons 2PCl_5(g) \quad K_2^2$

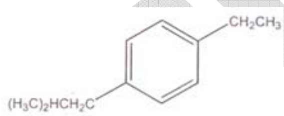
Net reaction : $2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g) \quad K = K_1K_2^2$

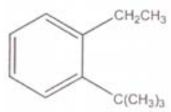
59. The major product of the following reaction is –



(A)


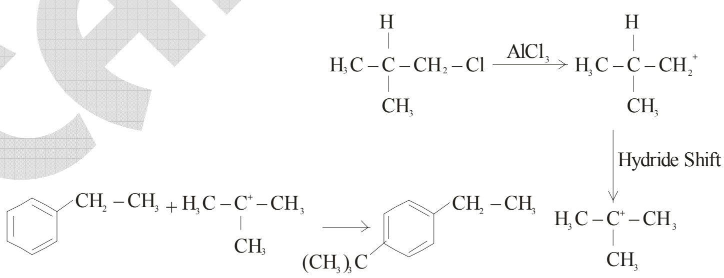
(B)


(C)


(D)


Ans. **(A)**

Sol.



60. Doping silicon with boron produces a –

(A) n-type semiconductor (B) Metallic conductor (C) p-type semiconductor (D) Insulator

Ans. **(C)**

BIOLOGY

61. The disorders that arise when the immune system destroys self cells are called autoimmune disorders. Which of the following would be classified under this?

(A) rheumatoid arthritis (B) asthma (C) rhinitis (D) eczema

Ans. **(A)**

Sol. Rheumatoid arthritis is a chronic, systemic inflammatory disorder that may affect many tissues and organs, but principally attacks flexible joints. RA is considered a systemic autoimmune disease arises from an inappropriate immune response of the body against substances and tissues normally present in the body.

62. When of the following class of immunoglobulins can trigger the complement cascade ?

(A) IgA (B) IgM (C) IgD (D) IgE

Ans. **(B)**

Sol. The complement system helps the ability of antibodies and phagocytic cells to clear pathogens from an organism. it si a part of innate immune system. it is operational via classical pathways. it is triggered by antigen bound antibody molecule preferably IgG or IgM. Although IgM is move effective

63. Diabetes insipidus is due to –

(A) hypersecretion of vasopressin (B) hyposecretion of insulin

(C) hypersecretion of insulin (D) hyposecretion of vasopressin

Ans. **(D)**

Sol. Diabetes incipidus (DI) is a condition characterized by excessive thirst and excretion of large amounts of severely diluted urine. DI is caused by a deficiency of vasopressin also known as antidiuretic hormone.

64. Fossils are most often found in which kind of rocks ?

(A) meteorites (B) sedimentary rocks (C) igneous rocks (D) metamorphic rocks

Ans. **(B)**

Sol. Sedimentary rocks are type of rock that are formed by deposition of material at the earth's surface and within bodies of water. It forms only 8% of total volume of crust fossils are mostly found in these.

65. Peptic ulcers are caused by –

(A) a fungus, Candida albicans (B) a virus, cytomegalovirus

(C) a parasite, Trypanosoma brucei (D) a bacterium, Helicobacter pylori

Ans. **(D)**

Sol. Peptic ulcer is mucosal erosion equal to or greater than 0.5 cm of GIT. 70 - 90% of peptic ulcer are associated with helicobacter pylori spiral shaped bacterium that lives in the acidic environment of stomach.

66. Transfer RNA (tRNA) –

(A) is present in the ribosomes and provides structural integrity

(B) usually has clover leaf-like structure

(C) carries genetic information form DNA to ribosomes

(D) codes for proteins

Ans. **(B)**

Sol. The clover leaf model is the 2-D model. Given by Holley. Its 3-D model is a L-shaped model.

67. Some animals excrete uric acid in urine (uricotelic) as it requires very little water. This is an adaptation to conserve water loss. Which animals among the following are most likely to be uricotelic ?

(A) fishes (B) amphibians (C) birds (D) mammals

Ans. **(C)**

Sol. Uricotelic organism excrete uric acid or its salts as a result of deamination. Uricotelic organisms included terrestrial arthropods, Lizards, Snakes and Birds.

68. A ripe mango, kept with unripe mangoes causes their ripening. This is due to the release of a gaseous plant hormone–

(A) auxin (B) gibberlin (C) cytokinine (D) ethylene

Ans. **(D)**

Sol. The only gaseous hormone out of these four options is ethylene. Its main function is ripening.

69. Human chromosomes undergo structural changes during the cell cycle. Chromosomal structure can be best visualized if a chromosome is isolated from a cell at –

(A) G1 phase (B) S phase (C) G2 phase (D) M phase

Ans. **(D)**

Sol. Chromosome structure is best seen at metaphase which is a sub stage of M-phase of the cell cycle.

70. By which of the following mechanisms is glucose reabsorbed from the glomerular filtrate by the kidney tubule ?

(A) osmosis (B) diffusion (C) active transport (D) passive transport

Ans. **(C)**

Sol. Glucose from glomerular filtrate is reabsorbed from proximal convulated tubule via active transport.

71. In mammals, the hormones secreted by the pituitary, the master gland, is itself regulated by –

(A) hypothalamus (B) median cortex (C) pineal gland (D) cerebrum

Ans. **(A)**

72. Which of the following is true for TCA cycle in eukaryotes ?

(A) takes place in mitochondrion (B) produces no ATP

(C) takes place in golgi complex (D) independent of electron transport chain

Ans. **(A)**

Sol. TCA (Tricarboxylic acid cycle), also known as Krebs' cycle, takes place in matrix of mitochondrion.

