# CAREER POINT

# MOCK TEST PAPER

## CENTRAL BOARD OF SENIOR SECONDARY EXAMINATION

## MATHEMATICS

### SOLUTIONS

**Sol.1** $R = \{(3, 12), (3, 15), (4, 4), (4, 12), (5, 15)\}$

$R^{-1} = \{(12, 3), (15, 3), (4, 4), (12, 4), (15, 5)\}$

**Sol.2** $\cos^{-1} \cos \left(2\pi - \frac{\pi}{3}\right)$

$= \cos^{-1} \cos \frac{\pi}{3} = \frac{\pi}{3}$

**Sol.3** $(8, 3) \in R, (3, 2) \in R$, but $(8, 2) \notin R$.

**Sol.4** Let $\Delta = \begin{vmatrix} \cos 15^o & \sin 15^o \\ \sin 75^o & \cos 75^o \end{vmatrix}$

$= \cos 15^o \cos 75^o - \sin 15^o \sin 75^o$

$= \cos (15^o + 75^o) = \cos 90^o = 0.$

**Sol.5** $\sqrt{b^2 + c^2}$

**Sol.6** $|\text{adj } A| = |A|^{3-1} = 25$

$\therefore |A.\text{adj}.A| = 5 \times 25 = 125.$

**Sol.7** We have $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow 2 + y = 5$ and $2x + 2 = 8$

$\Rightarrow y = 3$ and $x = 3$

**Sol.8** Zero, because $\sin^5 x$ is an odd function of x.

**Sol.9** Required position vector

$= \frac{1}{2} (5\hat{i} + 3\hat{j} + 3\hat{i} - \hat{j}) = 4\hat{i} + \hat{j}.$

**Sol.10** $\vec{a} \parallel \vec{b} \Rightarrow \vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow \hat{i}\ (-9 - 3\lambda) + \hat{k}\ (2\lambda + 6) = \vec{0}$

$\Rightarrow \lambda = -3.$

**Sol.11** Let A be the event of drawing a diamond card in the first draw and B be the event of drawing card in the second draw. Then

$$P(A) = \frac{^{13}C_1}{^{52}C_1} = \frac{1}{4}$$

After drawing a diamond card in first card 51 cards are left out of which 12 cards are diamond cards.

$P\left(\frac{B}{A}\right)$ = Prob. of drawing a diamond card in II[nd] draw when a diamond card has already been drawn in I[st] draw

$$= \frac{^{12}C_1}{^{51}C_1} = \frac{12}{51} = \frac{4}{17}$$

Required Prob. $= P(A \cap B) = P(A) \,.\, P\left(\frac{B}{A}\right)$

$$= \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

**Sol.12** L.H.S. $\tan^{-1}\left(\dfrac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}\right) = \tan^{-1}\left(\frac{17}{34}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\left(2\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{1}{2}\cos^{-1}\left[\frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}\right] = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ = R.H.S.

**Sol.13** LHS $= \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & c^2b & c(c^2+1) \end{vmatrix}$

$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$]

$= (1 + a^2 + b^2 + c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$= (1 + a^2 + b^2 + c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$

$= (1 + a^2 + b^2 + c^2).1 = (1 + a^2 + b^2 + c^2)$ = RHS

**Sol.14** LHL $= \lim_{x\to 1^{-1}} f(x) = 3a + b$

RHL $= \lim_{x\to 1^{+}} f(x) = 5a - 2b$, $f(1) = 11$

$\therefore$ $3a + b = 11$ …(i) $5a - 2b = 11$ …(ii)

Solving these equations, we get a = 3, b = 2.

