



CAREER POINT

KVPY QUESTION PAPER-2019 (STREAM SA)

Part – I

One - Mark Questions

Date : 03 / 11 / 2019

MATHEMATICS

1. Let ABC be an equilateral triangle with side length a. Let R and r denote the radii of the circumcircle and the incircle of triangle ABC respectively. Then, as a function of a, the ratio $\frac{R}{r}$
- (A) strictly increases
 (B) strictly decreases
 (C) remains constant
 (D) strictly increases for $a < 1$ and strictly decreases for $a > 1$

Ans. [C]

Sol. ΔABC equilateral Triangle
 side = a

$$R = \frac{abc}{4\Delta} \quad \dots(i) \quad \left\{ \begin{array}{l} (a=b=c)=a \\ \Delta = \frac{\sqrt{3}}{4} a^2 \end{array} \right.$$

$$r = \frac{\Delta}{s} \quad \dots(ii) \quad \left\{ s = \frac{a+b+c}{2} \right.$$

$$R = \frac{a^3}{4 \cdot \frac{\sqrt{3} a^2}{4}} = \frac{a}{\sqrt{3}} \quad \dots(iii)$$

$$r = \frac{\sqrt{3} a^2 / 4}{3a/2} = \frac{a}{\sqrt{3} \cdot 2} \quad \dots(iv)$$

$$\frac{R}{r} = \frac{a/\sqrt{3}}{a/2\sqrt{3}} = 2 \quad (\text{constant})$$

2. Let b be a non-zero real number. Suppose the quadratic equation $2x^2 + bx + \frac{1}{b} = 0$ has two distinct real roots. Then
- (A) $b + \frac{1}{b} > \frac{5}{2}$ (B) $b + \frac{1}{b} < \frac{5}{2}$ (C) $b^2 - 3b > -2$ (D) $b^2 + \frac{1}{b^2} < 4$

Ans. [C]

Sol. $2x^2 + bx + \frac{1}{b} = 0 \Rightarrow$ real roots

$$D > 0$$

$$b^2 - 4(2)\left(\frac{1}{b}\right) > 0$$

$$b^2 - \frac{8}{b} > 0$$

$$\frac{b^3 - 8}{b} > 0$$

$$\frac{(b-2)(b^2 + 4 + 2b)}{b} > 0$$

$$b^2 + 2b + 4 \text{ always +ve}$$

$$\frac{b-2}{b} > 0$$

$$b - 2 > 0, b > 0$$

$$b > 2$$

and

$$b - 2 < 0, b < 0$$

$$b < 0$$

$$b \in (-\infty, 0) \cup (2, \infty)$$

Check the options

(A) $b + \frac{1}{b} > \frac{5}{2}$

$b > 2$ satisfied but $b < 0$ Not satisfied

(B) $b + \frac{1}{b} < \frac{5}{2}$ $b > 2$ Not satisfied

(C) $b^2 - 3b > -2$

$$b^2 - 3b + 2 > 0$$

$$b^2 - 2b - b + 2 > 0$$

$$(b-2)(b-1) > 0$$

$b \in (-\infty, 1) \cup (2, \infty)$ satisfied both condition

3. Let $p(x) = x^2 + ax + b$ have two distinct real roots, where a, b are real numbers. Define $g(x) = p(x^3)$ for all real numbers x . Then which of the following statements are true ?

I. g has exactly two distinct real roots

II. g can have more than two distinct real roots

III. There exists a real number α such that $g(x) \geq \alpha$ for all real x

(A) Only I

(B) Only I and III

(C) Only II

(D) Only II and III

Ans. [B]

Sol. $p(x) = x^2 + ax + b$ $a, b \in \mathbb{R}$

roots are real

$$g(x) = p(x^3)$$

$$x^3 = t$$

$$g(x) = p(t)$$

$$p(t) = t^2 + at + b$$

roots are real

$$\text{but } t = x^3$$

Let $t = \beta$

$$x^3 = \beta$$

$$x = (\beta)^{1/3}, (\beta)^{1/3} \omega, (\beta)^{1/3} \omega^2$$

only 1 real root, 2-imag.

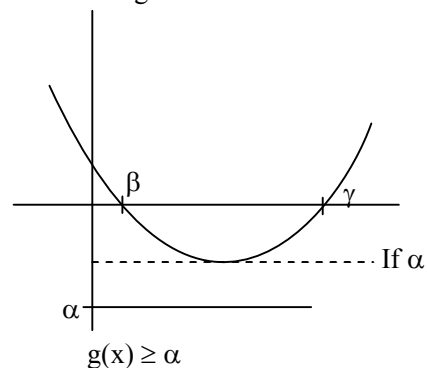
same are other root

$$t = \gamma$$

it has only one real root

Total 2-distinct real roots

and 4- Imag.



4. Let $a_n, n \geq 1$, be an arithmetic progression with first term 2 and common difference 4. Let M_n be the average of the first n terms. Then the sum $\sum_{n=1}^{10} M_n$ is

(A) 110

(B) 335

(C) 770

(D) 1100

Ans. [A]

Sol. A.P $a_n \quad n \geq 1$

first term $a_1 = 2$

common difference $d = 4$

$M_n =$ avg. of first n terms

$$M_n = \frac{S_n}{n} = \frac{\frac{n}{2}[2a + (n-1)d]}{n}$$

$$= \frac{1}{2} [2.2 + (n-1)4]$$

$$= 2 + 2(n-1)$$

$$M_n = 2 + 2n - 2 = 2n$$

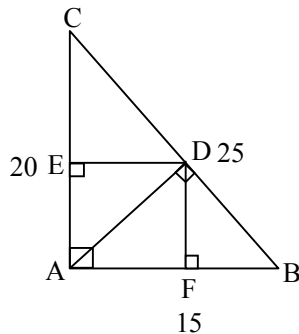
$$\Rightarrow \sum_{n=1}^{10} M_n = \sum_{n=1}^{10} (2n) = \frac{2(10) \times (11)}{2} = 110$$

5. In a triangle ABC, $\angle BAC = 90^\circ$; AD is the altitude from A onto BC. Draw DE perpendicular to AC and DF perpendicular to AB. Suppose AB = 15 and BC = 25. Then the length of EF is

(A) 12 (B) 10 (C) $5\sqrt{3}$ (D) $5\sqrt{5}$

Ans. [A]

Sol.



$$\angle BAC = 90^\circ$$

$$AD \perp BC$$

$$DE \perp AC$$

$$DF \perp AB$$

EDFA are rectangle

$$AD = EF$$

$$AB = 15, BC = 25$$

$$AC^2 + AB^2 = BC^2$$

$$AC^2 + 225 = 625$$

$$AC = 20$$

$$\text{Let } BD = (25 - x) \quad \text{So } CD = x$$

$\triangle ADB$

$$AD^2 = (15)^2 - (25 - x)^2 \quad \dots (i)$$

$\triangle ADC$

$$AD^2 = (20)^2 - x^2 \quad \dots (ii)$$

$$\text{eq (i)} = \text{eq (ii)}$$

$$225 - (625 + x^2 - 50x) = 400 - x^2$$

$$50x = 800$$

$$CD = x = 16$$

$$BD = 25 - 16 = 9$$

$\triangle ADC$

$$AD^2 = 20^2 - 16^2$$

$$= 400 - 256$$

$$AD^2 = 144$$

$$EF = AD = 12$$

6. The sides a, b, c of a triangle satisfy the relations $c^2 = 2ab$ and $a^2 + c^2 = 3b^2$. Then the measure of $\angle BAC$, in degrees, is
 (A) 30 (B) 45 (C) 60 (D) 90

Ans. [B]

Sol. $\triangle ABC$

Sides a, b, c

$$c^2 = 2ab \quad \dots(i)$$

$$a^2 + c^2 = 3b^2 \quad \dots(ii)$$

$$a^2 + 2ab = 3b^2$$

$$a^2 + 2ab + b^2 = 4b^2$$

$$(a + b)^2 = 4b^2$$

$$a + b = \pm 2b$$

$a, b, c \rightarrow$ sides always +ve

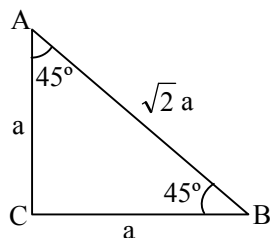
$$a + b = 2b$$

$$a = b$$

put in eq.(i)

$$c^2 = 2a^2$$

$$c = \sqrt{2} a$$



$$\angle BAC = 45^\circ$$

7. Let N be the least positive integer such that whenever a non-zero digit c is written after the last digit of N , the resulting number is divisible by c . Then sum of the digits of N is
 (A) 9 (B) 18 (C) 27 (D) 36

Ans. [A]

Sol. $N \rightarrow$ least positive integer and when a digit ' c ' is written after the last digit of N , the resulting number is divisible by ' c '

$$\text{So, } \frac{(N)c}{c}$$

Least positive integer (N) which is divisible by digit ' c ' i.e. (1-9)

So possible number is

$$\frac{(9 \times 8 \times 7 \times 5)c}{c}$$

Number, $9 \times 8 \times 7 \times 5$ is divisible by all the digits from 1 to 9

So, Number is $9 \times 8 \times 7 \times 5 = 2520 = N$

and sum of digits of number ' N ' is

$$2 + 5 + 2 = 9$$

8. Let x_1, x_2, \dots, x_{11} be 11 distinct positive integers. If we replace the largest of these integers by the median of the other 10 integers, then
- (A) the median remains the same (B) the mean increases
(C) the median decreases (D) the mean remains the same

Ans. [C]

Sol. let $x_1, x_2, x_3, x_4, \dots, x_{11}$ are distinct positive integers

Suppose the increasing order of the No's is .