73. A hormone molecule binds to a specific protein on the plasma membrane inducing a signal. The protein it binds to is called –

(A) ligand (B) antibody (C) receptor (D) histone

Ans. **(C)**

Sol. Hormone receptors for protein hormone are present on the surface of plasma membrane of cell.

74. DNA mutations that do not cause any functional change in the protein product are known as –

(A) nonsense mutations (B) missense mutations (C) deletion mutations (D) silent mutations

Ans. **(D)**

Sol. Silent mutations also called same - sense mutations are not lethal because in these mutations, the amino acid do not get changed.

75. Plant roots are usually devoid of chlorophyll and cannot perform photosynthesis. However, there are exceptions. Which of the following plant root can perform photosynthesis ?

(A) Arabidopsis (B) Tinospora (C) Rice (D) Hibiscus

Ans. **(B)**

Sol. Tinospora and Trapa are plants which have assimilatory or photosynthetic roots.

76. Vitamin A deficiency leads to night-blindness. Which of the following is the reason for the disease ?

(A) rod cells are not converted to cone cells

(B) rhodopsin pigment of rod cells is defective

(C) melanin pigment is not synthesized in cone cells

(D) cornea of eye gets dried

Ans. **(B)**

Sol. Aldehyde form of vitamin A (Retinal) is required for synthesis of rhodopsin pigments of rod cells. deficiency of vitamin A will lead to defective formation of rhodopsin.

77. In Dengue virus infection, patients often develop haemorrhagic fever due to internal bleeding. This happens due to the reduction of –

(A) platelets (B) RBCs (C) WBCs (D) lymphocytes

Ans. **(A)**

Sol. Dengue fever also known as break bone fever is an infections tropical disease caused by dengue virus. This results in bleeding, low levels of blood platelets and blood plams leakage.

78. If the sequence of bases in sense strand of DNA is 5'-GTTCATCG-3', then the sequence of bases in its RNA transcript would be –

(A) 5'-GTTCATCG-3' (B) 5'-GUUCAUCG-3' (C) 5'-CAAGTAGC-3' (D) 5'-CAAGUAGC-3'

Ans. **(B)**

Sol. The direction of RNA sequence is also from 5'-3'. The sequence of sense strand is -

5'-GTTCATCG-3'

We know the sequence of m-RNA is similar to sense strand. Only uracil is present instead of Thymine.

Hence the m-RNA sequence will be -

5'-GUUCAUCG-3'

79. A reflex action is a quick involuntary response to stimulus. Which of the following is an example of BOTH, unconditioned and conditioned reflex ?

(A) knee Jerk reflex (B) secretion of saliva in response to the aroma of food

(C) sneezing reflex (D) contraction of the pupil in response to bright light

Ans. **(B)**

80. In a food chain such as grass $\rightarrow$ deer $\rightarrow$ lion, the energy cost of respiration as a proportion of total assimilated energy at each level would be –

(A) 60% - 30 % - 20% (B) 20% - 30 % - 60% (C) 20% - 60 % - 30% (D) 30% - 30 % - 30%

Ans. **(B)**

Sol. Actually around one half of the energy is lost through respiration.

Hence best option is 20% - 30 % - 60%

Part – 2

Two - Mark Question

MATHEMATICS

81. Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has–

(A) Two distinct positive real roots (B) Two equal roots

(C) One positive and one negative root (D) No real roots

Ans. **(D)**

Sol. $x^2 + 2ax + b^2 = 0$ $\qquad$ $x^2 + 2bx + c^2 = 0$

$D_1 > 0$ $\qquad$ $D_2 > 0$

$4a^2 + b^2 > 0$ $\qquad$ $4b^2 - 4c^2 > 0$

$a^2 > b^2$ ....(1) $\qquad$ $b^2 > c^2$...........(2)

From (1) and (2)

$a^2 > b^2 > c^2 \Rightarrow a^2 > c^2 \Rightarrow c^2 - a^2 < 0$

$x^2 + 2cx + a^2 = 0$

$D = 4c^2 - 4a^2 < 0$ No real roots

82. The coefficient of x^{2012} in $\dfrac{1+x}{(1+x^2)(1-x)}$ is –

(A) 2010 (B) 2011 (C) 2012 (D) 2013

Ans. **(Bonus)**

Sol. Coeff. Of x^{2012}

$$\frac{(1+x)^2}{(1+x^2)(1-x^2)}$$

$= (1+x)^2(1-x^4)^{-1}$

$= (1+2x+x^2)(1-x^4)^{-1}$

Coeff. Of x^{2012} + 2Coeff of x^{2011} + Coeff of x^{2010} in the expansion of $(1-x^4)^{-1}$

x^{2011} and x^{2010} not possible in $(1 - x^4)^{-1}$

= only coeff. Of x^{2012} in the expassion of $(1 - x^4)^{-1}$

${}^{1+503-1}C_{503} = 1$

83. Let (x, y) be a variable point on the curve $4x^2 + 9y^2 - 8x - 36y + 15 = 0$. Then

$\min(x^2 - 2x + y^2 - 4y + 5) + \max(x^2 - 2x + y^2 - 4y + 5)$ is–

(A) $\dfrac{325}{36}$ (B) $\dfrac{36}{325}$ (C) $\dfrac{13}{25}$ (D) $\dfrac{25}{13}$