**Sol.15** Put $x^y = u$ and $y^x = v$

$\therefore u + v = a^b \Rightarrow \frac{dy}{dx} + \frac{dv}{dx} = 0$ …(i)

Now, $u = x^y \Rightarrow \log u = y \log x$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} = y.x^{y-1} + x^y.\log x.\frac{dy}{dx}$

And, $u = y^x \Rightarrow \log v = y \log x$

$\Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$

$\Rightarrow \frac{dy}{dx} = xy^{x-1}.\frac{dy}{dx} + y^x.\log y$

$\therefore$ (i) becomes

$y.x^{y-1} + x^y \log x \frac{dy}{dx} + xy^{x-1}\frac{dy}{dx} + y^x \log y = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y.x^{y-1} + y^x.\log y}{x^y.\log x + xy^{x-1}}$

**Sol.16** $f'(x) = 0 \Rightarrow -9 + 12x - 3x^2 = 0$

$\Rightarrow -3(x^2 - 4x + 9) = 0$

$\Rightarrow -(3)(x - 1)(x - 3) = 0$

$\Rightarrow x = 1, x = 3$

$\therefore$ The intervals are $(-\infty, 1), (1, 3), (3, \infty)$

So, f(x) is strictly decreasing in

$(-\infty, 1) \cup (3, \infty)$ and strictly increasing in (1, 3).

**Sol.17** $I = \int_{-1}^{\frac{1}{2}} |x\cos(\pi x)|\,dx$

Here three cases arise :

**Case I :** $-1 < x < \frac{-1}{2} \Rightarrow -\pi < \pi x < -\frac{\pi}{2}$

$\Rightarrow \cos \pi x < 0 \Rightarrow x \cos \pi x > 0$

**Case II :** $-\frac{1}{2} < x < 0 \Rightarrow -\frac{\pi}{2} < \pi x < 0$

$\Rightarrow \cos(\pi x) > 0 \Rightarrow x \cos(\pi x) < 0$

**Case III :** $0 < x < \frac{1}{2} \Rightarrow 0 < \pi x < \frac{\pi}{2}$

$\Rightarrow \cos(\pi x) > 0 \Rightarrow x \cos \pi x > 0$

$\therefore I = \int_{-1}^{-\frac{1}{2}} x\cos\pi x dx + \int_{-\frac{1}{2}}^{0} -x\cos\pi x dx + \int_{0}^{\frac{1}{2}} x\cos\pi x dx$

$= \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-1}^{-\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-\frac{1}{2}}^{0}$

$+ \left[\frac{x\sin\pi x}{\pi} + \frac{cso\pi x}{\pi^2}\right]_{0}^{-\frac{1}{2}}$

$$= \left[\left(\frac{1}{2\pi}+0\right)-\left(0-\frac{1}{\pi^2}\right)\right]-\left[-\left(\frac{1}{2\pi}+0\right)+\left(0+\frac{1}{\pi^2}\right)\right] + \left[-\left(0+\frac{1}{\pi^2}\right)+\left(\frac{1}{2\pi}+0\right)\right]$$

$$= \frac{1}{2\pi} + \frac{1}{\pi^2} + \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{1}{2\pi} - \frac{1}{\pi^2}$$

$$= \frac{3}{2\pi} - \frac{1}{\pi^2}.$$

**Sol.18** $ye^{\frac{x}{y}}\,dx = \left(ye^{\frac{x}{y}} + y\right)dy \Rightarrow \frac{dy}{dx} = \frac{xe^{\frac{x}{y}}+y}{y.e^{\frac{x}{y}}}$ …(i)

Let x = vy $\Rightarrow \frac{dx}{dy}$ = v + y. $\frac{dv}{dy}$

$\therefore$ (i) becomes v + y $\frac{dv}{dy} = \frac{vy.e^v + y}{y.e^v}$

$\Rightarrow$ y $\frac{dv}{dy} = \frac{vye^y + y}{y.e^v} - v = \frac{vye^v + y - vye^v}{y.e^v}$

$= \frac{1}{e^v} \Rightarrow e^v dv = \frac{dy}{y}$

Integrating we get, $e^v$ = log y + log c = log cy

Substituting v = $\frac{x}{y}$, we get $e^{\frac{x}{y}}$ = log cy.