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}$$

and x_{11} is the largest no.

and the median is x_6 ... (1)

Now, the median of first 10 No.'s is $\frac{x_5 + x_6}{2}$ (Let a)

and we have to replace largest no. (x_{11}) by the median of first 10 No's i.e. (a)

So, New increasing order will be

$$x_1, x_2, x_3, x_4, x_5, a, x_6, x_7, x_8, x_9, x_{10},$$

and new median will be a i.e. $\frac{x_5 + x_6}{2}$.

Which is lesser than the median of first eleven No. ' x_6 '

So, median decreases.

9. The number of cubic polynomials $P(x)$ satisfying $P(1) = 2, P(2) = 4, P(3) = 6, P(4) = 8$ is
- (A) 0 (B) 1
(C) more than one but finitely many (D) infinitely many

Ans. [A]

Sol. Let a Cubic Polynomial $P(x) = ax^3 + bx^2 + cx + d$

Now,

$$P(1) = a + b + c + d = 2 \quad \dots(i)$$

$$P(2) = 8a + 4b + 2c + d = 4 \quad \dots(ii)$$

$$P(3) = 27a + 9b + 3c + d = 6 \quad \dots(iii)$$

$$P(4) = 64a + 16b + 4c + d = 8 \quad \dots(iv)$$

$$\begin{aligned} \text{Eq.(ii)} - (i) \\ \Rightarrow 7a + 3b + c = 2 \quad \dots(v) \end{aligned}$$

$$\begin{aligned} \text{Eq.(iii)} - (ii) \\ \Rightarrow 19a + 5b + c = 2 \quad \dots(vi) \end{aligned}$$

$$\begin{aligned} \text{Eq.(vi)} - (v) \\ \Rightarrow 12a + 2b = 0 \quad \dots(vii) \end{aligned}$$

$$\begin{aligned} \text{Eq.(iv)} - (iii) \\ \Rightarrow 37a + 7b + c = 2 \quad \dots(viii) \end{aligned}$$

$$\begin{aligned} \text{Eq.(viii)} - (vi) \\ \Rightarrow 18a + 2b = 0 \quad \dots(ix) \end{aligned}$$

from equation (ix) – (vii)

$$6a = 0 \Rightarrow a = 0$$

If $a = 0$ then polynomial will be

$$P(x) = bx^2 + cx + d$$

i.e., there is no cubic polynomial is possible.

10. A two-digit number \overline{ab} is called almost prime if one obtains a two-digit prime number by changing at most one of its digits a and b . (For example, 18 is an almost prime number because 13 is a prime number). Then the number of almost prime two-digit numbers is
- (A) 56 (B) 75 (C) 87 (D) 90

Ans. [D]

Sol. 10 – 20 ← always at least 1 prime no.

21 – 30 between them

31 – 40

91 – 99

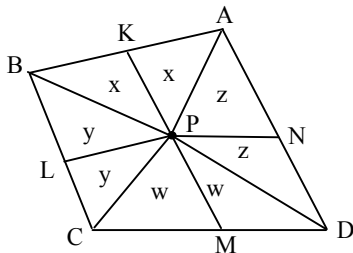
So, by changing one digit all No.'s would be prime no.

So, total almost prime two digit No.'s are = 90

11. Let P be an interior point of a convex quadrilateral $ABCD$ and K, L, M, N be the midpoints of AB, BC, CD, DA respectively. If Area $(PKAN) = 25$, Area $(PLBK) = 36$, and Area $(PMDN) = 41$, then Area $(PLCM)$ is
- (A) 20 (B) 29 (C) 52 (D) 54

Ans. [C]

Sol.



From mid point theorem given is that

$$\text{Area (PKAN)} = 25 = x + z \quad \dots(i)$$

$$\text{Area (PLBK)} = 36 = x + y \quad \dots(ii)$$

$$\text{Area (PMDN)} = 41 = w + z \quad \dots(iii)$$

$$\text{Area (PLCM)} = y + w$$

$$\text{Now (ii) + (iii) - (i)}$$

$$x + y + w + z - x - z = 36 + 41 - 25$$

$$\Rightarrow y + w = 52 = \text{Area (PLCM)}$$

12. The number of non-negative integer solutions of the equations $6x + 4y + z = 200$ and $x + y + z = 100$ is
- (A) 3 (B) 5 (C) 7 (D) infinite

Ans. [C]

Sol. $6x + 4y + z = 200$ $x, y, z \rightarrow \text{Non-Negative}$
 $x + y + z = 100$ \longrightarrow $z = 100 - (y + x) \geq 0$
 - - -

$$5x + 3y = 100$$

$$y = 0 \quad x = 20$$

$$y = 5 \quad x = 17$$

$$y = 10 \quad x = 14$$

$$y = 15 \quad x = 11$$

$$y = 20 \quad x = 8$$

$$y = 25 \quad x = 5$$

$$y = 30 \quad x = 2$$

$$\text{total solution} = 7$$

- 13.** Let $N_1 = 2^{55} + 1$ and $N_2 = 165$. Then
 (A) N_1 and N_2 are coprime
 (B) the HCF (Highest Common Factor) of N_1 and N_2 is 55
 (C) the HCF of N_1 and N_2 is 11
 (D) the HCF of N_1 and N_2 is 33

Ans. [D]

Sol. $N_1 = 2^{55} + 1$ $N_2 = 165 = 5 \times 3 \times 11 = 5 \times 33$
 $x^n + y^n$ divisible by $x + y$ when $n = \text{odd number}$
 $n = 55$
 $N_1 = 2^{55} + 1^{55}$ is divisible by $2 + 1 = 3$
 $= (2^5)^{11} + (1)^{11}$
 $N_1 = (32)^{11} + 1^{11}$ is divisible by $32 + 1 = 33$
 HCF of N_1 and $N_2 = 33$

- 14.** Let $\ell > 0$ be a real number, C denote a circle with circumference ℓ , and T denote a triangle with perimeter ℓ .
 Then

(A) given any positive real number α , we can choose C and T as above such that the ratio $\frac{\text{Area}(C)}{\text{Area}(T)}$ is greater than α

(B) given any positive real number α , we can choose C and T as above such that the ratio $\frac{\text{Area}(C)}{\text{Area}(T)}$ is less than α .

(C) given any C and T as above, the ratio $\frac{\text{Area}(C)}{\text{Area}(T)}$ is independent of C and T

(D) there exist real numbers a and b such that for any circle C and triangle T as above, we must have

$$a < \frac{\text{Area}(C)}{\text{Area}(T)} < b.$$

Ans. [A]

Sol. Circle perimeter = $\ell = C$

Triangle perimeter = $\ell = T$

For same perimeter circle's area is greater than Triangle's area

$$\text{area (C)} > \text{area (T)}$$

$$\frac{\text{area(C)}}{\text{area(T)}} > 1$$

$$\alpha = 1$$

15. The number of three digit numbers \overline{abc} such that the arithmetic mean of b and c and the square of their geometric mean are equal is

- (A) 9 (B) 18 (C) 36 (D) 54

Ans. [B]

Sol. \overline{abc} three digit numbers

$$\frac{b+c}{2} = bc$$

$$b + c = 2bc$$

Put $c = 0$ $b + 0 = 0 \Rightarrow b = 0$

$c = 1$ $b + 1 = 2b \Rightarrow b = 1$

$c = 2$ $b + 2 = 4b \Rightarrow b = \frac{2}{3}$

only $c = 0, b = 0$

and $c = 1, b = 1$

and $a = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$9 \times 2 \times 1 \text{ If } \begin{cases} b=0 \rightarrow c=0 \\ b=1 \rightarrow c=1 \end{cases}$$

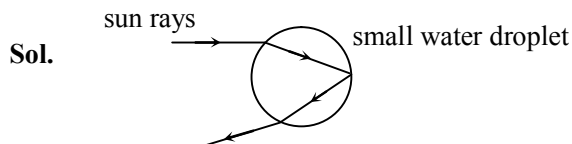
$$\text{Total} = 18$$

PHYSICS

16. Various optical processes are involved in the formation of a rainbow. Which of the following provides the correct order in time in which these processes occur ?

- (A) Refraction, total internal reflection, refraction
 (B) Total internal reflection, refraction, total internal reflection.
 (C) Total internal reflection, refraction, refraction.
 (D) Refraction, total internal reflection, total internal reflection.

Ans. [A]



When sunlight falls on small water droplet it suffers first refraction, then total internal reflection and then refraction.