Ans. **(A)**

Sol. $4x^2 + 9y^2 - 8x - 36y + 15 = 0$

$4(x^2 - 2x) + 9(y^2 - 4y) = -15$

$4(x^2 - 2x + 1) + 9(y^2 - 4y + 4 = -15 + 4 + 36$

$4(x-1)^2 + 9(y-2)^2 = 25$

$\frac{(x-1)^2}{\left(\frac{5}{2}\right)^2} + \frac{(y-2)^2}{\left(\frac{5}{3}\right)^2} = 1$..........................(1)

$x^2 - 2x + y^2 - 4y + 5$

$(x-1)^2 + (y-2)^2$

$\text{min of } ((x-1)^2 + (y-2)^2) = \frac{25}{9}$

$\text{max of } ((x-1)^2 + (y-2)^2) = \frac{25}{4}$

$= \frac{25}{9} + \frac{25}{4} = \frac{325}{36}$

84. The sum of all $x \in [0, \pi]$ which satisfy the equation $\sin x + \frac{1}{2}\cos x = \sin^2(x + \frac{\pi}{4})$ is –

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) π (D) 2π

Ans. **(C)**

Sol.

$\sin x + \frac{1}{2}\cos x = \sin^2(x + \frac{\pi}{4})$

$\sin x + \frac{1}{2}\cos x = \frac{1}{2}(1 - \cos\left(\frac{\pi}{2} + 2x\right)$

$\sin x + \frac{1}{2}\cos x = \frac{1}{2}(1 + \sin 2x)$

$2\sin x + \cos x = 1 + \sin x \cos x$

$2\sin x \cos x - 2\sin x + 1 - \cos x = 0$

$(1 - \cos x) - 2\sin x(1 - \cos x) = 0$

$(1 - \cos x)(1 - 2\sin x) = 0$

$1 - \cos x = 0$ $\quad$ $1 - 2\sin x = 0$

$\cos x = 1$ $\quad$ $\sin x = \frac{1}{2}$

$x = 0,$ $\quad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\text{sum} = 0 + \frac{\pi}{6} + \frac{5\pi}{6} = \pi$

85. A polynomial P(x) with real coefficients has the property that $P''(x) \neq 0$ for all x. Suppose $P(0) = 1$ and $P'(0) = -1$. What can you say about P(1) ?

(A) $P(1) \geq 0$ (B) $P(1) \neq 0$ (C) $P(1) \leq 0$ (D) $-1/2 < P(1) < 1/2$

Ans. **(B)**

Sol. As $P''(x) \neq 0$ for all x. So P(x) can be a polynomial of degree greater than or equal to 2.

but situational benefit must be given to P(x) as quadratic expression.

So let $P(x) = ax^2 + bx + c$

$P(0) = 1 \Rightarrow c = 1$

$P'(0) = -1 \Rightarrow b = -1$

$P''(0) \neq 2a \neq 0$

So $P(x) = ax^2 - x + 1$

$\therefore$ $P(1) = a \neq 0$

So $P(1) \neq 0$

but if we consider P(x) as polynomial of degree than 2. then all options (A), (B), (C), (D) can be correct answers, as given information will not be sufficient to find the result. So option (B) is the most appropriate answer. Although KVPY has mentioned only option (C) as answer in its answer key.

86. Define a sequence (a_n) by $a_1 = 5, a_n = a_1 a_2 ....a_{n-1} + 4$ for n > 1. Then $\lim_{n\to\infty} \frac{\sqrt{a_n}}{a_{n-1}}$

(A) Equals 1/2 (B) equals 1 (C) equals 2/5 (D)does not exist

Ans. **(C)**

Sol. $a_1 = 5$

$a_n = a_1a_2 ..... a_{n-1} + 4$

$a_2 = a_1 + 4 = 9$

$a_3 = a_1.a_2 + 4 = 5 \times 9 + 4 = 49$

$a_4 = a_1a_2a_3 + 4 = 2209$

$a_5 = a_1a_2a_3a_4 + 4 = 4870849 = (2207)^2$

$a_5 = (a_4 - 2)^2$

$a_4 = (49 - 2)^2 = (a_3 - 2)^2$

$a_3 = (9-2)^2 = (a_2 - 2)^2$

$an = (a_{n-1} - 2)^2$

$\sqrt{a_n} = a_{n-1} - 2$

$\frac{\sqrt{a_n}}{a_{n-1}} = \frac{a_{n-1} - 2}{a_{n-1}} = 1 - \lim_{n\to\infty} \frac{2}{a_{n-1}}$

$\because \lim_{n\to\infty} a_{n-1} = \infty$

$= 1$

87. The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx$, where a > 0, is –

(A) π (B) $a\pi$ (C) $\pi/2$ (D) 2π

Ans. **(C)**

Sol.