**Sol.19** $(1 + y + x^2y)dx + (x + x^3)dy = 0$

$\Rightarrow x(1 + x^2)\,dy = -[1 + y(1+x^2)]dx$

$\Rightarrow \frac{dy}{dx} = \frac{-1-y(1+x^2)}{x(1+x^2)} = \frac{-1}{x}.y - \frac{1}{x(1+x^2)}$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}.y = -\frac{1}{x(1+x^2)}$

It is a linear differential equation

$\therefore$ I.f. = $e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ The solution is given by

$y.x = -\int \frac{1}{x(1+x^2)}.x\,dx = -\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

When x = 1, y = 0

$\therefore 0 = -\tan^{-1}(1) + c \Rightarrow c = \frac{\pi}{4}$.

$\therefore xy = -\tan^{-1} x + \frac{\pi}{4}$.

**Sol.20** $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}\cdot\vec{c} = 0$

$\Rightarrow \vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$

$\therefore \vec{a}$ is $\perp$ to the plane of $\vec{b}$ and $\vec{c}$

$\vec{a}$ is parallel to $\vec{b} \times \vec{c}$

Let $\vec{a} = k(\vec{b} \times \vec{c})$, where k is a scalar.

$\Rightarrow |\vec{a}| = |k|\,|\vec{b}\times\vec{c}| = |k|\,|\vec{b}|\,|\vec{c}|\,\sin\frac{\pi}{6}$

$\Rightarrow 1 = |k|\,\frac{1}{2} \Rightarrow |k| = 2 \Rightarrow k = \pm 2$

$\therefore \vec{a} = \pm 2\,(\vec{b} \times \vec{c})$.

**Sol.21** Equation of plane passing through (0, –1, –1) is

a(x – 0) + b(y + 1) + c(z + 1) = 0 …(i)

(i) passes through (4, 5, 1) and (3, 9, 4)

$\Rightarrow$ 4a + 6b + 2c = 0 or 2a + 3b + c = 0 …(ii)

and 3a + 10b + 5c = 0 …(iii)

From (ii) and (iii), we get

$$\frac{a}{15-10} = \frac{-b}{10-3} = \frac{c}{20-9}$$

$$\Rightarrow \frac{a}{5} = \frac{-b}{7} = \frac{c}{11} = k(\text{say})$$

$\therefore$ a = 5k, b= – 7k, c = 11k …(iv)

Putting these values of a, b, c in (i) we get

5kx – 7k(y + 1) + 11k(z + 1) = 0

$\Rightarrow$ 5x – 7y + 11z + 4 = 0

Putting the point (–4, 4, 4) in (v), we get

–20 – 28 + 44 + 4 = 0, which is satisfied

$\therefore$ The given points are co-planar and equation of plane is 5x – 7y + 11z + 4 = 0.

**Sol.22** For any $(a, b) \in N \times N$ ; $ab = ba$

$\Rightarrow (a, b)\ R\ (a, b)$ Thus $R$ is reflexive

Let $(a, b)\ R\ (c, d)$ for any $a, b, c, d \in N$

$\therefore\ \ ad = bc$

$\Rightarrow cb = da \Rightarrow (c, d)\ R\ (a, b)$

$\therefore$ $R$ is symmetric

Let $(a, b)\ R\ (c,d),d$ and $(c, d)\ R\ (e, f)$ for $a, b, c, d, e, f \in N$

Then $ad = bc$ and $cf = de$

$\Rightarrow\ a\,d\,c\,f = b\,c\,d\,e$ or $af = be \Rightarrow (a, b)\ R\ (e, f)$

$\therefore$ $R$ is transitive

So $R$ is an equivalence Relation

**Sol.23** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore\ P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.6 - 0.8 = 0.3$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.3}{0.6} = \frac{1}{2}$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{0.3}{0.5} = \frac{3}{5}$$

**Sol.24** Let, number of executive class tickets to be sold, be $x$ and that of economy class be $y$.

$\therefore$ LPP becomes : Maximize Profit ($P$)