17. A specially designed vernier calliper has the main scale least count of 1 mm. On the vernier scale there are 10 equal divisions and they match with 11 main scale divisions. Then, the least count of the vernier calliper is

- (A) 0.1 mm (B) 0.909 mm (C) 1.1 mm (D) 0.09 mm

Ans. [A]

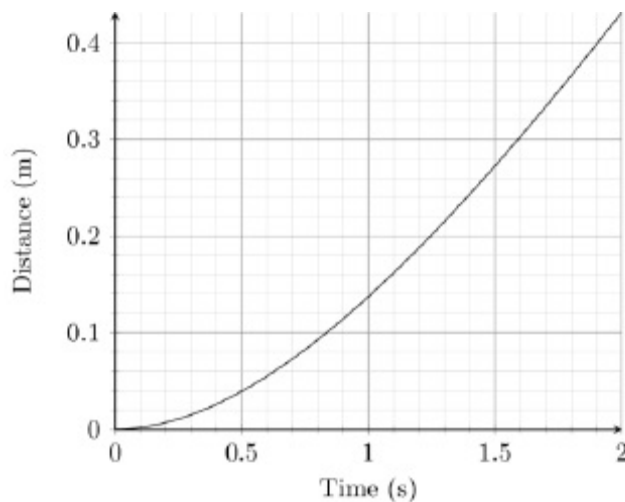
Sol. Least count = 1 V.S.D. – 1 M.S.D.

V.S.D. → Vernier scale division

M.S.D. → Main scale division

$$\begin{aligned} \text{L.C} &= \frac{11}{10} - 1 \\ &= \frac{11-10}{10} = \frac{1}{10} = 0.1 \text{ mm} \end{aligned}$$

18. A steel ball is dropped in a viscous liquid. The distance of the steel ball from the top of the liquid is shown below. The terminal velocity of the ball is closet to.



- (A) 0.26 m/s (B) 0.33 m/s (C) 0.45 m/s (D) 0.21 m/s

Ans. [B]

Sol. For terminal velocity, slope should be constant. If we look carefully slope is constant from $t = 1.6$ sec to $t = 2$ sec.

$$V_t = \frac{0.4 - 0.3}{1.9 - 1.6} = \frac{0.1}{0.3} = 0.33 \text{ m/s}$$

19. A student in a town in India, where the price per unit (1 unit = 1 kW-Hr) of electricity is Rs. 5.00. Purchases a 1 kVA UPS (uninterrupted power supply) battery. A day before the exam, 10 friends arrive to the student's home with their laptops and all connect their laptops to the UPS. Assume that each laptop has a constant power requirement of 90 W. Consider the following statements.
- All the 10 laptops can be powered by the UPS if connected directly.
 - All the 10 laptops can be powered if connected using an extension box with a 3 A fuse.
 - If all the 10 friends use the laptop for 5 hours, then the cost of the consumed electricity is about Rs.22.50.
- Select the correct option with the true statements.
- (A) I only (B) I and II only (C) I and III only (D) II and III only

Ans. [C]

Sol. Power delivered by the battery is 1 kVA = 1000 W

Total power requirement of = $90 \times 10 = 900$ W

So, statement I is correct

$P = VI$

$$I = \frac{900}{220} = 4.1 \text{ A}$$

So, statement II is incorrect

If all the laptops are used for 5 hours, then cost of consumed electricity is

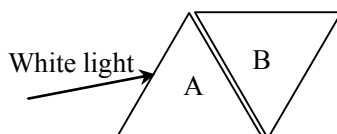
$$\frac{900 \times 5 \times 3600}{3.6 \times 10^6} \times 5 = 22.5 \text{ Rs.}$$

20. Frosted glass is widely used for translucent windows. The region where a transparent adhesive tape is stuck over the frosted glass becomes transparent. The most reasonable explanation for this is
- Diffusion of adhesive glue into glass.
 - Chemical reaction at adhesive tape - glass interface.
 - Refractive index of adhesive glue is close to that of glass.
 - Adhesive tape is more transparent than glass.

Ans. [C]

Sol. If refractive index of adhesive glue is different from that of glass there is a chance of total internal reflection. Then the adhesive tape will not be transparent.

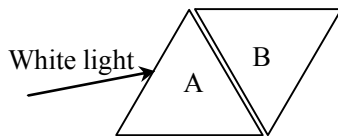
21. Consider two equivalent, triangular hollow prisms A and B made of thin glass plates and arranged with negligible spacing as shown in the figure. A beam of white light is incident on prism A from the left. Given that the refractive index of water is inversely related to temperature, the beam to the right of prism B would NOT appear white if –



- both prisms are filled with hot water (70°C)
- both prisms are filled with cold water (7°C)
- both prisms are empty
- prism A is filled with hot water (70°C) and prism B with cold water (7°C)

Ans. [D]

Sol.



As given,

μ_w is inversely related to temperature (T).

If both prisms are filled with water of equal temperature, then light emerging from prism B will not suffer dispersion and hence, there is white light. For dispersion, refractive indices should be different.

22. A ball is moving uniformly in a circular path of radius 1 m with a time period of 1.5 s. If the ball is suddenly stopped at $t = 8.3$ s, the magnitude of the displacement of the ball with respect to its position at $t = 0$ s is closest to -

(A) 1 m (B) 33 m (C) 3 m (D) 2 m

Ans. [D]

Sol. Time period = 1.5 s

at $t = 8.3$ s

$$\text{So, } \frac{8.3}{1.5} = 5.8$$

Five times it crosses the position at $t = 0$.

In 0.8 sec

$$\theta = \frac{2\pi}{1.5} \times 0.8 = \frac{16}{15}\pi$$

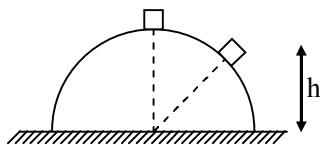
$$\frac{16\pi}{15} \text{ which is } 180 + 12^\circ = 192^\circ$$

So, displacement is greater than radius ($R = 1$ m) approx 2m.

23. A particle slides from the top of a smooth hemispherical surface of radius R which is fixed on a horizontal surface. If it separates from the hemisphere at a height h from the horizontal surface then the speed of the particle is -

(A) $\sqrt{2g(R-h)}$ (B) $\sqrt{2g(R+h)}$ (C) $\sqrt{2gR}$ (D) $\sqrt{2gh}$

Ans. [A]



Sol.

From energy conservation

$$mgR = mgh + \frac{1}{2}mv^2$$

$$v^2 = 2g(R-h)$$

$$v = \sqrt{2g(R-h)}$$

24. The nuclear radius is given by $R = r_0 A^{1/3}$, where r_0 is constant and A is the atomic mass number. Then -
- (A) The nuclear mass density of U^{238} is twice that of Sn^{119}
 (B) The nuclear mass density of U^{238} is thrice that of Sn^{119}
 (C) The nuclear mass density of U^{238} is the same as that of Sn^{119}
 (D) The nuclear mass density of U^{238} is half that of Sn^{119}

Ans. [C]

Sol. $R = r_0 A^{1/3}$

$$R^3 = r_0^3 A$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi r_0^3 A} = \frac{3m_p}{4\pi r_0^3}$$

Hence mass density does not depend on mass number (A) or atomic number (Z)

25. The electrostatic energy of a nucleus of charge Ze is equal to kZ^2e^2/R , where k is a constant and R is the nuclear radius. The nucleus divides into two daughter nuclei of charges $Ze/2$ and equal radii. The change in electrostatic energy in the process when they are far apart is -
- (A) $0.375 kZ^2e^2/R$ (B) $0.125 kZ^2e^2/R$ (C) kZ^2e^2/R (D) $0.5 kZ^2e^2/R$

Ans. [A]

Sol. Initial energy of nucleus $E_i = \frac{kZ^2e^2}{R}$

Now by volume conservation

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi R_0^3 \Rightarrow R_0 = \frac{R}{2^{1/3}}$$

$$E_f = \frac{2k\left(\frac{Ze}{2}\right)^2}{R_0} = \frac{2kZ^2e^2}{4R} 2^{1/3} \approx 0.63 \frac{kZ^2e^2}{R}$$

$$E_f - E_i = 0.37 \frac{kZ^2e^2}{R}$$

26. Two masses M_1 and M_2 carry positive charges Q_1 and Q_2 , respectively. They are dropped to the floor in a laboratory setup from the same height where there is a constant electric field vertically upwards. M_1 hits the floor before M_2 . Then -
- (A) $Q_1 > Q_2$ (B) $Q_1 < Q_2$ (C) $M_1Q_1 > M_2Q_2$ (D) $M_1Q_2 > M_2Q_1$

Ans. [D]

Sol. Let \vec{E} is electric field

$$M_1g - Q_1E = M_1a_1$$

$$a_1 = g - \frac{Q_1E}{M_1}$$

Similarly,

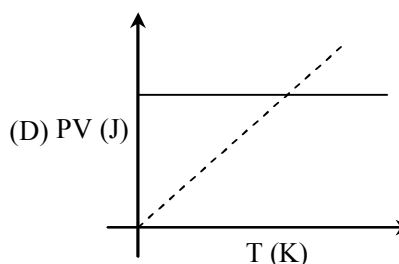
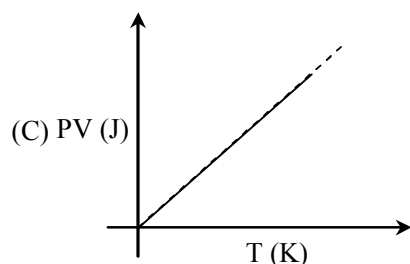
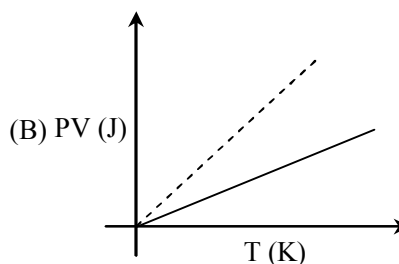
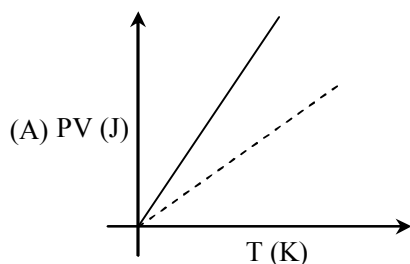
$$a_2 = g - \frac{Q_2 E}{M_2}$$

So, if M_1 hits the floor before M_2 , then

$$\frac{Q_1 E}{M_1} < \frac{Q_2 E}{M_2}$$

$$\text{or } M_1 Q_2 > M_2 Q_1$$

27. Which one of the following schematic graphs best represents the variation of PV (in Joules) versus T (in Kelvin) of one mole of an ideal gas? (The dotted line represents $PV = T$).