$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$.....................(1) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}} dx$

$I = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx$...........................(2)

add equation (1) and (2)

$2I = \int_{-\pi}^{\pi} \cos^2 x \left(\frac{1}{1+a^x} + \frac{a^x}{1+a^x}\right) dx$

$I = \int_0^{\pi} \cos^2 x \ dx = 2\int_0^{\pi/2} \cos^2 x \ dx = \pi/2$

88. Consider

$L = \sqrt[3]{2012} + \sqrt[3]{2013} + ... + \sqrt[3]{3011}$

$R = \sqrt[3]{2013} + \sqrt[3]{2014} + ... + \sqrt[3]{3012}$

and $I = \int_{2012}^{3012} \sqrt[3]{x} dx$ Then –

(A) $L + R < 2I$ (B) $L + R > 2I$ (C) $L + R = 2I$ (D) $\sqrt{LR} = I$

Ans. **(A)**

Sol. $L = \sqrt[3]{2012} + \sqrt[3]{2013} + ... + \sqrt[3]{3011}$ ………………(1)

$R = \sqrt[3]{2013} + \sqrt[3]{2014} + ... + \sqrt[3]{3012}$ ………………(2)

$I = \int_{2012}^{3012} x^{1/3} dx$ Let $f(x) = x^{1/3}$

$n = \frac{b-a}{h} = \frac{3012-2012}{1} = 1000$

$I = \frac{(b-a)}{n}[f(a) + f(a+h) + f(a+2h) + ... + f(a+(n-1)h)]$

$= [f(2012) + f(2013) + ... + f(3011)]$

$I = (2012)^{1/3} + (2013)^{1/3} + ... + (3011)^{1/3}$

$2I = 2(2012)^{1/3} + 2(2013)^{1/3} + ... + 2(3011)^{1/3}$

$= (2012)^{1/3} + (2012)^{1/3} + 2(2013)^{1/3} + ... + (2)(3011)^{1/3} + (3012)^{1/3} + (3012)^{1/3}$

$= (2012)^{1/3} + L + R + (3012)^{1/3}$

$\Rightarrow L + R < 2I$

89. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let P(K) be the probability of scoring at least K points. The largest value of K such that P(K) > ½ is –

(A) 14 (B) 15 (C) 16 (D) 17

Ans. **(B)**

Sol. Ways to make the sum K is coefficient of x^K in $(x + x^2)^{10}$

Coefficient of x^K in $x^{10}(1+x)^{10}$

Coefficient of x^{K-10} in $(1+x)^{10}$

Which is ${}^{10}C_{K-10}$

So ways to make sum minimum K is

${}^{10}C_{K-10} + {}^{10}C_{K-9} + {}^{10}C_{K-8} + \ldots + {}^{10}C_{10}$

Probability

$$P(K) = \frac{{}^{10}C_{K-10} + {}^{10}C_{K-9} + \ldots + {}^{10}C_{10}}{2^{10}}$$

$$P(K) = \frac{2^{10} - ({}^{10}C_0 + \ldots + {}^{10}C_{K-11})}{2^{10}}$$

$$= 1 - \frac{{}^{10}C_0 + \ldots + {}^{10}C_{K-11}}{2^{10}} > \frac{1}{2}$$

So ${}^{10}C_0 + {}^{10}C_1 + \ldots\ {}^{10}C_{K-11} < 2^9$

So K – 11 = 4

$\Rightarrow$ K = 15

90. Let $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$. Let

$f^1(x) = f(x), f^2(x) = f(f(x))$ and generally

$f^n(x) = f(f^{n-1}(x))$ for $n > 1$

Let $P = f^1(2)f^2(3)f^3(4)f^4(5)$

Which of the following is a multiple of P –

(A) 125 (B) 375 (C) 250 (D) 147

Ans. **(B)**

Sol. $f(x) = \frac{x+1}{x-1}$

$$f^2(x) = f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = x$$

$f^3(x) = f(x) = \frac{x+1}{x-1}$

$f^4(x) = x$

$P = f(2).f^3(3)f^3(4)f^4(5)$

$P = 3 \times 3 \times \frac{5}{3} \times 5 = 75$

Multiple of P is 375

PHYSICS

91. The total energy of a black body radiation source is collected for five minutes and used to heat water. The temperature of the water increases from 10.0º C to 11.0ºC. The absolute temperature of the black body is doubled and its surface area halved and the experiment repeated for the same time. Which of the following statements would be most nearly correct ?

(A) The temperature of the water would increase from 10.0º C to a final temperature of 12º C

(B) The temperature of the water would increase from 10.0º C to a final temperature of 18º C

(C) The temperature of the water would increase from 10.0º C to a final temperature of 14ºC

(D) The temperature of the water would increases from 10.0º C to a final temperature of 11º C

Ans. **(B)**

Sol. Energy radiated, $U \propto AT^4t$

$$\Rightarrow \frac{U_2}{U_1} = \frac{A/2(2T)^4.t}{AT^4.t} = 8$$

$$\Rightarrow U_2 = 8U_1$$

$$\Rightarrow mS\Delta t_2 = 8mS\Delta t_1$$

$$\Rightarrow \Delta t_2 = 8\Delta t_1$$

92. A small asteroid is orbiting around the sun in a circular orbit of radius r_0 with speed V_0. A rocket is launched from the asteroid with speed $V = \alpha V_0$, where V is the speed relative to the sun. The highest value of α for which the rocket will remain bound to the solar system is (ignoring gravity due to the asteroid and effects of other planets) –

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) 1

Ans. **(A)**

Sol. Rocket will remain bound to the solar system till its mechanical energy becomes zero.