$= 1000\,x + 600y$

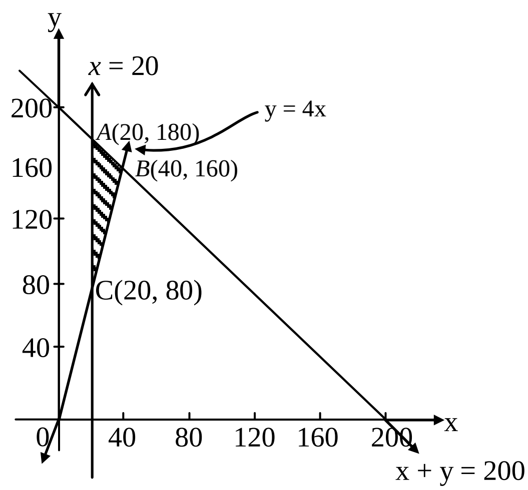
Subject to : $x \geq 0, y \geq 0$

$x + y \leq 200$

$y \geq 4x$ or $4x - y \leq 0$

$x \geq 20$

For correct graph




Getting vertices of feasible region as

$A(20, 180)$, $B(40, 160)$, $C(20, 80)$

Profit at $A$ = Rs 128000

Profit at $B$ = Rs 136000

Profit at $C$ = Rs 68000

$\therefore$ Max profit = Rs. 136000 for 40 executive and 160 economy tickets

**Sol.25** $|A| = 2(-1) - 1(4) + 3(1) = -3 \neq 0$

$\Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|}\ \text{adj}A$$

The cofactors are

$A_{11} = -1,\ A_{12} = -4,\ A_{13} = 1$

$A_{21} = 5,\ A_{22} = 23,\ A_{23} = -11$

$A_{31} = 3,\ A_{32} = 12,\ A_{33} = -6$

$$\text{adj } A = \begin{pmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{pmatrix}^T = \begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\ \text{adj } A = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}$$

Given equations can be written as

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{ or } A.X = B$$

$\Rightarrow X = A^{-1}.B$

$$\therefore\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ -27 \\ 14 \end{pmatrix}$$

$\therefore$ x = –6,y = –27, z = 14.

**Sol.26** In the figure, AD = DC = BC = 10 cm

DM ⊥ AB and CN ⊥ AB.

∵ ΔADM ≅ ΔBCN ⇒ AM = BN = x (say)

∴ DM = $\sqrt{10^2 - x^2}$

Area of trapezium (A) = $\frac{1}{2}$ (AB + DC) × DM

= $\frac{1}{2}$ (10 + 10 + 2x) $\sqrt{100 - x^2}$

= (10 + x) $\sqrt{100 - x^2}$

Let S = $A^2 = (10 + x)^2 (100 - x^2)$

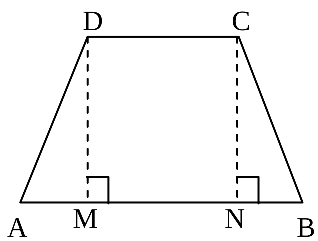
$\frac{dS}{dx} = 0$

⇒ $(10 + x)^2 (-2x) + (100 - x^2)\, 2(10 + x) = 0$

⇒ $(10 + x)^2 (-2x + 20 - 2x) = 0$ ⇒ x = 5

$\frac{d^2S}{dx^2} = (10 + x)^2 (-4) + (20 - 4x)\, 2\, (10 + x) < 0$

at x= 5




∴ S is maximum when x = 5.