Ans. [A]

Sol. From gas equation

$$PV = nRT$$

In the graph, dotted line represents

$$PV = T$$

Here, Slope = 1

$$PV = (nR) T$$

Slope should be greater than 1

28. Mumbai needs 1.4×10^{12} litres of water annually. Its effective surface area is 600 km^2 and it receives an average rainfall of 2.4 m annually. If 10% of this rain water is conserved it will meet approximately -
- (A) 1% of Mumbai's water needs
 (B) 10% of Mumbai's water needs
 (C) 50% of Mumbai's water needs
 (D) 100% of Mumbai's water needs

Ans. [B]

Sol. Area $A = 600 \times 10^6 \text{ m}^2$

Height of rainfall annually $h = 2.4 \text{ m}$

$$\begin{aligned} \text{Volume } V &= 600 \times 10^6 \times 2.4 \\ &= 1440 \times 10^6 \text{ m}^3 \end{aligned}$$

10% is conserved

$$\begin{aligned} V' &= 1440 \times 10^6 \times \frac{1}{10} \\ &= 144 \times 10^6 \text{ m}^3 \end{aligned}$$

or $V' = 1.4 \times 10^8 \text{ m}^3$

Water needs

$$\begin{aligned} V'' &= 1.4 \times 10^{12} \times 10^{-3} \text{ m}^3 \\ &= 1.4 \times 10^9 \text{ m}^3 \end{aligned}$$

$$\therefore \frac{1.4 \times 10^8}{1.4 \times 10^9} \times 100$$

$$= \frac{1}{10} \times 100 = 10\%$$

29. A mass M moving with a certain speed V collides elastically with another stationary mass m . After the collision the masses M and m move with speeds V' and v respectively. All motion is in one dimension. Then-

(A) $V = V' + v$ (B) $V' = V + v$ (C) $V' = (V + v) / 2$ (D) $v = V + V'$

Ans. [D]

Sol. From momentum conservation

$$MV = MV' + mv \quad \dots\text{(i)}$$

From energy conservation

$$\frac{1}{2} MV^2 = \frac{1}{2} MV'^2 + \frac{1}{2} mv^2 \quad \dots\text{(ii)}$$

$$V = V' + \frac{m}{M} v$$

$$\frac{V^2}{2} = \frac{V'^2}{2} + \frac{1}{2} \frac{m}{M} v^2$$

$$\frac{m}{M} = \frac{V^2 - V'^2}{v^2}$$

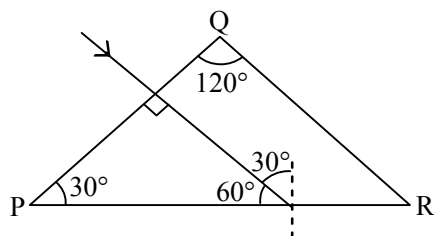
$$V = \frac{V'v + (V^2 - V'^2)}{v}$$

$$Vv - V'v = (V + V')(V - V')$$

$$v = V + v'$$

30. Four rays, 1, 2, 3 and 4 are incident normally on the face PQ of an isosceles prism PQR with apex angle $\angle Q = 120^\circ$. The refractive indices of the material of the prism for the above rays 1, 2, 3 and 4 are 1.85, 1.95, 2.05 and 2.15, respectively and the surrounding medium is air. Then the rays emerging from the face QR are-
- (A) 4 only (B) 1 and 2 only (C) 3 and 4 only (D) 1, 2, 3 and 4

Ans. [C]



Sol.

For critical angle

$$\mu \sin 30^\circ = 1$$

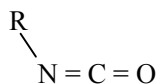
$$\frac{1}{2} = \frac{1}{\mu}$$

$$\therefore \mu = 2$$

For $\mu > 2$, all rays will suffer T.I.R. from face PR, so all these rays will emerge from face QR.

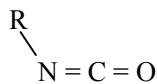
CHEMISTRY

31. The hybridizations of N, C and O shown in the following compound respectively, are -



- (A) sp^2 , sp, sp^2 (B) sp^2 , sp^2 , sp^2 (C) sp^2 , sp, sp (D) sp, sp, sp^2

Ans. [A]



Sol.

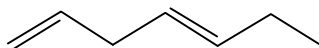
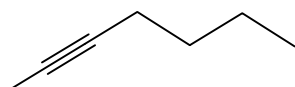
Hybridization of

$$\text{N} = 2\text{B.P.} + 1 \ell.\text{P.} = 3 = sp^2$$

$$\text{C} = 2 \text{B.P.} = 2 = sp$$

$$\text{O} = 1 \text{B.P.} + 2 \ell.\text{P.} = 3 = sp^2$$

32. The following compounds are -



- (A) geometrical isomers (B) positional isomers (C) optical isomers (D) functional group isomers

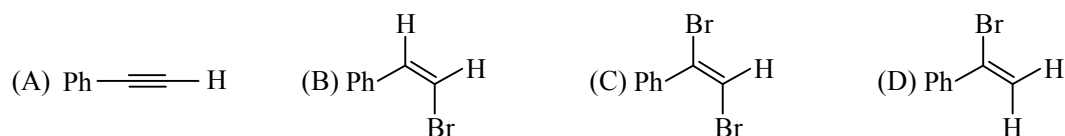
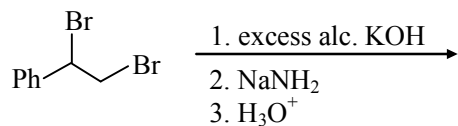
Ans. [D]

Sol. (I) $\text{C} - \text{C} \equiv \text{C} - \text{C} - \text{C} - \text{C} - \text{C} \Rightarrow$ -yne compound

(II) $\text{C} = \text{C} - \text{C} - \text{C} = \text{C} - \text{C} - \text{C} \Rightarrow$ -diene compound

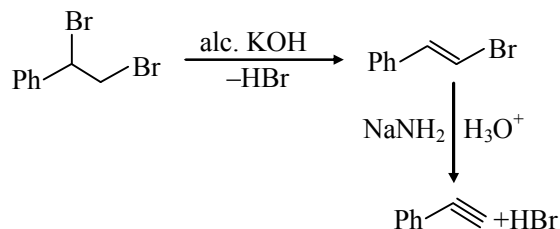
Functional group isomers.

33. The major product of the following reaction is -

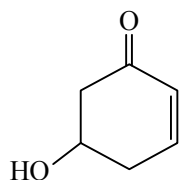


Ans. [A]

It is a method of preparation of alkynes

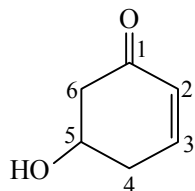


34. IUPAC name of the following compound is -



- (A) 1-hydroxycyclohex-4-en-3-one (B) 1-hydroxycyclohex-3-en-5-one
(C) 3-hydroxycyclohex-5-en-1-one (D) 5-hydroxycyclohex-2-en-1-one

Ans. [D]



Prefix : hydroxy

Word root : cyclohex

Primary suffix : - en

Secondary suffix : - one

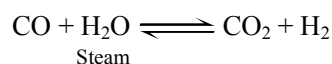
So, IUPAC name : 5-hydroxycyclohex-2-en-1-one

35. In water-gas shift reaction, hydrogen gas is produced from the reaction of steam with -

- (A) Methane (B) Coke (C) Carbon Monoxide (D) Carbon Dioxide

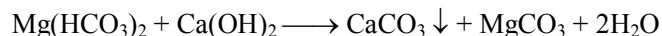
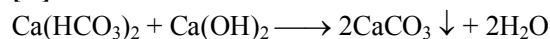
Ans. [C]

Sol. The water-gas shift reaction is reaction of carbon monoxide and water vapor to form carbon dioxide and hydrogen



36. Treatment with lime can remove hardness of water caused by -
 (A) CaCl_2 (B) CaSO_4 (C) $\text{Ca}(\text{HCO}_3)_2$ (D) CaCO_3

Ans. [C]



Lime is used to remove temporary hardness of water caused by bicarbonates salts of Ca and Mg.