i.e. $\frac{GMm}{r_0} - \frac{1}{2}mv^2 = 0$

$$\frac{1}{2}mv^2 = \frac{GMm}{r_0} \Rightarrow v^2 = \frac{2GM}{r_0}$$

$$\alpha^2 v_0^2 = \frac{2GM}{r_0}$$

$$\alpha^2 \times \frac{GM}{r_0} = \frac{2GM}{r_0}$$

$$\alpha = \sqrt{2}$$

93. A radioactive nucleus A has a single decay mode with half life τ_A. Another radioactive nucleus B has two decay modes 1 and 2. If decay mode 2 were absent, the half life of B would have been $\tau_A/2$. If decay mode 1 were absent, the half life of B would have been $3\tau_A$, then the ratio τ_B/τ_A is–

(A) 3/7 (B) 7/2 (C) 7/3 (D) 1

Ans. **(A)**

Sol. $\tau_B = \frac{\tau_{A/2}.3\tau_A}{\tau_{A/2} + 3\tau_A}$

$$\frac{\tau_B}{\tau_A} = \frac{3}{7}$$

94. A stream of photons having energy 3 eV each impinges on a potassium surface. The work function of potassium is 2.3 eV. The emerging photo-electrons are slowed down by a copper plate placed 5 mm away. If the potential difference between the two metal plates is 1 V, the maximum distance the electrons can move away from the potassium surface before being turned back is–

(A) 3.5 mm (B) 1.5 mm (C) 2.5 mm (D) 5.0 mm

Ans. **(A)**

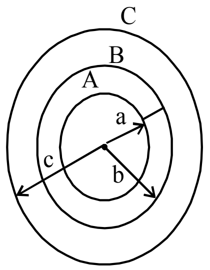
Sol. $K = 3 - 2.3 = 0.7\text{eV}$, $S = \dfrac{K}{eE}$ and $E = V/d$

95. Consider three concentric metallic spheres A, B and C of radii a, b, c respectively where a < b < c, A and B are connected whereas C is grounded. The potential of the middle sphere B is raised to V then the charge on the sphere C is–

(A) $-4\pi\varepsilon_0 V\dfrac{bc}{c-b}$ (B) $-4\pi\varepsilon_0 V\dfrac{ac}{c-a}$ (C) $+4\pi\varepsilon_0 V\dfrac{bc}{c-b}$ (D) zero

Ans. **(A)**

Sol.



$V_B = \dfrac{kq}{b} + \dfrac{k(-q)}{c} = V$ (Given)

$q = \dfrac{4\pi\varepsilon_0.bc}{c-b}.V$

Charge on C = –q

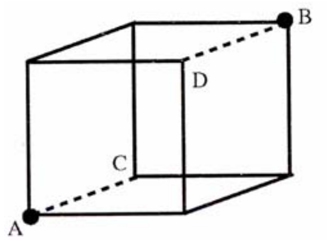
96. On a bright sunny day a diver of height h stands at the bottom of a lake of depth H. Looking upward, he can see objects outside the lake in a circular region of radius R. Beyond this circle he sees the image of objects lying on the floor of the lake. If refractive index of water is 4/3, then the value of R is–

(A) $3(H-h)/\sqrt{7}$ (B) $(H-h)/\sqrt{7/3}$ (C) $3h\sqrt{7}$ (D) $(H-h)/\sqrt{5/3}$

Ans. **(A)**

Sol. $R = \dfrac{h'}{\sqrt{\mu^2 - 1}}$

97. As shown in the figure below, a cube is formed with ten identical resistance R (thick lines) and two shorteing wires (dotted lines) along the arms AC and BD.



Resistance between point A and B is–

(A) R/2 (B) 5R/6 (C) 3R/4 (D) R

Ans. **(A)**

Sol. The given circuit can be simplified into two wheatstone bridge in parallel

98. A standing wave in a pipe with a length L = 1.2 m is described by

$y(x,t) = y_0 \sin[(2\pi/L)x] \sin[(2\pi/L)x + \pi/4]$

Based on above information, which one of the following statements is incorrect.

(Speed of sound in air is 300 m s^{-1})–

(A) The pipe is closed at both ends

(B) The wavelength of the wave could be 1.2 m

(C) There could be a node at x = 0 and antinode at x = L/2

(D) The frequency of the fundamental mode of vibrations is 137.5 Hz

Ans. **(B, D)**

Sol. * If v = 330 the "D" will also be correct option.

If we consider option "B" that seems to be incorrect as per given condition. So question is about incorrect statement.

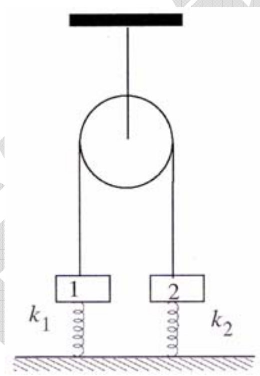
So answer should be "B" and "D"

(i) $\frac{2\pi}{\lambda} = \left(\frac{4\pi}{\ell}\right)$ $\qquad \frac{2\pi}{\lambda} = k =$ Propagation constant

or $\lambda = \ell/2$ or $\lambda = \frac{1.2}{2} = 0.6$

(ii) $v = f\lambda \Rightarrow f = v/\lambda$

99. Two blocks (1 and 2) of equal mass m are connected by an ideal string (see figure shown) over a frictionless pulley. The blocks are attached to the ground by springs having spring constants k_1 and k_2 such that $k_1 > k_2$



Initially, both springs are unstretched. The block 1 is slowly pulled down a distance x and released. Just after the release the possible values of the magnitude of the acceleration of the blocks a_1 and a_2 can be–