**Sol.27** I = $\int \frac{I}{\sin x(5 - 4\cos x)}$ dx

= $\int \frac{\sin x}{\sin^2 x(5 - 4\cos x)}$ dx

= $\int \frac{\sin x}{(1 - \cos^2 x)(5 - 4\cos x)}$ dx

= $-\int \frac{dt}{(1 - t^2)(5 - 4t)}$, where cos x = t

⇒ dt = – sin x dx

= $-\int \frac{dt}{(1 - t)(1 + t)(5 - 4t)}$

Let $\frac{1}{(1 - t)(1 + t)(5 - 4t)} = \frac{A}{1 - t} + \frac{B}{1 + t} + \frac{C}{5 - 4t}$

⇒ $1 = A(1 + t)(5 - 4t) + B(1 - t)(5 - 4t) + C(1 - t^2)$ ….(i)

Putting t = 1 in (i) we get A = $\frac{1}{2}$

Putting t = – 1 in (i) we get B = $\frac{1}{18}$

Putting t = $\frac{5}{4}$ in (i) we get C = $-\frac{16}{9}$

∴ $\frac{1}{(1 - t)(1 + t)(5 - 4t)} = \frac{1}{2(1 - t)} + \frac{1}{18(1 + t)} - \frac{16}{9(5 - 4t)}$

∴ I = $-\left[\frac{1}{2}\int \frac{dt}{1 - t} + \frac{1}{18}\int \frac{dt}{1 + t} - \frac{16}{9}\int \frac{dt}{5 - 4t}\right]$

$-\left[-\frac{1}{2}\log|1 - t| + \frac{1}{18}\log|1 + t| + \frac{17}{9 \times 4}\log|5 - 4t|\right] + c$

= $\frac{1}{2}$ log |1 – cosx | $-\frac{1}{18}$ log | 1 + cos x |

$-\frac{4}{9}$ log | 5 – 4 cos x | + c

**Sol.28** Equations of the curves are

$x^2 + y^2 = 5$ and y = $\begin{Bmatrix} 1 - x, & x < 1 \\ x - 1, & x > 1 \end{Bmatrix}$ …(i)

$x^2 + y^2 = 5$ is a circle with centre (0, 0) and radius $\sqrt{5}$. These two curves intersect at C(2, 1) and D(–1, 2)

Required area = area of (EABCDE)

– area of (ADEA) – area of (ABCA)

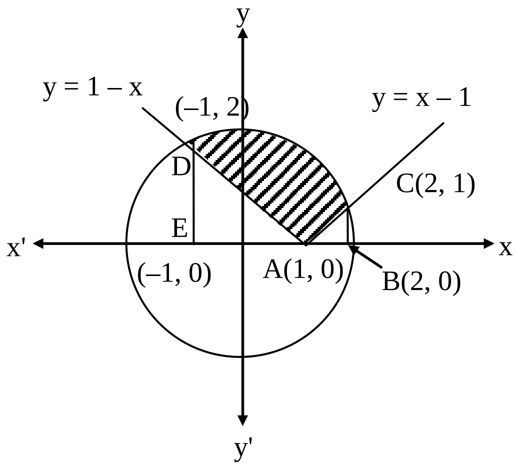
= $\int_{-1}^{2} \sqrt{5 - x^2}\, dx - \int_{-1}^{1} (1 - x)dx - \int_{1}^{2} (x - 1)\, dx$

= $\left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1}$

$-\left[\frac{x^2}{2} - x\right]_{1}^{2}$

$$= \left[\left\{1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right\}-\left\{\frac{1}{2}\times 2+\frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right\}\right]$$

$$-\left[\left(1-\frac{1}{2}\right)-\left(-1-\frac{1}{2}\right)\right] - \left[(2-2)-\left(\frac{1}{2}-1\right)\right]$$




$$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} - \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]$$

**Sol.29** Lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and

$\frac{x-x_2}{l_2} = \frac{y-y_2}{l_2} = \frac{z-z_2}{n_2}$ are

Coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

In this case $\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

= – 2(5 – 10) + 1 (–15 + 5) + 0 – 10 – 10 = 0

$\therefore$ lines are coplanar

Equation of the required plane is

$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$

$\Rightarrow$ 5x – 10 y + 5z = 0

$\Rightarrow$ x – 2y + z = 0.