37. The most polarizable ion among the following is
 (A) F^- (B) I^- (C) Na^+ (D) Cl^-

Ans. [B]

Sol. According to Fajan's rule, the size of anion should be larger to get polarize easily.

As per the given options :



I^- has the largest size among four and therefore largest polarisibility.

38. For a multi-electron atom, the highest energy level among the following is -

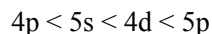
- (A) $n = 5, l = 0, m = 0, s = +\frac{1}{2}$ (B) $n = 4, l = 2, m = 0, s = +\frac{1}{2}$
 (C) $n = 4, l = 1, m = 0, s = +\frac{1}{2}$ (D) $n = 5, l = 1, m = 0, s = +\frac{1}{2}$

Ans. [D]

In options :



According to $(n + l)$ rule, increasing order of energy level :



So, the highest energy level is 5p.

39. The oxide which is neither acidic nor basic is -
 (A) As_2O_3 (B) Sb_4O_{10} (C) N_2O (D) Na_2O

Ans. [C]

Sol. Some non-metal oxides does not shows any acidic or basic nature are called neutral oxides.

Example : N_2O , CO , NO , H_2O etc.

- (A) As_2O_3 } Amphoteric
 (B) Sb_4O_{10} }
 (D) Na_2O } Basic

40. The element whose salts cannot be detected by flame test is -
 (A) Mg (B) Na (C) Cu (D) Sr

Ans. [A]

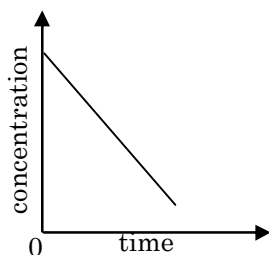
Sol. (A) Mg \longrightarrow None for salts [but for burning Mg metal (intense white)]

(B) Na \longrightarrow Yellow

(C) Cu \longrightarrow $\left\{ \begin{array}{l} \text{Green} \\ \text{Blue-green [in case of halides]} \end{array} \right.$

(D) Sr \longrightarrow Crimson to scarlet

41. The plot of concentration of a reactant vs. time for a chemical reaction is shown below :



The order of this reaction with respect to the reactant is

- (A) 0 (B) 1
(C) 2 (D) not possible to determine from this plot

Ans. [A]

Sol. consider a rxn (zero order rxn)



$$\text{at } t = 0 \quad C_0 \quad 0$$

$$\text{at } t = t \quad C_t \quad C_0 - C_t$$

$$\text{Rate} \propto [A]_{\text{at } t=t}^0 \text{ (acc. to LO. M.A)}$$

$$r = k \quad [\because [A]^0 = 1]$$

$$\text{Also rate} = \frac{\text{change in conc.}}{\text{change in time}}$$

$$\therefore \frac{-dc}{dt} = k$$

$$\Rightarrow -dc = kdt$$

on integration, and applying limits,

$$-\int_{c_0}^{c_t} dc = \int_0^t kdt$$

$$[c]_{c_0}^{c_t} = -k[t]_0^t$$

$$c_t - c_0 = -kt$$

$$\Rightarrow c_t = c_0 - kt$$

where c_t = conc. of reactant at time 't' = t

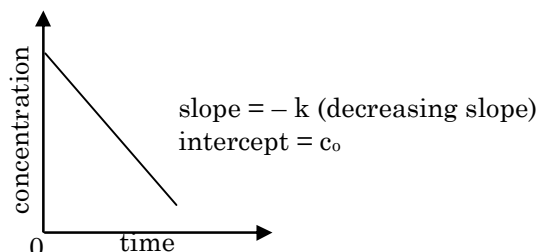
c_0 = conc. of reactant at time t = 0

k = rate constant

compare it with , $y = mx + c$, straight line equation

$$c_t = -kt + c_0$$

plot of c_t vis t



42. During the free expansion of an ideal gas in an isolated chamber,
 (A) internal energy remains constant (B) internal energy decreases
 (C) work done on the system is negative (D) temperature increases

Ans. [A]

Sol. Free expansion → irreversible process in which gas expands freely in adiabatic process

$$\omega = -P_{\text{ext}} \cdot \Delta V$$

$$(P_{\text{ext}} = 0 \text{ for free expansion})$$

$$\therefore \omega = 0$$

$$q = 0 \text{ (isolated chamber)}$$

$$\Delta U = q + \omega$$

$$\therefore \Delta U = 0$$

Hence, Internal Energy is Constant.

43. The number of moles of water present in a spherical water droplet of radius 1.0 cm is
 [Given density of water in the droplet = 1.0 g cm⁻³]

(A) $\frac{\pi}{18}$ (B) $\frac{2\pi}{27}$ (C) 24π (D) $\frac{2\pi}{9}$

Ans. [B]

Sol. Given = r = 1.0cm

$$d = 1.0\text{g/cm}^3$$

$$\text{density (d)} = \frac{\text{Mass (M)}}{\text{Volume (V)}} \Rightarrow 1 = \frac{M}{V} \Rightarrow M = V$$

$$\text{number of moles} = \frac{\text{mass given}}{\text{Molecular mass}}$$

$$= \frac{\text{volume given}}{\text{molecular mass}} \quad [\because m = v]$$

$$= \frac{\frac{4}{3} \pi r^3}{18} \quad [\because \text{molecular mass of H}_2\text{O} = 2 \times 1 + 16 = 18]$$

$$= \frac{\frac{4}{3} \times \pi \times 1^3}{18}$$

$$\text{number of moles} = \frac{2\pi}{27}$$

44. Among the following, the correct statement about cathode ray discharge tube is
 (A) the electrical discharge can only be observed at high pressure and at low voltages
 (B) in the absence of external electrical or magnetic field, cathode rays travel in straight lines
 (C) the characteristics of cathode rays depend upon the material of electrodes
 (D) the characteristics of cathode rays depend upon the gas present in the cathode ray tube

Ans. [B]

Sol. In absence of external electric or magnetic field, cathode rays travel in a straight line in cathode ray tube due to high voltage (10,000-20,000V) and low pressure (10⁻⁴ atm).

45. For a spontaneous process
(A) enthalpy change of the system must be negative
(B) entropy change of the system must be positive
(C) entropy change of the surrounding must be positive
(D) entropy change of the system and surrounding (universe) must be positive

Ans. [D]

Sol. spontaneous reaction $\Delta H \Rightarrow$ -ive

$\Delta S \Rightarrow$ + ive

A spontaneous reaction is a reaction that occurs in a given set of condition without intervention
spontaneous reaction one accompanied by an increase in overall entropy or disorder ness of universe.

BIOLOGY

46. Which one of the following is a CORRECT statement about primates evolution?
(A) Chimpanzees and gorillas evolved from macaques
(B) Humans and chimpanzees evolved from gorillas
(C) Humans, chimpanzees and gorillas evolved from a common ancestor
(D) Humans and gorillas evolved from chimpanzees

Ans. [C]

Sol. Humans, chimpanzees and gorillas evolved from a common ancestor according to evolutionary study.
Primatologists know from fossils that humans, chimpanzees, and gorilla shared an ancient ancestor.

47. The crypts of Lieberkuhn are found in which one of the following parts of the human digestive tract ?
(A) Oesophagus (B) Small intestine
(C) Stomach (D) Rectum

Ans. [B]

Sol. Crypts of lieberkuhn found in the small intestine.

A gland which is found between villi in the intestinal epithelium of living organisms.

48. Removal of the pancreas impairs the breakdown of
(A) Lipids and carbohydrates only (B) Lipids and proteins only
(C) Lipids, proteins and carbohydrates (D) Proteins and carbohydrates only

Ans. [C]

Sol. The pancreas produce pancreatic juice that contains various enzyme, amylase, lipase, trypsin which are necessary for the complete digestion or breakdown of lipids, proteins and carbohydrates.

49. Microscopic examination of a blood smear reveals an abnormal increase in the number of granular cells with multiple nuclear lobes. Which one of the following cells types has increased in number ?
(A) Lymphocytes (B) Monocytes
(C) Neutrophils (D) Thrombocytes

Ans. [C]

Sol. In this type of granules, granulocytes has very tiny granules and nucleus is multilobed with lobed connected by thin strands of nuclear material.

50. Which one of the following genetic phenomena is represented by the blood group AB ?

- (A) Codominance (B) Dominance
(C) Overdominance (D) Semidominance

Ans. [A]

Sol. Codominance occurs when both alleles show dominance in case of blood group AB type (I^A & I^B) in humans.

51. The mode of speciation mediated by geographical isolation is referred to as

- (A) Adaptive radiation (B) Allopatric speciation
(C) Parapatric speciation (D) Sympatric speciation

Ans. [B]

Sol. Allopatric speciation also known as geographical speciation. this speciation occur when biological populations of the same species become isolated due to geographical changes.

52. Which one of the following metabolic conversions requires oxygen ?

- (A) Glucose to pyruvate (B) Glucose to CO_2 and ethanol
(C) Glucose to lactate (D) Glucose to CO_2 and H_2O

Ans. [D]

Sol. Option (A) is glycolysis there is no oxygen requirement.