(A) Either $\left(a_1 = a_2 = \frac{(k_1 + k_2)x}{2m}\right)$ or $\left(a_1 \frac{k_1 x}{m} - g \text{ and } a_2 = \frac{k_2 x}{m} + g\right)$

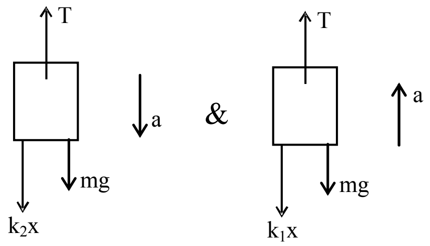
(B) $\left(a_1 = a_2 \frac{(k_1 + k_2)x}{2m}\right)$ only

(C) $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m}\right)$ only

(D) Either $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m}\right)$ or $\left(a_1 = a_2 = \frac{(k_1 k_2)x}{(k_1 + k_2)m} - g\right)$

Ans. **(A)**

Sol.



$T + k_1x - mg = ma$ ....(i)

$k_2x + mg - T = ma$ ....(ii)

From equation (i) & (ii)

$T = \left(\frac{k_2 - k_1}{2}\right)x + mg$ and

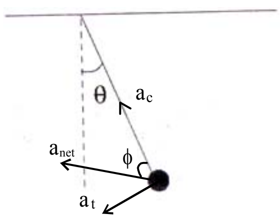
$a = \left(\frac{k_1 + k_2}{2m}\right)x$ if T is non-zero.

If T is zero, then

$a_1 = \frac{k_1x - mg}{m} = \frac{k_1x}{m} - g$

$a_2 = \frac{k_2x + mg}{m} = \frac{k_2x}{m} + g$

100. A simple pendulum is released from rest at the horizontally stretched position. When the string makes an angle θ with the vertical, the angle ϕ which the acceleration vector of the bob makes with the string is given by–



(A) $\phi = 0$ (B) $\phi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$ (C) $\phi = \tan^{-1}(2\tan\theta)$ (D) $\phi = \pi/2$

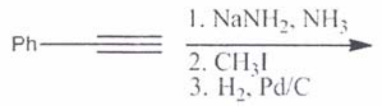
Ans. **(B)**

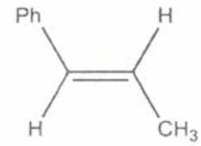
Sol. By energy conservation

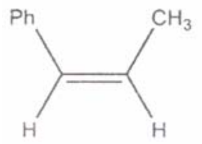
$\tan\phi = a_t/a_c$

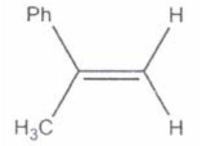
CHEMISTRY

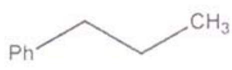
101. The final major product obtained in the following sequence of reactions is –



(A) 

(B) 

(C) 

(D) 

Ans. **(D)**

Sol. $PhC \equiv CH \xrightarrow{NaNH_2/NH_3} PhC \equiv C^- \xrightarrow{CH_3I} PhC \equiv C - CH_3 \xrightarrow{H_2,\ Pd/C} Ph - CH_2 - CH_2 - CH_3$

102. In the DNA of E. Coli the mole ratio of adenine to cytosine is 0.7. If the number of moles of adenine in the DNA is 350000, the number of moles of guanine is equal to –

(A) 350000 (B) 500000 (C) 225000 (D) 700000

Ans. **(B)**

Sol.

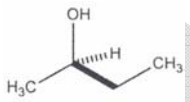
$$\frac{A}{C} = 0.7$$

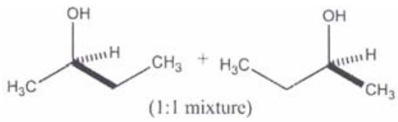
$$\frac{35 \times 10^4}{C} = 0.7$$

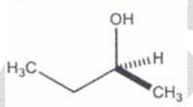
$$C = \frac{35 \times 10^4}{0.7} = 5 \times 10^5$$

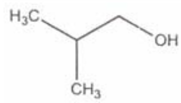
Moles of cytosine = moles of guanine

103. (R)-2-bromobutane upon treatment with aq. NaOH gives –

(A) 

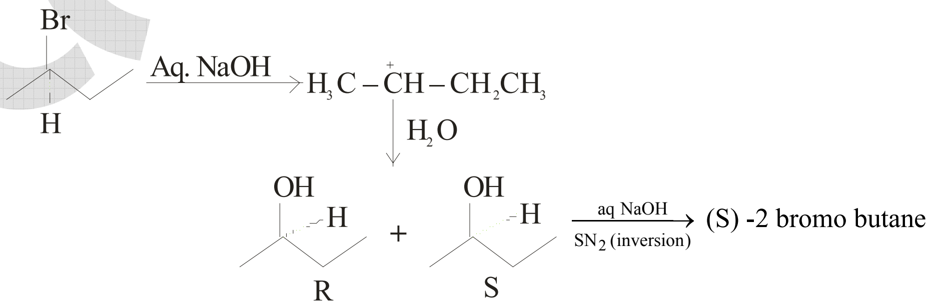
(B) 

(C) 

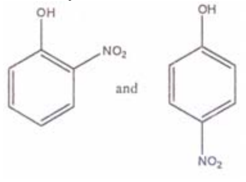
(D) 

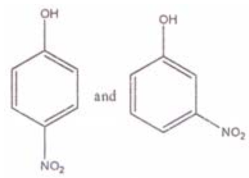
Ans. **(C)**

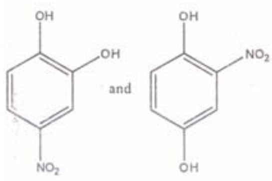
Sol.