Option B and C are fermentation, glycolysis and fermentation both are anaerobic process in which no requirement of oxygen.

Option D is oxygenic breakdown of glucose in which glucose molecule is broken down into CO_2 and H_2O to release energy.



53. Where are the proximal and distal convoluted tubules located within the human body ?

- (A) Adrenal cortex (B) Adrenal medulla
(C) Renal cortex (D) Renal medulla

Ans. [C]

Sol. The proximal convoluted tubule is found in the renal cortex of loop of Henle's (mostly in medulla) and distal convoluted tubule found in renal cortex.

54. In a diploid organism, when the locus X is inactivated, transcription of the locus Y is triggered. Based on this observation, which one of the following statements is CORRECT ?

- (A) X is dominant over Y (B) X is epistatic to Y
(C) Y is dominant over X (D) Y is epistatic to X

Ans. [B]

Sol. A gene is said to be epistatic when it's presence suppresses the effect of a gene at another locus so here X is suppressing Y.

Hence X is epistatic to Y.

55. Which one of the following sequences represents the CORRECT taxonomical hierarchy ?

- (A) Species, genus, family, order (B) Order, genus, family, species
(C) Species, order, genus, family (D) Species, genus, order, family

Ans. [A]

Sol. Correct taxonomic category, showing hierarchical arrangement

species \rightarrow genus \rightarrow family \rightarrow order \rightarrow class \rightarrow division \rightarrow kingdom

56. Which one of the following organs is NOT a site for the production of white blood cells ?
 (A) Bone marrow (B) Kidney (C) Liver (D) Spleen
Ans. [B]
Sol. Kidney is not a site for production of WBC, which are produced mainly in bone marrow spleen and partially in liver.
57. Which one of the following anatomical structures is involved in guttation ?
 (A) Cuticle (B) Hydathodes (C) Lenticles (D) Stomata
Ans. [B]
Sol. Guttation is the exudation of drops of xylem sap from the tip or edges of leaves. These edge structure are – Hydathodes. The root pressure force some water to exude through special edge structure.
58. Which one of the following parts of the eye is affected in cataract ?
 (A) Cornea (B) Conjunctiva (C) Retina (D) Lens
Ans. [D]
Sol. Cataract is clouding of the lens in eye. Most cataract develops slowly and don't disturb your eyesight early on
59. Which one of the following organisms is a bryophyte ?
 (A) Liverwort (B) Volvox (C) Chlamydomonas (D) Fern
Ans. [A]
Sol. Liverwort is class of bryophytes. The liverwort grow usually in moist and shady habitat.
60. During oogenesis in mammals, the second meiotic division occurs
 (A) Before fertilization (B) After implantation (C) Before ovulation (D) After fertilization
Ans. [D]
Sol. During oogenesis in mammals the second meiotic division started after fertilization.

Part – II

Two - Mark Questions

MATHEMATICS

61. Let a, b, c, d be distinct real numbers such that a, b are roots of $x^2 - 5cx - 6d = 0$, and c, d are roots of $x^2 - 5ax - 6b = 0$. Then $b + d$ is
 (A) 180 (B) 162 (C) 144 (D) 126
Ans. [C]
Sol. $x^2 - 5cx - 6d = 0$ $\begin{cases} a \\ b \end{cases}$ $x^2 - 5ax - 6b = 0$ $\begin{cases} c \\ d \end{cases}$
 $x = a$ $a^2 - 5ac - 6d = 0$... (i)
 $c^2 - 5ac - 6b = 0$... (ii)
 add (i) and (ii)
 $a^2 + c^2 - 10ac - 6(b + d) = 0$
 $(a + c)^2 - 12ac - 6(b + d) = 0$... (iii)
 $a + b = 5c$ and $c + d = 5a$
 $a + b + c + d = 5c + 5a$

$$b + d = 4(a + c) \quad \dots \text{(iv)}$$

$$ab = -6d \quad cd = -6b$$

$$abcd = 36bd$$

$$ac = 36 \quad \dots \text{(v)}$$

put in equation (iii)

$$(a + c)^2 - 12 \times 36 - 6(4)(a + c) = 0$$

$$(a + c)^2 - 24(a + c) - 12 \times 36 = 0$$

$$(a + c)^2 - 36(a + c) + 12(a + c) - 12 \times 36 = 0$$

$$(a + c)[(a + c) - 36] + 12[(a + c) - 36] = 0$$

$$(a + c + 12)(a + c - 36) = 0$$

$$a + c = 36 \quad a + c = -12$$

$$b + d = 36 \times 4$$

$$b + d = 144$$

62. Let $S = \{1, 2, 3, \dots, 100\}$. Suppose b and c are chosen at random from the set S . The probability that $4x^2 + bx + c$ has equal roots is

- (A) 0.001 (B) 0.004 (C) 0.007 (D) 0.01

Ans. [A]

Sol. Let $S = \{1, 2, 3, \dots, 100\}$

For equal roots of equation $4x^2 + bx + c = 0$

$$D = 0$$

$$b^2 - 16c = 0$$

$$b^2 = 16c$$

Possible values	b 4 8 ⋮ 40	c 1 4 ⋮ 100	}	10 Possibility
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And

Total No. of Events, are 100×100

[For b and c random values from set S]

So, the probability is, $\frac{10}{100 \times 100} = \frac{1}{1000} = 0.001$

63. Let N be the set of positive integers. For all $n \in N$, let $f_n = (n + 1)^{1/3} - n^{1/3}$ and $A = \left\{ n \in N : f_{n+1} < \frac{1}{3(n+1)^{2/3}} < f_n \right\}$.

Then

- (A) $A = N$
 (B) A is a finite set
 (C) the complement of A in N is nonempty, but finite
 (D) A and its complement in N are both infinite

Ans. [A]

Sol. $f_n = (n+1)^{1/3} - n^{1/3}$

$$f_n = \frac{(n+1) - n}{(n+1)^{2/3} + n^{2/3} + n^{1/3}(n+1)^{1/3}}$$

$$f_n = \frac{1}{(n+1)^{2/3} + n^{2/3} + (n(n+1))^{1/3}}$$

$$(n+1)^{2/3} = (n+1)^{2/3}$$

$$n^{2/3} < (n+1)^{2/3}$$

$$n^{1/3}(n+1)^{1/3} < (n+1)^{2/3}$$

$$(n+1)^{2/3} + n^{2/3} + n^{1/3}(n+1)^{1/3} < 3(n+1)^{2/3}$$

$$\frac{1}{(n+1)^{2/3} + n^{2/3} + n^{1/3}(n+1)^{1/3}} > \frac{1}{3(n+1)^{2/3}}$$

$$f_n > \frac{1}{3(n+1)^{2/3}} \quad \dots\dots(i)$$

same as

$$f_{n+1} = (n+2)^{1/3} - (n+1)^{1/3}$$

$$f_{n+1} = \frac{(n+2) - (n+1)}{(n+2)^{2/3} + (n+1)^{2/3} + ((n+1)(n+2))^{1/3}}$$

$$f_{n+1} = \frac{1}{(n+2)^{2/3} + (n+1)^{2/3} + ((n+1)(n+2))^{1/3}}$$

$$(n+1)^{2/3} = (n+1)^{2/3}$$

$$(n+2)^{2/3} > (n+1)^{2/3}$$

$$((n+1)(n+2))^{2/3} > (n+1)^{2/3}$$

$$(n+1)^{2/3} + (n+2)^{2/3} + ((n+1)(n+2))^{1/3} > 3(n+1)^{2/3}$$

$$\frac{1}{(n+1)^{2/3} + (n+2)^{2/3} + ((n+1)(n+2))^{1/3}} > \frac{1}{3(n+1)^{2/3}}$$

$$f_{n+1} < \frac{1}{3(n+1)^{2/3}} \quad \dots\dots(ii)$$

for eq. (i) and (ii)

$$f_{n+1} < \frac{1}{3(n+1)^{2/3}} < f_n$$

for all $n \in \mathbb{N}$

$A = \mathbb{N}$

64. A prime number p is called special if there exist prime p_1, p_2, p_3, p_4 such that

$$p = p_1 + p_2 = p_3 - p_4.$$

The number of special primes is

(A) 0

(B) 1

(C) more than one but finite

(D) infinite

Ans. [B]

Sol. p, p_1, p_2, p_3, p_4 are prime No.

$$\text{i.e. } p = \underbrace{p_1 + p_2}_{\text{Prime No.}} = \underbrace{p_3 - p_4}_{\text{Prime No.}}$$

Only Possible when

$$p = \underset{\downarrow 2}{p_1} + p_2 = p_3 - \underset{\downarrow 2}{p_4}$$

$$p = 2 + p_2 = p_3 - 2$$

Now, only Possible value for p_2 and p_3 are 3 and 7 Respectively.

So, only one Possibility exist.

65. Let ABC be a triangle in which $AB = BC$. Let X be a point on AB such that $AX : XB = AB : AX$. If $AC = AX$, then the measure of $\angle ABC$ equals

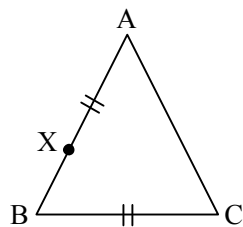
- (A) 18° (B) 36° (C) 54° (D) 72°

Ans. [B]

Sol. $\frac{AX}{XB} = \frac{AB}{AX}$

$$\frac{AX}{XA + XB} = \frac{AB}{AX + AB}$$

$$\frac{AX}{AB} = \frac{AB}{AX + AB}$$



$$(AX)^2 + AX \cdot AB = (AB)^2$$

$$(AX)^2 + 2 \cdot AX \cdot \frac{AB}{2} + \left(\frac{AB}{2}\right)^2 = (AB)^2 + \left(\frac{AB}{2}\right)^2$$

$$\left(AX + \frac{AB}{2}\right)^2 = \frac{5}{4}(AB)^2$$

$$AX = \frac{\sqrt{5}}{2} AB - \frac{AB}{2}$$

Let $AB = a$

$AB = BC = a$

$$AC = AX = \left(\frac{\sqrt{5}-1}{2}\right)a$$

For $\angle ABC$

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2 \cdot AB \cdot BC}$$

$$\cos B = \frac{a^2 + a^2 - \left(\frac{\sqrt{5}-1}{2}\right)^2 a^2}{2 \cdot a \cdot a}$$

$$\cos B = \frac{2a^2 - \left(\frac{5+1-2\sqrt{5}}{4}\right)a^2}{2a^2}$$

$$= \frac{2 \times 4 - 6 + 2\sqrt{5}}{2 \times 4}$$

$$\cos B = \frac{2 + 2\sqrt{5}}{8}$$

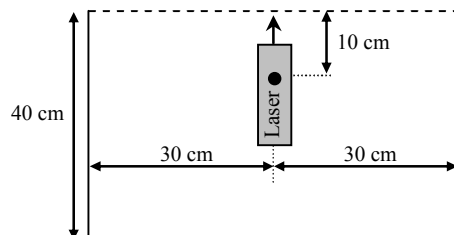
$$\cos B = \frac{1 + \sqrt{5}}{4}$$

$$B = 36^\circ$$

$$\angle ABC = 36^\circ$$

PHYSICS

66. A water-proof laser pointer of length 10 cm placed in a water tank rotates about a horizontal axis passing through its center of mass in a vertical plane as shown in the figure. The time period of rotation is 60s. Assuming the water to be still and no reflections from the surface of the tank, the duration for which the light beam escapes the tank in one time period is close to (Refractive index of water = 1.33)



(A) 8.13 s

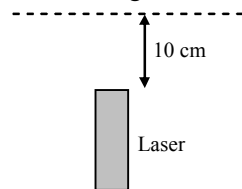
(B) 14.05 s

(C) 16.27 s

(D) 23.86 s

Ans. [C]

Sol. Critical angle



$$\frac{4}{3} \sin \theta_c = 1 \times 1$$

$$\sin \theta_c = \frac{3}{4}$$

$$\theta_c \approx 50^\circ$$

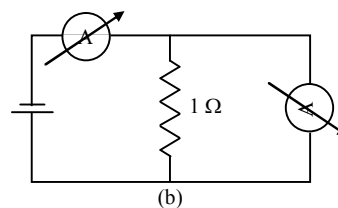
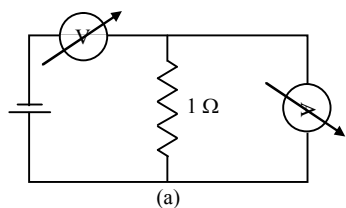
$$2\theta_c = \frac{60}{2\pi} \times 2 \times 50 \times \frac{\pi}{180} \approx 16.27 \text{ sec}$$

67. In an hour-glass approximately 100 grains of sand fall per second (starting from rest), and it takes 2 sec for each sand particle to reach the bottom of the hour-glass. If the average mass of each sand particle is 0.2 g then the average force exerted by the falling sand on the bottom of the hour-glass is close to
 (A) 0.4 N (B) 0.8 N (C) 1.2 N (D) 1.6 N

Ans. [A]

Sol. $v = u + at$
 $v = 0 + 10 \times 2 = 20 \text{ m/s}$
 $p_i = 0.2 \times 10^{-3} \times 20; \quad p_f = 0$
 $\Rightarrow \Delta p = 4 \times 10^{-3} \text{ kg m/s}$
 $f = \frac{\Delta p}{\Delta t} \times n = 100 \times 4 \times 10^{-3} = 0.4 \text{ N}$

68. A student uses the resistance of a known resistor (1 Ω) to calibrate a voltmeter and an ammeter using the circuits shown below. The student measures the ratio of the voltage to current to be $1 \times 10^3 \Omega$ in circuit (a) and 0.999Ω in circuit (b). From these measurements, the resistances (in Ω) of the voltmeter and ammeter are found to be close to:



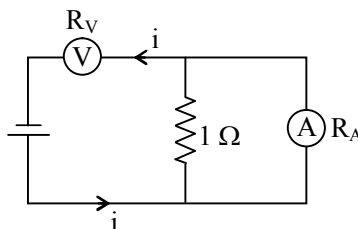
- (A) 10^2 and 10^{-2} (B) 10^3 and 10^{-3} (C) 10^{-2} and 10^2 (D) 10^{-3} and 10^3

Ans. [B]

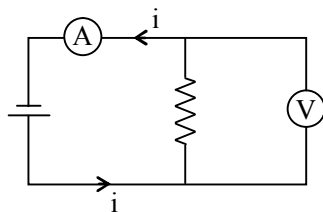
Sol. Case - 1

$$\frac{V}{A} = \frac{i R_V}{\frac{1 \times i}{1 + R_A}}$$

$$1000 = R_V (1 + R_A)$$



Case - 2



$$\frac{V}{A} = \frac{\left(\frac{i \times 1}{1 + R_V} \right) R_V}{i}$$

$$0.999 = \frac{R_V}{1 + R_V}$$

$$\Rightarrow R_V = 999 \Omega \approx 10^3 \Omega$$

$$R_A \approx 10^{-3} \Omega$$

69. A hot air balloon with a payload rises in the air. Assume that the balloon is spherical in shape with diameter of 11.7 m and the mass of the balloon and the payload (without the hot air inside) is 210 kg. Temperature and pressure of outside air are 27 °C and 1 atm = 10^5 N/m² respectively. Molar mass of dry air is 30 g. The temperature of the hot air inside is close to, [The gas constant R = 8.31 J/K/mol]
- (A) 27 °C (B) 52 °C (C) 105 °C (D) 171 °C

Ans. [C]

Sol. $210 \text{ g} + V \rho_{\text{in}} g = V \rho_{\text{out}} g$

$$V (\rho_{\text{out}} - \rho_{\text{in}}) g = 210 \text{ g}$$

$$\rho_{\text{out}} - \rho_{\text{in}} = \frac{210 \times 3}{4\pi r^3}$$

$$\frac{PM}{R} \left(\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}} \right) = \frac{210 \times 3}{4\pi r^3}$$

$$\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}} = \frac{210 \times 3}{4\pi \left(\frac{11.7}{2} \right)^3} \times \frac{8.31}{10^5 \times 30 \times 10^{-3}}$$

$$\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}} = 0.007$$

$$\Rightarrow T_{\text{in}} = 380.22 \text{ K} = 107.22^\circ\text{C}$$

70. A healthy adult of height 1.7 m has an average blood pressure (BP) of 100 mm of Hg. The heart is typically at a height of 1.3 m from the foot. Take the density of blood to be 10^3 kg/m³ and note that 100 mm of Hg is equivalent to 13.3 kPa (kilo Pascals). The ratio of BP in the foot region to that in the head region is close to
- (A) one (B) two (C) three (D) four

Ans. [C]

Sol. $p_{\text{Foot}} = p_{\text{Heart}} + \rho gh$
 $= 13.3 \times 10^3 + 1000 \times 10 \times 1.3 \Rightarrow 26.3 \text{ kPa}$

$$p_{\text{Heart}} = p_{\text{Head}} + \rho gh$$

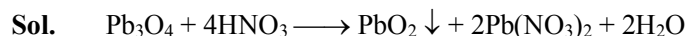
$$p_{\text{Heart}} = 13.3 \times 10^3 - 10^3 \times 10 \times 0.4 = 9.3$$

$$\frac{p_{\text{Foot}}}{p_{\text{Head}}} = \frac{26.3}{9.3} \approx 3$$

CHEMISTRY

71. PbO₂ is obtained from
- (A) the reaction of PbO with HCl
 (B) thermal decomposition of Pb(NO₃)₂ at 200 °C
 (C) the reaction of Pb₃O₄ with HNO₃
 (D) the reaction of Pb with air at room temperature

Ans. [C]



72. For one mole of a van der Waals gas, the compressibility factor $Z \left(= \frac{PV}{RT} \right)$ at a fixed volume will certainly decrease if
 [Given: "a", "b" are standard parameter for van der Waals gas]
 (A) "b" increases and "a" decreases at constant temperature
 (B) "b" decreases and "a" increases at constant temperature
 (C) temperature increases at constant "a" and "b" values
 (D) "b" increases at constant "a" and temperature

Ans. [B]

Sol. "b" decreases and "a" increases at constant temperature

a → strength of intermolecular forces

b → excluded volume

$$Z = \frac{V_{\text{real}}}{V_{\text{ideal}}} = \frac{PV}{RT}$$

To decrease Z value, V_{real} , (numerator) should decrease which decreases when 'a' increase & 'b' decreases.