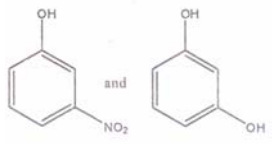


104. Phenol on treatment with dil. HNO_3 gives two products **P** and **Q.** **P** is steam volatile but **Q** is not. **P** and **Q** are, respectively–

(A) 

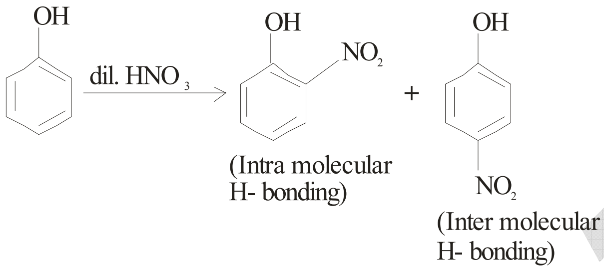
(B) 

(C) 

(D) 

Ans. **(A)**

Sol.



105. A metal is irradiated with light of wavelength 660 nm. Given that the work that the work function of the metal is 1.0 eV, the de Broglie wavelength of the ejected electron is close to –

(A) 6.6×10^{-7} m (B) 8.9×10^{-11} m (C) 1.3×10^{-9} m (D) 6.6×10^{-13} m

Ans. **(C)**

Sol.

$$E = \phi + K.E.$$

$$\because E = \frac{hC}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}}$$

$$= 3\times10^{-19}\ J$$

$$\phi = 1ev = 1.6\times10^{-19}\ J$$

$$K.E. = 3\times10^{-19} - 1.6\times10^{-19} = 1.4\times10^{-19}\ J$$

for wave length of emitted electron

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.4\times10^{-19}}} = \frac{6.6\times10^{-34}}{5\times10^{-25}} = 1.32\times10^{-9}\ \text{meter}$$

106. The inter-planar spacing between the (2 2 1) planes of a cubic lattice of length 450 pm is –

(A) 50 pm (B) 150 pm (C) 300 pm (D) 450 pm

Ans. **(B)**

Sol. $$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{450}{\sqrt{4+4+1}} = \frac{450}{\sqrt{9}} = 150\ pm$$

107. The ΔH for vaporization of a liquid is 20 kJ/mol. Assuming ideal behaviour, the change in internal energy for the vaporization of 1 mol of the liquid at 60°C and 1 bar is close to –

(A) 13.2 kJ/mol (B) 17.2 kJ/mol (C) 19.5 kJ/mol (D) 20.0 kJ/mol

Ans. **(B)**

Sol. $\Delta H = \Delta E + \Delta n_g RT$

$20 = \Delta E + 8.314 + 10^{-3} \times 333 \Rightarrow \Delta E = 17.2$ kJ/mol

108. Among the following, the species that is both tetrahedral and diamagnetic is –

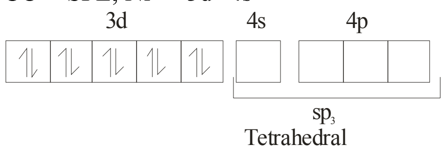
(A) $[NiCl_4]^{2-}$ (B) $[Ni(CN)_4]^{2-}$ (C) $Ni(CO)_4$ (D) $[Ni(H_2O)_6]^{2+}$

Ans. **(C)**

Sol. $[Ni(CO)_4]$

$Ni^o = 3d^8 4s^2$

$CO = SFL, Ni^o = 3d^{10} 4s^0$



Unpaired electron = 0, Diamagnetic

109. Three moles of an ideal gas expands reversibly under isothermal condition form 2 L to 20 L at 300 K. The amount of heat-change (in kJ/mol) in the process is –

(A) 0 (B) 7.2 (C) 10.2 (D) 17.2

Ans. **(D)**

Sol.

$$W = -2.303nRT\log\frac{V_2}{V_1}$$

$$= -2.303 \times 3 \times 8.314 \times 10^{-3} \times 300 \log\frac{20}{2}$$

$$= -17.2 kJ / mol$$

110. The following data are obtained for a reaction, X + Y → Products.

Expt.	$[X_0]$/mol	$[Y_0]$/mol	rate/mol $L^{-1}s^{-1}$
1	0.25	0.25	1.0×10^{-6}
2	0.50	0.25	4.0×10^{-6}
3	0.25	0.50	8.0×10^{-6}

The overall order of the reaction is

(A) 2 (B) 4 (C) 3 (D) 5

Ans. **(D)**

Sol.