73. The correct statements among the following
 i. $E_{2s}(\text{H}) > E_{2s}(\text{Li}) > E_{2s}(\text{Na}) > E_{2s}(\text{K})$
 ii. The maximum number of electrons in the shell with principal quantum number n is equal to $2n^2$
 iii. Extra stability of half-filled subshell is due to smaller exchange energy.
 iv. Only two electrons, irrespective of their spin, may exist in the same orbital.
 (A) i and ii (B) ii and iii (C) iii and iv (D) i and iv

Ans. [A]

Sol. (i) Fact (General Trend)

(ii) Fact

(iii) ×

(iv) ×

74. An organic compounds contains 46.78% of a halogen X. When 2.00 g of this compound is heated with fuming HNO_3 in the presence of AgNO_3 , 2.21 g AgX was formed. The halogen X is
 [Given: atomic weight of Ag = 108, F = 19, Cl = 35.5, Br = 80, I = 127]
 (A) F (B) Cl (C) Br (D) I

Ans. [C]

Sol. Organic compound + $\text{HNO}_3 \xrightarrow{\text{AgNO}_3} \text{AgX}$
 (2 gm) 2.21 g,

$$\text{Moles of 'X' in organic compound} = \frac{2 \times \frac{46.78}{100}}{x} \quad \dots (1)$$

[x = Mol. Weight of X]

$$\text{Moles of 'X' in AgX} = \frac{2.21}{108 + x} \quad \dots (2)$$

On equating equation (1) and (2) -

$$\frac{2 \times \frac{46.78}{100}}{x} = \frac{2.21}{108 + x}$$

$$(0.9356)(108 + x) = 2.21 x$$

$$x = 79.55 \approx 80 = \text{Mol. Weight of Br}$$

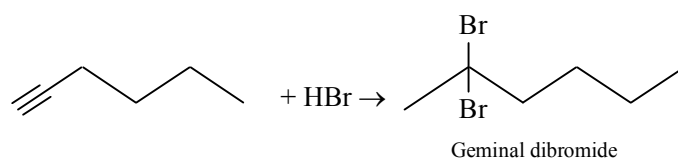
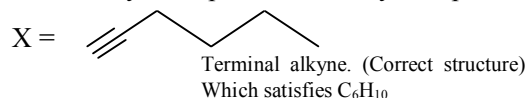
75. An organic compound X with molecular formula C_6H_{10} , when treated with HBr, forms a gem dibromide. The compound X upon warming with $HgSO_4$ and dil. H_2SO_4 , produces a ketone which gives a positive iodoform test. The compound X is



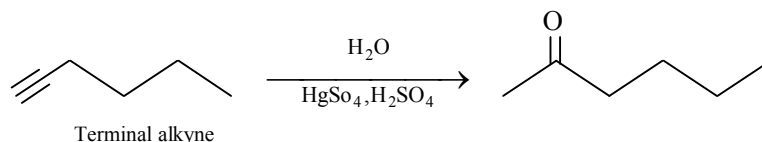
Ans. [D]

Sol. Given Molecular formula = C_6H_{10} i.e. C_nH_{2n-2}
Which simply represents the alkyne.

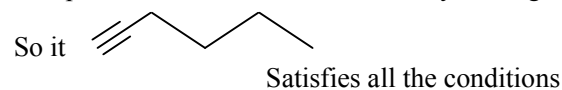
And in only one option i.e. 'D' alkyne is present.



Also, terminal alkyne $\xrightarrow[H_2O]{HgSO_4, H_2SO_4}$ Ketone.



Also positive Iodoform test is shown by those group. Which contain $CH_3-C(=O)-$ group in their structure.



BIOLOGY

76. A cell weighing 1 mg grows to double its initial mass before dividing into two daughter cells of equal mass. Assuming no death, at the end of 100 divisions what will be the ratio of the mass of the entire population of these cells to that of the mass of the Earth? Assume that mass of the Earth is 10^{24} kg and 2^{10} is approximately equal to 1000.

(A) 10^{-28} (B) 10^{-3} (C) 1 (D) 10^3

Ans. [C]

Sol. After 1st division No. of cells = 2
After 2nd division No. of cells = 4
After 3rd division No. of cells = 8
After 100th division No. of cells = $2 \cdot 2^{100-1}$
After 100th division No. of cells = 2^{100}
Total Numbers of cells = 2
Mass of one cell = $1 \text{ mg} = 10^{-6} \text{ kg}$.

$$\begin{aligned} \text{Total mass of cell's} &= 2^{100} \times 10^{-6} \text{ kg} && \{\because 2^{10} = 10^3\} \\ &= (10^3)^{10} \times 10^{-6} \text{ kg} \\ &= 10^{24} \text{ kg} \end{aligned}$$

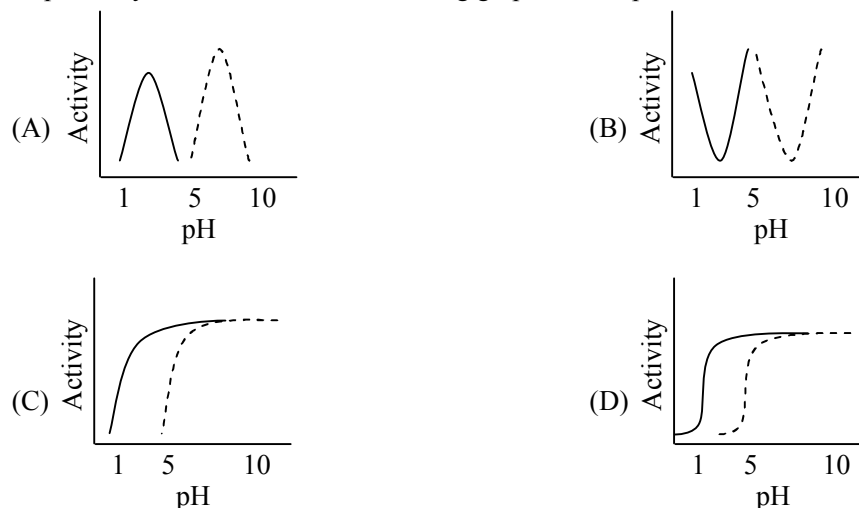
$$\text{Ratio of mass} = \frac{10^{24}}{10^{24}} = 1$$

77. Papaya is a dioecious species with XY sexual genotype for male and XX for female. What will be the genotype of the embryos and endosperm nuclei after double fertilization?
- (A) 50% ovules would have XXX endosperm and XY embryo, while the other 50% would have XXY endosperm and XX embryo
- (B) 100% ovules would have XXX endosperm and XY embryo
- (C) 100% ovules would have XXY endosperm and XY embryo
- (D) 50% ovules would have XXX endosperm and XX embryo, while the other 50% would have XXY endosperm and XY embryo

Ans. [D]

Sol. After double fertilization when male gametes enter in embryo sac and fused with XX of ♀ & XY of ♂ then the genotype of endosperm will be –
XXX and XXY and 50% embryo of XX and 50% embryo of XY.

78. Solid and dotted lines represent the activities of pepsin and salivary amylase enzymes of the digestive tract, respectively. Which one of the following graphs best represents their activity vs pH?



Ans. [A]

Sol. In the graph (A) indicating activity of pepsin at low pH (2.8). It will be highest and similarly activity of salivary amylase will be highest at the pH of 6.8 and in other graphs.
(B) graph is representing minimum activity
(C) and (D) graphs showing constant activity of both enzyme at increasing pH.

79. If the gene pool of the locus X in the human genome is 4, then what would be the highest possible number of genotypes in a large population?
- (A) 6 (B) 8 (C) 10 (D) 16

Ans. [C]

Sol. For locus 'X' total gene = 4
For example → a, b, c and d
Possible genotype

aa	bb	cc	dd
ab	bc	cd	
ac	bd		
ad			

Formula – for given gene alleles

Total number = n

$$\begin{aligned}
 \text{Possible genotype} &= \frac{n}{2}(n + 1) \\
 &= \frac{4}{2}(4 + 1) \\
 &= 2(5) \\
 &= 2 \times 5 \\
 &= 10
 \end{aligned}$$

80. Match the plant hormones in Column I with their primary function in Column II.

Column I

- P. Abscisic acid
Q. Ethylene
R. Cytokinin
S. Gibberellin

Column II

- (i) Promotes disease resistance
(ii) Maintains seed dormancy
(iii) Promotes seed germination
(iv) Promotes fruit ripening
(v) Inhibits leaf senescence

Choose the CORRECT combination.

- (A) P-iii, Q-iv, R-i, S-ii
(C) P-v, Q-iii, R-ii, S-i

- (B) P-ii, Q-iv, R-v, S-iii
(D) P-iv, Q-ii, R-iii, S-v

Ans. [B]

Sol. ABA → Maintain seed dormancy
Ethylene → Promote Fruit ripening
Cytokinin → Inhibit leaf senescence
Gibberellin → Promotes seed germination