$r = K[X]^x[Y]^y$

Total order = n = x + y

By exp.(1) & (2)

$$\frac{r_1}{r_2} = \frac{K[.25]^x[.25]^y}{K[.50]^x[.25]^y} = \frac{1.0\times10^{-6}}{4.0\times10^{-6}}$$

$$\frac{1}{(2)^x} = \frac{1}{4}, x = 2$$

By exp.(1) & (3)

$$\frac{r_1}{r_3} = \frac{K[.25]^x[.25]^y}{K[.25]^x[.50]^y} = \frac{1\times10^{-6}}{8\times10^{-6}}$$

$$\frac{1}{(2)^y} = \frac{1}{8}, y = 3$$

So Total order = 2 + 3 = 5

BIOLOGY

111. Why hydrogen peroxide is applied on the wound as a disinfectant, there is frothing at the site of injury, which is due to the presence of an enzyme in the skin that used hydrogen peroxide as a substrate to produce–

(A) Hydrogen (B) Carbon Dioxide (C) Water (D) Oxygen

Ans. **(D)**

112. Persons suffering from hypertension (high blood pressure) are advised a low-salt diet because–

(A) More salt is absorbed in the body of a patient with hypertension

(B) High salt leads to water retention in the blood that further increases the blood pressure

(C) High salt increases nerve conduction and increases blood pressure

(D) High salt causes adrenaline release that increases blood pressure

Ans. **(B)**

113. Insectivorous plants that mostly grow on swampy soil use insects as a source of–

(A) Carbon (B) Nitrogen (C) Phosporous (D) Magnesium

Ans. **(B)**

Sol. Insectivorous plants are grown in nitrogen deficient soil and they get nitrogen by trapping insect e.g. *Utricularia, Dionea, Nepenthes.*

114. In cattle, the coat colour red and white are two dominant traits, which express equally F_1 to produce roan (red and white colour in equal proportion). If F_1 progeny are selfbred, the resulting progency in F_2 will have phenotypic ration (red : roan: white) is –

(A) 1: 1:1 (B) 3 : 9 : 3 (C) 1 : 2 : 1 (D) 3 : 9 : 4

Ans. **(C)**

Sol. This is an example of Co-dominance. (Result is 1 Red: 2 Roan: 1 white).

115. The restriction endonuclease EcoR-I recognizes and cleaves DNA sequence as shown below

5' -G A A T T C-3'

3' -C T T A A G-5'

What is the probable number of cleavage sites that can occur in a 10 kb long random DNA sequence ?

(A) 10 (B) 2 (C) 100 (D) 50

Ans. **(B)**

Sol. Eco RI is an example of six-cutter restriction endonuclease. It usually cleaves once in every 4096 bp.

$$\left(\frac{1}{4}\right)^6 = \frac{1}{4096}$$

Given length of DNA fragment= 10 Kb = 10000 bp

Hence,

Probable no. of cleaving sites = $\frac{10000}{4096} = 2.44$

116. Which one of the following is true about enzyme catalysis ?

(A) The enzyme changes at the end of the reaction

(B) The activation barrier of the process is lower in the presence of an enzyme

(C) The rate of the reaction is retarded in the presence of an enzyme

(D) The rate of the reaction is independent of substrate concentration

Ans. **(B)**

117. Vibrio cholerae causes cholera in humans. Ganga water was once used successfully to combat the infection. The possible reason could be–

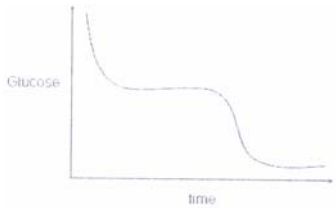
(A) High salt content of Ganga water (B) Low salt content of Ganga water

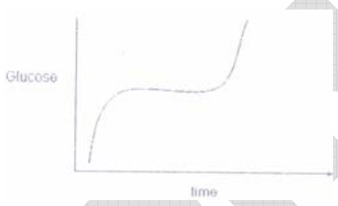
(C) Presence of bacterophases in Ganga water (D) Presence of antibiotics in Ganga Water

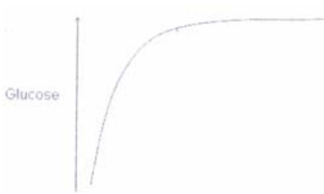
Ans. **(C)**

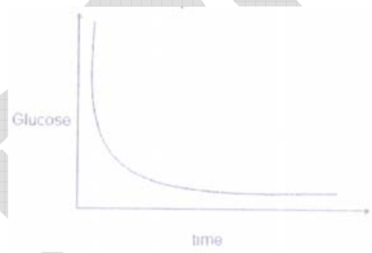
Sol. Large number of bacteriophage are present in Ganga water which destroy bacteria.

118. When a person beings to fast, after some time glycongen stored in the liver is mobilized as a source of glucose. Which of the following graphs best represents the change of glucose level (y axis) in his blood, starting from the time (x -axis) when the beings to fast ?

(A) 

(B) 

(C) 

(D) 

Ans. **(A)**

119. The following sequence contains the open reading frame of a polypeptide. How many amino acids will the polypeptide consists of ?

5'- AGCATATGATCGTTTCTCTGCTTTGAACT-3'

(A) 4 (B) 2 (C) 10 (D) 7

Ans. **(D)**

Sol. 5' AGCAT A TG A TC G TT T CT C TG C TT T GA T GAA CT' 3'

1 2 3 4 5 6 7

5' AGC AU A U G A U C G U U U C U C U G C U U U G A ACU 3'

1 2 3 4 5 6 Stop

So this sequence is present in mRNA and translation start from start codon 'AUG' so 6 Amino acid will be formed close to 7

So answer can (D)

120. Insects constitute the largest animal group on earth. About 25-30% of the insect species are known to be herbivores. In spite of such huge herbivore pressure, globally, green plants have persisted. one possible reason for this persistence is –

(A) Food preference of insects has tended to change with time

(B) Herbivore insects have become inefficient feeders of green plants

(C) Herbivore population has been kept in control by predators

(D) Decline in reproduction of herbivores with time

Ans. **(C)**